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SOME FIXED POINT THEOREMS IN SYMMETRIC G-CONE METRIC SPACES

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Abstract. In this paper we obtain a unique common fixed point theorem for two pairs of weakly compatible mappings in symmetric G-cone metric spaces.

1 Introduction and Preliminaries

Mustafa and Sims [18] and Naidu et. al [21, 22, 23] demonstrated that most of the claims concerning the fundamental topological structure of D-metric introduced by Dhage [6, 7, 8, 9] and hence all theorems are incorrect. As an alternative Mustafa and Sims [19] introduced the concept of G-metric space (a generalized metric space).

In recent years many authors have obtained different fixed point and common fixed point theorems for mappings satisfing various contractive conditions on G-metric spaces. For a survey of fixed point theory, its applications, comparision of different contractive conditions and related topics in G-metric spaces we refer the reader to [3, 2, 24, 1, 5, 26, 13-20] and the references therein.

Based on cone metric spaces introduced by [10] and on G-metric spaces introduced by [19], I. Beg et. al [4] introduced generalized cone metric spaces as follows:

Let E be a real Banach space and P be a subset of E. Then P is called a cone if it has the following properties:
(i)P is non empty, closed and $P \neq \{0\}$;
(ii) $0 \leqslant a, b \in R$ and $x, y \in P \Rightarrow ax + by \in P$;
(iii) $P \bigcap (-P) = \{0\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leqslant$ on E with respect to P by $x \leqslant y$ if and only if $y - x \in P$. We will write $x < y$ if $x \leqslant y$ and $x \neq y$, while $x << y$ will stands for $y - x \in P^o$, where P^o denotes the interior of P.

Proposition 1.1. [12]. *Let P be a cone in a real Banach space E. If $a \in P$ and $a \leqslant \lambda a$ for some $\lambda \in [0, 1)$, then $a = 0$.*

Proposition 1.2. $[25, Corollary\ 1.4]$ *Let P be a cone in a real Banach space E.*
(i) If $a \leqslant b$ and $b << c$, then $a << c$.
(ii) If $a \in E$ and $a << c$ for all $c \in P^o$, then $a = 0$.

Remark 1. [25] $\lambda\ P^o \subseteq P^o$ *for* $\lambda > 0$ *and* $P^o + P^o \subseteq P^o$.

Definition 1. [4] *Let* X *be a nonempty set and let* $G : X \times X \times X \to E$ *be a function satisfying the following properties :*
(G_1): $G(x,y,z) = 0$ *if* $x = y = z$,
(G_2): $0 < G(x,x,y)$ *for all* $x,y \in X$ *with* $x \neq y$,
(G_3): $G(x,x,y) \leqslant G(x,y,z)$ *for all* $x,y,z \in X$ *with* $y \neq z$,
(G_4): $G(x,y,z) = G(x,z,y) = G(y,z,x) = ...$*(symmetry in three variables),*
(G_5): $G(x,y,z) \leqslant G(x,a,a) + G(a,y,z)$ *for all* $x,y,z,a \in X$.
Then the function G *is called a generalized cone metric on* X *and* X *is called a generalized cone metric space or a* G*- cone metric space.*

It is clear that if $G(x,y,z) = 0$ then $x = y = z$ for any $x,y,z \in X$.

Definition 2. [4] *A* G*-cone metric space* X *is called symmetric if* $G(x,x,y) = G(x,y,y)$ *for all* $x,y \in X$.

Definition 3. [4] *Let* X *be a* G*-cone metric space and* $\{x_n\}$ *be a sequence in* X. *The sequence* $\{x_n\}$ *is said to converge to a point* $x \in X$ *if for every* $c \in E$ *with* $0 << c$ *there is* N *such that* $G(x_n, x_m, x) << c$ *for all* $n,m > N$. *In this case, we write* $x_n \to x$ *as* $n \to \infty$.

The sequence $\{x_n\}$ *is said to be a* G*-Cauchy sequence in* X *if for every* $c \in E$ *with* $0 << c$ *there is* N *such that* $G(x_n, x_m, x_l) << c$ *for all* $n,m,l > N$.

X *is said to be complete if every* G*-Cauchy sequence in* X *is convergent in* X.

Proposition 1.3. [4, *Lemma* 2.8] *Let* X *be a* G*-cone metric space. Then for a sequence* $\{x_n\} \subseteq X$ *and a point* $x \in X$, *the following statements are equivalent:*
(i) $\{x_n\}$ *is* G*-convergent to* x,
(ii) $G(x_n, x_n, x) \to 0$ *as* $n \to \infty$,
(iii) $G(x_n, x, x) \to 0$ *as* $n \to \infty$,
(iv) $G(x_m, x_n, x) \to 0$ *as* $m,n \to \infty$.

Proposition 1.4. [4, *Lemma* 2.9] *Let* X *be a* G*-cone metric space. Then the function* $G(x,y,z)$ *is jointly continuous in all three of its variables.*

Remark 2. [12] *If* $c \in P^o, 0 \leqslant a_n$ *and* $a_n \to 0$, *then there exists* $n_0 \in \mathbb{N}$ *such that for all* $n > n_0$ *we have* $a_n << c$.

Ismat Beg et. al [4] proved the following

Theorem 1.1. [4, *Theorem* 3.1] *Let* X *be a complete symmetric* G*-cone metric space and* $T : X \to X$ *be a mapping satisfying one of the following conditions*

$$G(Tx,Ty,Tz) \leqslant aG(x,y,z) + bG(x,Tx,Tx) + cG(y,Ty,Ty) + dG(z,Tz,Tz)$$

or

$$G(Tx,Ty,Tz) \leqslant aG(x,y,z) + bG(x,x,Tx) + cG(y,y,Ty) + dG(z,z,Tz)$$

for all $x,y,z \in X$, *where* $0 \leqslant a + b + c + d < 1$.

Then T *has a unique fixed point in* X.

Now, we state the lemma for G-cone metric spaces which is similar to the one for cone metric spaces proved by Jain et. al [11].

Lemma 1.1. *Let X be a G-cone metric space, P be a cone in a real Banach space E, let $k_1, k_2, k_3, k_4 \geqslant 0$ be such that $k_1 + k_2 + k_3 + k_4 > 0$, and let $k > 0$. If $x_n \to x, y_n \to y, z_n \to z$ and $p_n \to p$ in X and*

$$ka \leqslant k_1 G(x_n, x_m, x) + k_2 G(y_n, y_m, y) + k_3 G(z_n, z_m, z) + k_4 G(p_n, p_m, p), \tag{1.1}$$

then $a = 0$.

Proof. Since $x_n \to x, y_n \to y, z_n \to z$ and $p_n \to p$, we have for $c \in P^o$, there exists a positive integer N_c such that

$$\tfrac{c}{k_1+k_2+k_3+k_4} - G(x_n, x_m, x), \tfrac{c}{k_1+k_2+k_3+k_4} - G(y_n, y_m, y),$$

$$\tfrac{c}{k_1+k_2+k_3+k_4} - G(z_n, z_m, z), \tfrac{c}{k_1+k_2+k_3+k_4} - G(p_n, p_m, p) \in P^o \ \ \forall \ \ n > N_c.$$

By Remark 1.3, we have

$$\tfrac{k_1 c}{k_1+k_2+k_3+k_4} - k_1 G(x_n, x_m, x), \tfrac{k_2 c}{k_1+k_2+k_3+k_4} - k_2 G(y_n, y_m, y),$$

$$\tfrac{k_3 c}{k_1+k_2+k_3+k_4} - k_3 G(z_n, z_m, z), \tfrac{k_4 c}{k_1+k_2+k_3+k_4} - k_4 G(p_n, p_m, p) \in P^o \ \ \forall \ \ n > N_c.$$

Adding these four elements, by Remark 1.3, we have

$$c - [k_1 G(x_n, x_m, x) + k_2 G(y_n, y_m, y) + k_3 G(z_n, z_m, z) + k_4 G(p_n, p_m, p)] \in P^o \ \ \forall \ \ n > N_c.$$

Now by (1.1) and Proposition 1.2(i), we have $ka << c$ for all $c \in P^o$. By Proposition 1.2(ii), we have $a = 0$ since $k > 0$. □

2 The main result

Theorem 2.1. *Let (X, G) be a symmetric G-cone metric space, P be a cone and $S, T, f, g : X \to X$ be mappings satisfying*
(2.1.1) *$S(X) \subseteq g(X)$ and $T(X) \subseteq f(X)$,*
(2.1.2) *one of $f(X)$ and $g(X)$ is a G-complete subspace of X,*
(2.1.3) *the pairs (S, f) and (T, g) are weakly compatible,*
(2.1.4) *$G(Sx, Ty, z) \leqslant q \ M(x, y, z)$ for all $x, y, z \in X$ with $z = Sx$ or Ty, where $0 \leqslant q < 1$ and*

$$M(x,y,z) \in \left\{ \begin{array}{c} G(fx, gy, z), G(fx, Sx, z), G(gy, Ty, z), \\ \frac{1}{2}[G(fx, Ty, z) + G(gy, Sx, z)], \\ G(fx, Sx, Sx), G(gy, Ty, Ty), G(fx, gy, gy) \end{array} \right\}.$$

Then the maps S, T, f and g have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be an arbitrary point, then by (2.1.1) there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$y_{2n} = Sx_{2n} = gx_{2n+1}$, $y_{2n+1} = Tx_{2n+1} = fx_{2n+2}$ for $n = 0, 1, 2, ...$

Case (i): Suppose that $y_{2n+1} = y_{2n+2}$ for some n. Then $fu = Su$, where $u = x_{2n+2}$. Since $S(X) \subseteq g(X)$, there exists $v \in X$ such that $Su = gv$. Suppose $Su \neq Tv$. Note that

$$G(Su, Tv, Su) \leqslant q\ M(u, v, u),$$

where

$$\begin{aligned} M(u,v,u) &\in \left\{ \begin{array}{c} G(fu, gv, Su), G(fu, Su, Su), G(gv, Tv, Su), \\ \frac{1}{2}[G(fu, Tv, Su) + G(gv, Su, Su)], \\ G(fu, Su, Su), G(gv, Tv, Tv), G(fu, gv, gv) \end{array} \right\} \\ &= \left\{ \begin{array}{c} 0, 0, G(Su, Tv, Su), \frac{1}{2}G(Su, Tv, Su), \\ 0, G(Su, Tv, Tv), 0 \end{array} \right\} \\ &= G(Su, Tv, Su), \end{aligned}$$

since (X,G) is symmetric. Thus

$$G(Su, Tv, Su) \leqslant q\ G(Su, Tv, Su).$$

By Proposition 1.1 , we have $G(Su, Tv, Su) = 0$, so that $Su = Tv$. Thus

$$fu = Su = Tv = gv = p, \tag{2.1}$$

for some $p \in X$.

Since the pairs (S, f) and (T, g) are weakly compatible , we have

$$fp = Sp \ \text{ and } \ Tp = gp. \tag{2.2}$$

Moreover,

$$G(Sp, p, p) = G(Sp, Tv, Tv) \leqslant q\ M(p, u, v),$$

where

$$\begin{aligned} M(p,u,v) &\in \left\{ \begin{array}{c} G(fp, gv, Tv), G(fp, Sp, Tv), G(gv, Tv, Tv), \\ \frac{1}{2}[G(fp, Tv, Tv) + G(gv, Sp, Tv)], \\ G(fp, Sp, Sp), G(gv, Tv, Tv), G(fp, gv, gv) \end{array} \right\} \\ &= \left\{ \begin{array}{c} G(Sp, p, p), G(Sp, Sp, p), 0, \\ \frac{1}{2}[G(Sp, p, p) + G(p, Sp, p)], 0, 0, G(Sp, p, p) \end{array} \right\} \\ &= G(p, p, Sp), \end{aligned}$$

since (X, G) is symmetric. Thus

$$G(Sp, p, p) \leqslant qG(p, p, Sp).$$

By Proposition 1.1, we have $G(p, p, Sp) = 0$, so that $Sp = p$. Hence

$$fp = Sp = p. \tag{2.3}$$

Next

$$G(p,p,Tp) = G(Sp,Sp,Tp) = G(Sp,Tp,Sp) \leqslant q\ M(p,p,p),$$

where

$$\begin{aligned} M(p,p,p) &\in \left\{ \begin{array}{c} G(fp,gp,Sp), G(fp,Sp,Sp), G(gp,Tp,Sp), \\ \frac{1}{2}[G(fp,Tp,Sp)+G(gp,Sp,Sp)], \\ G(fp,Sp,Sp), G(gp,Tp,Tp), G(fp,gp,gp) \end{array} \right\} \\ &= \left\{ \begin{array}{c} G(p,Tp,p), 0, G(Tp,Tp,p), \\ \frac{1}{2}[G(p,Tp,p)+G(Tp,p,p)], 0, 0, G(Tp,Tp,p) \end{array} \right\} \\ &= G(p,p,Tp), \end{aligned}$$

since (X,G) is symmetric. Thus

$$G(p,p,Tp) \leqslant qG(p,p,Tp).$$

By Proposition 1.1 , we have $G(p,p,Tp) = 0$, so that $Tp = p$. Hence

$$gp = Tp = p. \tag{2.4}$$

By (2.3) and (2.4), p is a common fixed point of f, g, S and T.

Suppose $p\,'$ is another common fixed point of f, g, S and T. Note that

$$G(p,p\,',p) = G(Sp,Tp\,',Sp) \leqslant q\ M(p,p\,',p),$$

where

$$\begin{aligned} M(p,p\,',p) &\in \left\{ \begin{array}{c} G(p,p\,',p), 0, G(p\,',p\,',p), \\ \frac{1}{2}[G(p,p\,',p)+G(p\,',p,p)], \\ 0, 0, G(p,p\,',p\,') \end{array} \right\} \\ &= G(p,p\,',p), \end{aligned}$$

since (X,G) is symmetric. Thus

$$G(p,p\,',p) \leqslant qG(p,p\,',p).$$

By Proposition 1.1, we have $G(p,p\,',p) = 0$, so that $p\,' = p$. Thus p is the unique common fixed point of f, g, S and T.

Case (ii): Suppose that $y_{2n} = y_{2n+1}$ for some n. Then $gv = Tv$, where $v = x_{2n+1}$. The rest of the proof follows in the similar lines as in Case (i).

Case (iii): Suppose that $y_n \neq y_{n+1}$ for all n. Denote $d_n = G(y_n, y_{n+1}, y_{n+1})$. Then

$$d_{2n} = G(y_{2n}, y_{2n+1}, y_{2n+1}) = G(y_{2n}, y_{2n+1}, y_{2n}),\ \textit{since } (X,G) \textit{ is symmetric.}$$

$$d_{2n} = G(Sx_{2n}, Tx_{2n+1}, Sx_{2n}) \leqslant q\ M(x_{2n}, x_{2n+1}, x_{2n}),$$

where

$$M(x_{2n}, x_{2n+1}, x_{2n}) \in \left\{ \begin{array}{c} G(y_{2n-1}, y_{2n}, y_{2n}), G(y_{2n-1}, y_{2n}, y_{2n}), G(y_{2n}, y_{2n+1}, y_{2n}), \\ \frac{1}{2}[G(y_{2n-1}, y_{2n+1}, y_{2n}) + G(y_{2n}, y_{2n}, y_{2n})], \\ G(y_{2n-1}, y_{2n}, y_{2n}), G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n-1}, y_{2n}, y_{2n}) \end{array} \right\}$$

$$= \left\{\ d_{2n-1}, d_{2n-1}, d_{2n}, \tfrac{1}{2}[d_{2n-1} + d_{2n} + 0], d_{2n-1}, d_{2n}, d_{2n-1}\ \right\}.$$

Thus $d_{2n} \leqslant \alpha d_{2n-1}$ where $\alpha = \max\left\{q, \frac{\frac{q}{2}}{1-\frac{q}{2}}\right\} < 1$.

Moreover,

$$d_{2n+1} = G(y_{2n+1}, y_{2n+2}, y_{2n+2}) = G(y_{2n+2}, y_{2n+1}, y_{2n+1}).$$

Since (X, G) is symmetric

$$d_{2n+1} = G(Sx_{2n+2}, Tx_{2n+1}, Tx_{2n+1}) \leqslant q\ M(x_{2n+2}, x_{2n+1}, x_{2n+1}),$$

where

$$M(x_{2n+2}, x_{2n+1}, x_{2n+1}) \in A$$

and

$$A = \left\{ \begin{array}{c} G(y_{2n+1}, y_{2n}, y_{2n+1}), G(y_{2n+1}, y_{2n+2}, y_{2n+1}), G(y_{2n}, y_{2n+1}, y_{2n+1}), \\ \frac{1}{2}[G(y_{2n+1}, y_{2n+1}, y_{2n+1}) + G(y_{2n}, y_{2n+2}, y_{2n+1})], \\ G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n+1}, y_{2n}, y_{2n}) \end{array} \right\}$$

$$= \left\{\ d_{2n}, d_{2n+1}, d_{2n}, \tfrac{1}{2}[0 + d_{2n} + d_{2n+1}], d_{2n+1}, d_{2n}, d_{2n}\ \right\}.$$

Thus $d_{2n+1} \leqslant \alpha d_{2n}$. Hence

$$d_n \leqslant \alpha\ d_{n-1} \leqslant \alpha^2\ d_{n-2} \cdots \leqslant \alpha^n d_0 = \alpha^n\ G(y_0, y_1, y_1).$$

Now for $m > n$

$$\begin{aligned} G(y_n, y_n, y_m) &\leqslant G(y_n, y_n, y_{n+1}) + G(y_{n+1}, y_{n+1}, y_{n+2}) + ... + G(y_{m-1}, y_{m-1}, y_m) \\ &\leqslant \alpha^n\ G(y_0, y_1, y_1) + \alpha^{n+1}\ G(y_0, y_1, y_1) + ... + \alpha^{m-1}\ G(y_0, y_1, y_1) \\ &\leqslant \tfrac{\alpha^n}{1-\alpha}\ G(y_0, y_1, y_1) \to 0 \text{ as } n \to \infty. \end{aligned}$$

By Remark 2, it follows that for $0 << c$ and large n, we have $\frac{\alpha^n}{1-\alpha}\ G(y_0, y_1, y_1) << c$. Now, from Proposition 1.2(i), we have $G(y_n, y_n, y_m) << c$ for $m > n$. Hence $\{y_n\}$ is a G-Cauchy sequence.

Suppose $f(X)$ is G-complete. Then there exist $p, t \in X$ such that $y_{2n+1} \to p = ft$. Since $\{y_n\}$ is G-Cauchy, it follows that $y_{2n} \to p$ as $n \to \infty$.

Next

$$\begin{aligned} G(p, p, St) &= G(p, St, St) \\ &\leqslant G(p, Tx_{2n+1}, Tx_{2n+1}) + G(Tx_{2n+1}, St, St) \\ &= G(p, p, Tx_{2n+1}) + G(St, Tx_{2n+1}, St) \\ &= G(p, p, y_{2n+1}) + G(St, Tx_{2n+1}, St) \\ &\leqslant G(p, p, y_{2n+1}) + q\ M(t, x_{2n+1}, t), \end{aligned}$$

where

$$M(t, x_{2n+1}, t) \in B$$

and

$$B = \left\{ \begin{array}{c} G(p, y_{2n}, St), G(p, St, St), G(y_{2n}, y_{2n+1}, St), \\ \frac{1}{2}[G(p, y_{2n+1}, St) + G(y_{2n}, St, St)], \\ G(p, St, St), G(y_{2n}, y_{2n+1}, y_{2n+1}), G(p, y_{2n}, y_{2n}) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} G(p, y_{2n}, p) + G(p, St, St), G(p, St, St), \\ G(y_{2n}, y_{2n+1}, p) + G(p, St, St), \\ \frac{1}{2}[G(p, y_{2n+1}, p) + G(p, St, St) + G(y_{2n}, y_{2n}, p) + G(p, St, St)], \\ G(p, St, St), G(y_{2n}, y_{2n+1}, p) + G(p, y_{2n+1}, y_{2n+1}), \\ G(p, y_{2n}, y_{2n}) \end{array} \right\}.$$

Now we have
$(1-q)G(p,p,St) \leqslant G(p,p,y_{2n+1}) + q\ G(p,p,y_{2n})$
or
$(1-q)G(p,p,St) \leqslant G(p,p,y_{2n+1})$
or
$(1-q)G(p,p,St) \leqslant G(p,p,y_{2n+1}) + q\ G(y_{2n},y_{2n+1},p)$
or
$(1-q)G(p,p,St) \leqslant (1+\frac{q}{2})\ G(p,p,y_{2n+1}) + \frac{q}{2}\ G(p,p,y_{2n})$
or
$G(p,p,St) \leqslant (1+q)G(p,p,y_{2n+1})$
or
$G(p,p,St) \leqslant G(p,p,y_{2n+1}) + q\ G(p,y_{2n},y_{2n})$.

By Proposition 1.3 and Lemma 1.1, we have $St = p$. Thus $ft = p = St$. Since the pair (S,f) is weakly compatible, we have $fp = Sp$.

The rest of the proof follows in the similar lines as in Case (i).

Similarly, we can prove the theorem when $g(X)$ is G- complete. □

Corollary 2.1. *Let (X,G) be a symmetric G-cone metric space and $T,f,g : X \to X$ be mappings satisfying*
(2.2.1) $T(X) \subseteq f(X)$ *and* $T(X) \subseteq g(X)$,
(2.2.2) $f(X)$ *or* $g(X)$ *is G-complete,*
(2.2.3) *the pairs* (T,f) *and* (T,g) *are weakly compatible,*
(2.2.4) $G(Tx,Ty,Ty) \leqslant q\ M(x,y,y)$, *where* $0 \leqslant q < 1$ *and*

$$M(x,y,y) \in \left\{ \begin{array}{c} G(fx,gy,Ty), G(fx,Tx,Ty), G(gy,Ty,Ty), G(fx,Tx,Tx), \\ G(fx,gy,gy), \frac{1}{2}[G(fx,Ty,Ty) + G(gy,Tx,Ty)] \end{array} \right\}$$

for all $x,y \in X$.

Then the maps T,f and g have a unique common fixed point.

Corollary 2.2. . *Let (X,G) be a symmetric G-cone metric space and $T,f : X \to X$ be mappings satisfying*
(2.3.1) $T(X) \subseteq f(X)$,
(2.3.2) $f(X)$ *is G-complete,*
(2.3.3) *the pair* (T,f) *is weakly compatible,*
(2.3.4) $G(Tx,Ty,Ty) \leqslant q\ M(x,y,y)$, *where* $0 \leqslant q < 1$ *and*

$$M(x,y,y) \in \left\{ \begin{array}{c} G(fx,fy,Ty), G(fx,Tx,Ty), G(fy,Ty,Ty), G(fx,Tx,Tx), \\ G(fx,fy,fy), \frac{1}{2}[G(fx,Ty,Ty) + G(fy,Tx,Ty)] \end{array} \right\}$$

for all $x,y \in X$.

Then the maps T and f have a unique common fixed point.

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