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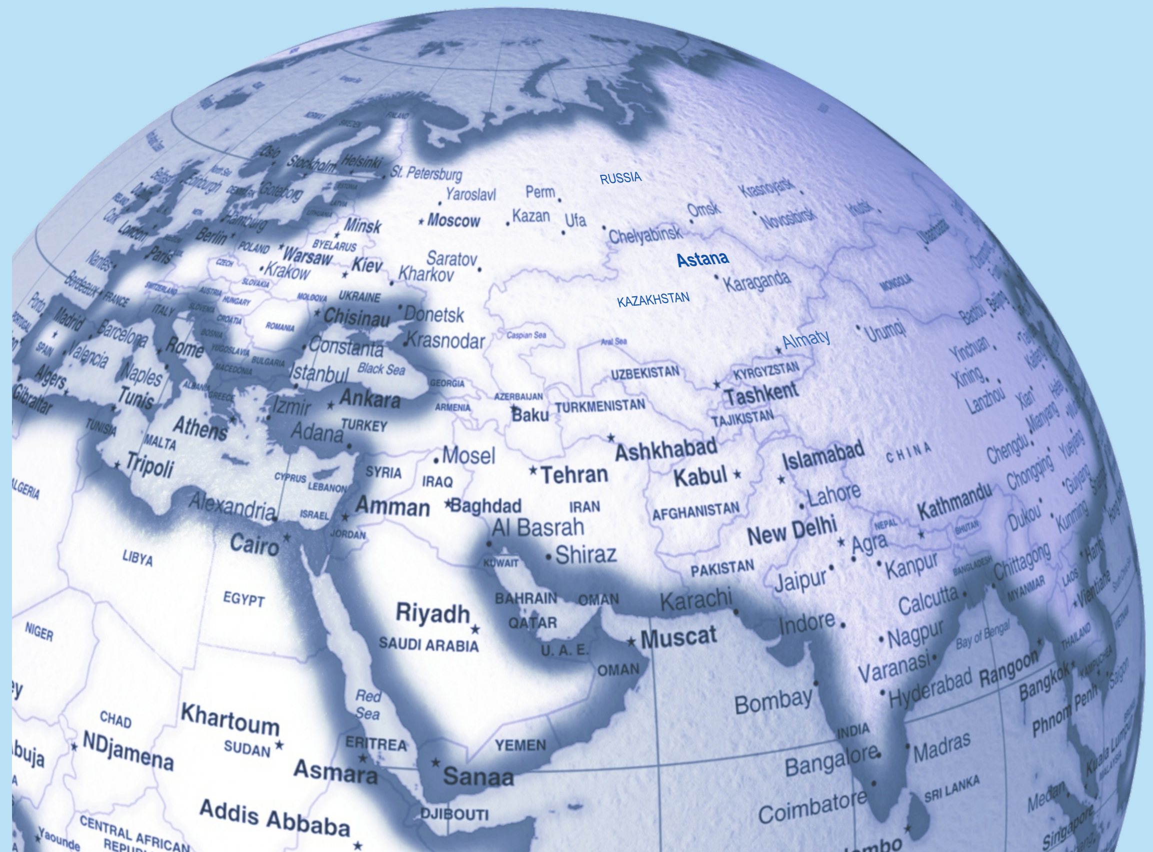

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LOCAL AND 2-LOCAL $\frac{1}{2}$-DERIVATIONS OF SOLVABLE LEIBNIZ ALGEBRAS

U. Mamadaliyev, A. Sattarov, B. Yusupov

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Key words: Leibniz algebras, solvable algebras, nilpotent algebras, $\frac{1}{2}$-derivation, local $\frac{1}{2}$-derivation, 2-local $\frac{1}{2}$-derivation.

AMS Mathematics Subject Classification: 17A32, 17B30, 17B10.

Abstract. We show that any local $\frac{1}{2}$-derivation on solvable Leibniz algebras with model or abelian nilradicals, whose dimensions of complementary spaces are maximal, is a $\frac{1}{2}$-derivation. We show that solvable Leibniz algebras with abelian nilradicals, which have 1-dimensional complementary spaces are $\frac{1}{2}$-derivations. Moreover, a similar problem concerning 2-local $\frac{1}{2}$-derivations of such algebras is investigated.

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1 Introduction

In recent years non-associative analogues of classical constructions have become of interest in connection with their applications in many branches of mathematics and physics. The notions of local and 2-local derivations have also become popular for some non-associative algebras such as Lie and Leibniz algebras.

The notions of local derivations were introduced in 1990 by R.V. Kadison [17] and D.R. Larson, A.R. Sourour [18]. Later in 1997, P. Šemrl introduced the notions of 2-local derivations and 2-local automorphisms on algebras [16]. The main problems concerning these notions are to find conditions under which all local (2-local) derivations become (global) derivations and to present examples of algebras with local (2-local) derivations that are not derivations.

Investigation of local derivations on Lie algebras was initiated in papers [7] and [14]. Sh.A. Ayupov and K.K. Kudaybergenov have proved that every local derivation on a semi-simple Lie algebra is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. In [8] local derivations and automorphisms of complex finite-dimensional simple Leibniz algebras are investigated. They proved that all local derivations on finite-dimensional complex simple Leibniz algebras are automatically derivations and it is shown that filiform Leibniz algebras admit local derivations which are not derivations.

Several papers have been devoted to similar notions and corresponding problems for 2-local derivations and automorphisms of finite-dimensional Lie and Leibniz algebras [5, 8, 9, 14]. Namely, in [9] it is proved that every 2-local derivation on a semi-simple Lie algebra is a derivation and that each finite-dimensional nilpotent Lie algebra, with dimension larger than two admits a 2-local derivation which is not a derivation. Concerning 2-local automorphisms, Z. Chen and D. Wang in [14] proved that if $\mathcal{L}$ is a simple Lie algebra of type A_l, D_l or $E_k, (k = 6, 7, 8)$ over an algebraically closed field of characteristic zero, then every 2-local automorphism of $\mathcal{L}$ is an automorphism. Finally, in [5]

Sh.A. Ayupov and K.K. Kudaybergenov generalized this result of [14] and proved that every 2-local automorphism of a finite-dimensional semi-simple Lie algebra over an algebraically closed field of characteristic zero is an automorphism. Moreover, they also showed that every nilpotent Lie algebra of finite dimension greater than two admits a 2-local automorphism which is not an automorphism.

In [3] local derivations of solvable Lie algebras are investigated and it is shown that in the class of solvable Lie algebras there exist algebras which admit local derivations which are not derivations and also algebras for which every local derivation is a derivation. Moreover, it is proved that every local derivation on a finite-dimensional solvable Lie algebra with model nilradical and maximal dimension of complementary space is a derivation. Sh.A. Ayupov, A.Kh. Khudoyberdiyev and B.B. Yusupov proved similar results concerning local derivations on solvable Leibniz algebras in their recent paper [4]. The results of paper [10] show that p-filiform Leibniz algebras as a rule admit local derivations which are not derivations. Similar results concerning local derivations on direct sum null-filiform Leibniz algebras were obtained in [2].

In [13], [21] Sh.A. Ayupov and B.B. Yusupov investigated 2-local derivations on infinite-dimensional Lie algebras over a field of characteristic zero. They proved that all 2-local derivations on a Witt algebra as well as on a positive Witt algebra are (global) derivations, and gave an example of an infinite-dimensional Lie algebra with a 2-local derivation which is not a derivation. In [11] they have proved that every 2-local derivation on a generalized Witt algebra $W_n(\mathbb{F})$ over a vector space $\mathbb{F}^n$ is a derivation. In [15] Y. Chen, K. Zhao and Y. Zhao studied local derivations on generalized Witt algebras. They proved that every local derivation a Witt algebra is a derivation and that every local derivation on a centerless generalized Virasoro algebra of higher rank is a derivation. In [12] Sh.A. Ayupov, K.K. Kudaybergenov and B.B. Yusupov studied local and 2-local derivations of locally finite split simple Lie algebras. They proved that every local and 2-local derivation on a locally finite split simple Lie algebra is a derivation.

In the present paper we study local and 2-local $\frac{1}{2}$-derivations of solvable Leibniz algebras. We show that any local $\frac{1}{2}$-derivation on a solvable Leibniz algebra with model or abelian nilradicals, whose dimension of the complementary space is maximal, is a $\frac{1}{2}$-derivation. Moreover, similar problems concerning 2-local $\frac{1}{2}$-derivations of such algebras are investigated.

2 Preliminaries

In this section we give some necessary definitions and preliminary results.

Definition 1. A vector space with a bilinear bracket $(\mathcal{L}, [\cdot,\cdot])$ is called a Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity

$$\big[x,[y,z]\big] = \big[[x,y],z\big] - \big[[x,z],y\big],$$

holds, or equivalently, $[[x,y],z] = [[x,z],y] + [x,[y,z]]$.

Here, we adopt the right Leibniz identity; since the bracket is not skew-symmetric, there exists the version corresponding to the left Leibniz identity,

$$\big[[x,y],z\big] = \big[x,[y,z]\big] - \big[y,[x,z]\big] .$$

Let $\mathcal{L}$ be a Leibniz algebra. For a Leibniz algebra $\mathcal{L}$ consider the following lower central and derived sequences:

$$\mathcal{L}^1 = \mathcal{L}, \quad \mathcal{L}^{k+1} = [\mathcal{L}^k, \mathcal{L}^1], \quad k \geq 1,$$
$$\mathcal{L}^{[1]} = \mathcal{L}, \quad \mathcal{L}^{[s+1]} = [\mathcal{L}^{[s]}, \mathcal{L}^{[s]}], \quad s \geq 1.$$

Definition 2. A Leibniz algebra $\mathcal{L}$ is called nilpotent (respectively, solvable), if there exists $k \in \mathbb{N}$ ($s \in \mathbb{N}$) such that $\mathcal{L}^k = 0$ (respectively, $\mathcal{L}^{[s]} = 0$).The minimal number k (respectively, s) with such property is said to be the index of nilpotency (respectively, of solvability) of the algebra $\mathcal{L}$.

Note that any Leibniz algebra $\mathcal{L}$ contains a unique maximal solvable (respectively nilpotent) ideal, called the radical (respectively nilradical) of the algebra.

A $\frac{1}{2}$-derivation on a Leibniz algebra $\mathcal{L}$ is a linear map $D : \mathcal{L} \to \mathcal{L}$ which satisfies the Leibniz rule:

$$D([x, y]) = \frac{1}{2}\left([D(x), y] + [x, D(y)]\right) \quad \text{for any} \quad x, y \in \mathcal{L}. \tag{2.1}$$

The set of all $\frac{1}{2}$-derivations of a Leibniz algebra $\mathcal{L}$ is a Lie algebra with respect to the usual matrix commutator and it is denoted by $\frac{1}{2}Der(\mathcal{L})$.

For a finite-dimensional nilpotent Leibniz algebra N and for the matrix of the linear operator ad_x denote by $C(x)$ the descending sequence of its Jordan blocks' dimensions. Consider the lexicographical order on the set $C(N) = \{C(x) \mid x \in N\}$.

Definition 3. The sequence

$$\left(\max_{x \in N \setminus N^2} C(x)\right)$$

is said to be the characteristic sequence of a nilpotent Leibniz algebra N.

Definition 4. A linear operator Δ is called a local $\frac{1}{2}$-derivation, if for any $x \in \mathcal{L}$, there exists a $\frac{1}{2}$-derivation $D_x : \mathcal{L} \to \mathcal{L}$ (depending on x) such that $\Delta(x) = D_x(x)$. The set of all local $\frac{1}{2}$-derivations on $\mathcal{L}$ we denote by $Loc\frac{1}{2}Der(\mathcal{L})$.

Definition 5. A map $\nabla : \mathcal{L} \to \mathcal{L}$ (not necessary linear) is called a 2-local $\frac{1}{2}$-derivation, if for any $x, y \in \mathcal{L}$, there exists a $\frac{1}{2}$-derivation $D_{x,y} \in \frac{1}{2}Der(\mathcal{L})$ such that

$$\nabla(x) = D_{x,y}(x), \quad \nabla(y) = D_{x,y}(y).$$

2.1 Solvable Leibniz algebras with abelian nilradical

Let $\mathbf{a}_n$ be an n-dimensional abelian algebra and let R be a solvable Leibniz algebra with the nilradical $\mathbf{a}_n$. Take a basis $\{f_1, f_2, \dots, f_n, x_1, x_2, \dots x_k\}$ of R, such that $\{f_1, f_2, \dots, f_n\}$ is a basis of $\mathbf{a}_n$. In [1] such solvable algebras in the case of $k = n$ are classified and it is proved that any $2n$-dimensional solvable Leibniz algebra with the nilradical $\mathbf{a}_n$ is isomorphic to the direct sum of two dimensional algebras, i.e., isomorphic to the algebra

$$\mathcal{L}_t : [f_j, x_j] = f_j, \quad [x_j, f_j] = \alpha_j f_j, \quad 1 \le j \le n,$$

where $\alpha_j \in \{-1, 0\}$ and t is the number of zero parameters α_j.

Moreover, in the following theorem a classification of $(n+1)$-dimensional solvable Leibniz algebras with n-dimensional abelian nilradical is given.

Theorem 2.1. *[1] Let R be an $(n+1)$-dimensional solvable Leibniz algebra with n-dimensional abelian nilradical. If R has a basis $\{f_1, f_2, \dots, f_n, x\}$ such that the operator $ad_x|_{\mathbf{a}_n}$ has Jordan block form, then it is isomorphic to one of the following two non-isomorphic algebras:*

$$R_1 : \begin{cases} [f_i, x] = f_i + f_{i+1}, & 1 \le i \le n-1, \\ [f_n, x] = f_n, \end{cases} \qquad R_2 : \begin{cases} [f_i, x] = f_i + f_{i+1}, & 1 \le i \le n-1, \\ [f_n, x] = f_n, \\ [x, f_i] = -f_i - f_{i+1}, & 1 \le i \le n-1, \\ [x, f_n] = -f_n. \end{cases}$$

2.2 Solvable Leibniz algebras with model nilradical

Let N be a nilpotent Leibniz algebra with the characteristic sequence $(m_1, \dots, m_s)$, and with the table of multiplication

$$N_{m_1,\dots,m_s} : [e_i^t, e_1^1] = e_{i+1}^t, \ 1 \le t \le s, \ 1 \le i \le m_t - 1.$$

The algebra $N_{m_1,\dots,m_s}$ is usually said to be a model Leibniz algebra. For solvable Leibniz algebras with nilradical $N_{m_1,\dots,m_s}$ and the complement dimension space equal to s, we will use the notation $R(N_{m_1,\dots,m_s}, s)$.

Theorem 2.2. *[20] A solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$ with nilradical $N_{m_1,\dots,m_s}$, such that $DimR(N_{m_1,\dots,m_s}, s) - DimN_{m_1,\dots,m_s} = s$, is isomorphic to the algebra:*

$$R(N_{m_1,\dots,m_s}, s) : \begin{cases} [e_i^t, e_1^1] = e_{i+1}^t, & 1 \le t \le s, \ 1 \le i \le m_t - 1, \\ [e_i^1, x_1] = ie_i^1, & 1 \le i \le m_1, \\ [e_i^t, x_1] = (i-1)e_i^t, & 2 \le t \le s, \ 2 \le i \le m_t, \\ [e_i^t, x_t] = e_i^t, & 2 \le t \le s, \ 1 \le i \le m_t, \\ [x_1, e_1^1] = -e_1^1, & \end{cases}$$

where $\{x_1, \dots x_s\}$ is a basis of the complementary vector space.

3 $\frac{1}{2}$-derivation of solvable Leibniz algebras

In the following propositions, we present a general form of the $\frac{1}{2}$-derivation of the algebras $R(N_{m_1,\dots,m_s}, s)$, $\mathcal{L}_t$, R_1 and R_2.

Proposition 3.1. *Any $\frac{1}{2}$-derivation D of the algebra $\frac{1}{2}Der(R(N_{m_1,\dots,m_s}, s))$ has the following form:*

$$\begin{aligned} &D(e_i^1) = \alpha_1 e_i^1, && 1 \le i \le m_1, \\ &D(e_i^t) = \tfrac{1}{2^{i-1}}((2^{i-1} - 1)\alpha_1 + \alpha_t)e_i^t, && 2 \le t \le s, \quad 1 \le i \le m_t, \\ &D(x_i) = \alpha_i x_i, \quad 1 \le i \le s. \end{aligned}$$

Proof. Let $\{e_1^1, e_1^2, \dots, e_1^s, x_1, \dots, x_s\}$ be a basis elements of the algebra $R(N_{m_1,\dots,m_s}, s)$.

Let d be a $\frac{1}{2}$-derivation of the algebra $R(N_{m_1,\dots,m_s}, s)$.

We put

$$D(e_1^p) = \sum_{t=1}^{s}\sum_{i=1}^{m_t} \alpha_{t,i}^p e_i^t + \sum_{i=1}^{s} \beta_{1,i}^p x_i, \quad D(x_p) = \sum_{t=1}^{s}\sum_{i=1}^{m_t} \gamma_{t,i}^p e_i^t + \sum_{i=1}^{s} \beta_{2,i}^p x_i, \quad 1 \le p \le s.$$

The following restrictions follow from the equality

$$D([e_1^1, x_1]) = \frac{1}{2}([D(e_1^1), x_1] + [e_1^1, D(x_1)]) :$$

$\alpha_{1,i}^1 = 0, \ \ 3 \le i \le m_1, \quad \gamma_{1,1}^1 = 0, \quad \beta_{2,1}^1 = \alpha_{1,1}^1,$
$\alpha_{t,i}^1 = 0, \ \ 2 \le t \le s, \quad i = 1, 2, \ \ 4 \le i \le m_t.$
$\beta_{1,i}^1 = 0, \ \ 1 \le i \le s.$

Consider the equality

$$D([e_1^p, x_1]) = \frac{1}{2}([D(e_p^1), x_1] + [e_p^1, D(x_1)]), \quad \text{for} \quad 2 \le p \le s.$$

Then we get

$$\begin{cases} \alpha^p_{1,i} = 0, & 1 \le i \le m_1, \\ \alpha^p_{t,i} = 0, & 2 \le t \le s, \qquad 2 \le i \le m_t, \\ \beta^1_{2,p} = 0. \end{cases}$$

Similarly, from the equality

$$0 = D([x_p, x_1]) = \frac{1}{2}([D(x_p), x_1] + [x_p, D(x_1)]),$$

with $1 \le p \le s$ we have

$$\begin{cases} \gamma^p_{t,i} = 0, & 1 \le t \le s, \quad 2 \le i \le m_t, \quad 1 \le p \le s, \\ \gamma^p_{1,1} = 0, & 2 \le p \le s. \end{cases}$$

The equality

$$D([e^1_1, x_p]) = \frac{1}{2}([D(e^1_1), x_p] + [e^1_1, D(x_p)]),$$

for $2 \le p \le s$ which imply

$$\alpha^1_{p,3} = \beta^p_{2,1}, \quad 2 \le p \le s.$$

Consequently,

$$D(e^1_1) = \alpha^1_{1,1}e^1_1 + \alpha^1_{1,2}e^1_2, \quad D(x_p) = \sum_{t=2}^{s} \gamma^p_{t,1}e^t_1 + \sum_{i=2}^{s} \beta^p_{2,i}x_i, \quad 2 \le p \le s.$$

From the equality

$$0 = D([x_p, x_j]) = \frac{1}{2}([D(x_p), x_j] + [x_p, D(x_j)]),$$

for $2 \le p, j \le s$ we obtain the following restrictions:

$$\gamma^p_{j,1} = 0, \quad 1 \le p, j \le s.$$

From the relations

$$D([x_1, e^1_1]) = \frac{1}{2}([D(x_1), e^1_1] + [x_1, D(e^1_1)]), \quad D([e^p_1, x_j]) = \frac{1}{2}([D(e^p_1), x_j] + [e^p_1, D(x_j)]),$$

for $2 \le p, j \le s$, we have

$$\begin{cases} \alpha^1_{1,2} = 0, & \gamma^1_{t,1} = 0, & 2 \le t \le s, \\ \alpha^p_{t,1} = 0, & 2 \le p, t \le s, & p \ne t, \\ \beta^p_{1,j} = 0, & \beta^p_{2,p} = \alpha^p_{p,1}, & 2 \le p \le s, \quad 1 \le j \le s, \\ \beta^j_{2,p} = 0, & 2 \le j, p \le s & j \ne p. \end{cases}$$

Consequently,

$$\begin{cases} D(e^p_1) = \alpha^p_{p,1}e^p_1, & 1 \le p \le s, \\ D(x_p) = \alpha^p_{p,1}x_p, & 1 \le p \le s. \end{cases}$$

From the chain of equalities

$$D(e_i^p) = D([e_{i-1}^p, e_1^1]) = \frac{1}{2}[D(e_{i-1}^p), e_1^1] + \frac{1}{2}[e_{i-1}^p, D(e_1^1)], \quad 1 \le p \le s, \quad 2 \le i \le m_p,$$

and the restrictions obtained above, it is easy to establish that

$$\begin{array}{l} D(e_i^1) = \alpha_{1,1}^1 e_i^1, \quad 1 \le i \le m_1, \\ D(e_i^p) = \frac{1}{2^{i-1}}((2^{i-1}-1)\alpha_{1,1}^1 + \alpha_{p,1}^p)e_i^p, \quad 2 \le p \le s, \quad 1 \le i \le m_p. \end{array}$$

□

Proposition 3.2. *Any $\frac{1}{2}$-derivation D of the algebra $\mathcal{L}_t$ has the following form:*

$$D(f_j) = a_j f_j, \quad D(x_j) = \alpha_j b_j f_j, \quad 1 \le j \le n.$$

Proof. The proof is similar to the proof of Proposition 3.1. □

Proposition 3.3. *Any $\frac{1}{2}$-derivation D of the algebras R_1 and R_2 have the following forms, respectively:*

$$Der(R_1) : \begin{cases} D(f_i) &= \alpha_1 f_i, \quad 1 \le i \le n, \\ D(x) &= \alpha_1 x. \end{cases}$$

$$Der(R_2) : \begin{cases} D(f_i) &= \alpha_1 f_i, \quad 1 \le i \le n, \\ D(x) &= \sum\limits_{j=1}^{n} \beta_j f_j + \alpha_1 x. \end{cases}$$

Proof. The proof is similar to the proof of Proposition 3.1. □

4 Local $\frac{1}{2}$-derivation of solvable Leibniz algebras

4.1 Local $\frac{1}{2}$-derivation of solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$

Now we shall give the main result concerning local $\frac{1}{2}$-derivations of the solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$.

Theorem 4.1. *Any local $\frac{1}{2}$-derivation on the solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$ is a $\frac{1}{2}$-derivation.*

Proof. Let Δ be a local $\frac{1}{2}$-derivation on $R(N_{m_1,\dots,m_s}, s)$, then we have

$$\Delta(x_i) = \sum_{j=1}^{s} a_{i,j} x_j + \sum_{p=1}^{s}\sum_{j=1}^{m_p} b_{i,j}^p e_j^p, \quad \Delta(e_i^t) = \sum_{j=1}^{s} c_{i,j}^t x_j + \sum_{p=1}^{s}\sum_{j=1}^{m_p} d_{i,j}^{t,p} e_j^p.$$

Let D be a $\frac{1}{2}$-derivation on $R(N_{m_1,\dots,m_s}, s)$, then by Proposition 3.1, we obtain

$$\begin{array}{ll} D(e_i^1) = \alpha_{1,e_i^1} e_i^1, & 1 \le i \le m_1, \\ D(e_i^t) = \frac{1}{2^{i-1}}((2^{i-1}-1)\alpha_{1,e_i^t} + \alpha_{t,e_i^t})e_i^t, & 2 \le t \le s, \quad 1 \le i \le m_t, \\ D(x_i) = \alpha_{i,x_i} x_i, \quad 1 \le i \le s. & \end{array}$$

Considering the equalities

$$\Delta(x_j) = D_{x_j}(x_j), \quad 1 \le j \le s,$$

$$\Delta(e_i^t) = D_{e_i^t}(e_i^t), \quad 1 \le t \le s, \ 1 \le i \le m_t,$$

we have

$$\begin{cases} \sum\limits_{j=1}^{s} c_{i,j}^{1}x_j + \sum\limits_{p=1}^{s}\sum\limits_{j=1}^{m_p} d_{i,j}^{1,p}e_j^p = \alpha_{1,e_i^1}e_i^1, & 1 \le i \le m_1 \\ \sum\limits_{j=1}^{s} c_{i,j}^{t}x_j + \sum\limits_{p=1}^{s}\sum\limits_{j=1}^{m_p} d_{i,j}^{t,p}e_j^p = \frac{1}{2^{i-1}}((2^{i-1}-1)\alpha_{1,e_i^t} + \alpha_{t,e_i^t})e_i^t, & 2 \le t \le s,\ 1 \le i \le m_t, \\ \sum\limits_{j=1}^{s} a_{i,j}x_j + \sum\limits_{p=1}^{s}\sum\limits_{j=1}^{m_p} b_{i,j}^{p}e_j^p = \alpha_{i,x_i}x_i, & 1 \le i \le n. \end{cases}$$

From the previous restrictions, we get that

$$\begin{aligned} \Delta(e_i^1) &= d_{i,i}^{1,1}e_i^1, \quad 1 \le i \le m_1, \\ \Delta(e_i^t) &= d_{i,i}^{t,t}e_i^t, \quad 2 \le t \le s, \quad 1 \le i \le m_t, \\ \Delta(x_i) &= a_{i,i}x_i \quad 1 \le i \le s. \end{aligned}$$

Considering $\Delta(e_1^1 + e_i^1)$ for $2 \le i \le m_1$, we have

$$\Delta(e_1^1 + e_i^1) = d_{1,1}^{1,1}e_1^1 + d_{i,i}^{1,1}e_i^1.$$

On the other hand,

$$\begin{aligned} \Delta(e_1^1 + e_i^1) = D_{e_1^1+e_i^1}(e_1^1 + e_i^1) &= D_{e_1^1+e_i^1}(e_1^1) + D_{e_1^1+e_i^1}(e_i^1) = \\ &= \alpha_{1,e_1^1+e_i^1}e_1^1 + \alpha_{i,e_1^1+e_i^1}e_i^1 \end{aligned}$$

Comparing the coefficients at the basis elements e_1^1 and e_i^1, we get the equalities $\alpha_{1,e_1^1+e_i^1} = d_{1,1}^{1,1}$, $\alpha_{1,e_1^1+e_i^1} = d_{i,i}^{1,1}$, which imply

$$d_{i,i}^{1,1} = d_{1,1}^{1,1}, \quad 2 \le i \le m_1.$$

Now for $2 \le t \le s$, $1 \le i \le m_t$, we consider

$$\Delta(e_i^t + e_1^1 + x_1 + x_t) = d_{i,i}^{t,t}e_i^t + d_{1,1}^{1,1}e_1^1 + a_{1,1}x_1 + a_{t,t}x_t.$$

On the other hand,

$$\begin{aligned} \Delta(e_i^t + e_1^1 + x_1 + x_t) &= D_{e_i^t+e_1^1+x_1+x_t}(e_i^t + e_1^1 + x_1 + x_t) = \\ &= \frac{1}{2^{i-1}}((2^{i-1}-1)\alpha_{1,e_i^t+e_1^1+x_1+x_t} + \alpha_{i,e_i^t+e_1^1+x_1+x_t})e_i^t + \\ &+ \alpha_{1,e_i^t+e_1^1+x_1+x_t}e_1^1 + \alpha_{1,e_i^t+e_1^1+x_1+x_t}x_1 + \alpha_{t,e_i^t+e_1^1+x_1+x_t}x_t \end{aligned}$$

Comparing the coefficients at the basis elements e_i^t, e_1^1, x_1 and x_t, we get the equalities

$$\alpha_{1,e_i^t+e_1^1+x_1+x_t} = d_{1,1}^{1,1} = a_{1,1}, \quad \frac{1}{2^{i-1}}((2^{i-1}-1)\alpha_{1,e_i^t+e_1^1+x_1+x_t} + \alpha_{i,e_i^t+e_1^1+x_1+x_t}) = d_{i,i}^{t,t},$$

$$\alpha_{t,e_i^t+e_1^1+x_1+x_t} = a_{t,t},$$

which imply

$$d_{i,i}^{t,t} = \frac{1}{2^{i-1}}((2^{i-1}-1)d_{1,1}^{1,1} + a_{t,t}), \quad a_{1,1} = d_{1,1}^{1,1}, \ 2 \le t \le s, \quad 1 \le i \le m_t.$$

Thus, we obtain that the local $\frac{1}{2}$-derivation Δ has the following form:

$$\begin{array}{lll}\Delta(e_i^1) = d_{1,1}^{1,1}e_i^1, & 1 \le i \le m_1, & \\ \Delta(e_i^t) = \frac{1}{2^{i-1}}((2^{i-1}-1)d_{1,1}^{1,1} + a_{t,t})e_i^t, & 2 \le t \le s, & 1 \le i \le m_t, \\ \Delta(x_1) = d_{1,1}^{1,1}x_1, & & \\ \Delta(x_i) = a_{t,t}x_i, & i \le t \le s. & \end{array}$$

Proposition 3.1 implies that Δ is a $\frac{1}{2}$-derivation. Hence, every local $\frac{1}{2}$-derivation on $R(N_{m_1,\dots,m_s}, s)$ is a $\frac{1}{2}$-derivation. □

4.2 Local $\frac{1}{2}$-derivation of solvable Leibniz algebras with abelian nilradical

Now we shall give the main result concerning local $\frac{1}{2}$-derivations on solvable Leibniz algebras with abelian nilradicals.

Theorem 4.2. *Any local $\frac{1}{2}$-derivation on the algebra $\mathcal{L}_t$ is a $\frac{1}{2}$-derivation.*

Proof. For any local $\frac{1}{2}$-derivation Δ on the algebra $\mathcal{L}_t$, we put the $\frac{1}{2}$-derivation D, such that:

$$D(f_j) = a_j f_j, \quad D(x_j) = \alpha_j b_j f_j, \quad 1 \le j \le n,$$

Then, we get

$$\Delta(f_j) = D_{f_j}(f_j) = a_j f_j, \quad \Delta(x_j) = D_{x_j}(x_j) = \alpha_j b_j f_j.$$

Hence, Δ is a $\frac{1}{2}$-derivation. □

In the following theorem, we show that $(n+1)$-dimensional solvable Leibniz algebras with n-dimensional abelian nilradical have a local derivation which is not a derivation.

Theorem 4.3. *Consider the $(n+1)$-dimensional solvable Leibniz algebras R_1 and R_2 (see Theorem 2.1). Any local $\frac{1}{2}$-derivation on the algebras R_1 and R_2 is a $\frac{1}{2}$-derivation.*

Proof. We prove the theorem for the algebra R_1, and for the algebra R_2 the proof is similar.

Let Δ be a local $\frac{1}{2}$-derivation on R_1, then we have

$$\begin{aligned} \Delta(f_i) &= \sum_{j=1}^{n} a_{i,j} f_j + c_i x, \quad 1 \le i \le n, \\ \Delta(x) &= \sum_{j=1}^{n} b_j f_j + dx. \end{aligned} \tag{4.1}$$

Let D be a $\frac{1}{2}$-derivation on R_1, then by Proposition 3.3, we obtain

$$\begin{cases} D(f_i) &= \alpha_{1,f_i} f_i, \quad 1 \le i \le n, \\ D(x) &= \alpha_{1,x} x. \end{cases}$$

Considering the equalities

$$\Delta(x) = D_x(x), \quad \Delta(f_i) = D_{f_i}(f_i), \quad 1 \le i \le n,$$

we have

$$\begin{cases} \sum\limits_{j=1}^{n} a_{i,j} f_j + c_i x = \alpha_{1,f_i} f_i, \quad 1 \le i \le n \\ \sum\limits_{j=1}^{n} b_j f_j + dx = \alpha_{1,x} x. \end{cases}$$

From the previous restrictions, we get that

$$\Delta(f_i) = a_{i,i}f_i, \quad 1 \leq i \leq n,$$
$$\Delta(x) = dx.$$

For $2 \leq i \leq n$, we have

$$\Delta(f_1 + f_i) = a_{1,1}f_1 + a_{i,i}f_i.$$

On the other hand,

$$\Delta(f_1 + f_i) = D_{f_1+f_i}(f_1 + f_i) = D_{f_1+f_i}(f_1) + D_{f_1+f_i}(f_i) = \\ = \alpha_{1,f_1+f_i}f_1 + \alpha_{1,f_1+f_i}f_i.$$

Comparing the coefficients at the basis elements f_1 and f_i, we get the equalities $\alpha_{1,f_1+f_i} = a_{1,1}$, $\alpha_{1,f_1+f_i} = a_{i,i}$, which imply

$$a_{i,i} = a_{1,1}, \quad 2 \leq i \leq n.$$

Similarly, the equalities

$$\begin{aligned}\Delta(f_1 + x) &= a_{1,1}f + dx \\ &= D_{f_1+x}(f_1 + x) = D_{f_1+x}(f_1) + D_{f_1+x}(x) \\ &= \alpha_{1,f_1+x}f_1 + \alpha_{1,f_1+x}x,\end{aligned}$$

imply

$$d = a_{1,1}.$$

Thus, we obtain that the local $\frac{1}{2}$-derivation Δ has the following form:

$$\Delta(f_i) = a_{1,1}f_i, \quad 1 \leq i \leq n,$$
$$\Delta(x) = a_{1,1}x$$

Proposition 3.3 implies that Δ is a $\frac{1}{2}$-derivation. Hence, every local $\frac{1}{2}$-derivation on R_1 is a $\frac{1}{2}$-derivation.

□

5 2-local $\frac{1}{2}$-derivation of solvable Leibniz algebras

5.1 2-local $\frac{1}{2}$-derivation of solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$

Now we shall give the main result concerning of the 2-local $\frac{1}{2}$-derivations of the solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$.

Consider an element $q = \sum\limits_{t=1}^{s} x_t$ of $R(N_{m_1,\dots,m_s}, s)$.

Theorem 5.1. *Any 2-local $\frac{1}{2}$-derivation of the solvable Leibniz algebra $R(N_{m_1,\dots,m_s}, s)$ is a $\frac{1}{2}$-derivation.*

Proof. Let ∇ be a 2-local $\frac{1}{2}$-derivation on $R(N_{m_1,\dots,m_s}, s)$ such that $\nabla(q) = 0$. Then for any element

$$p = \sum_{t=1}^{s}\sum_{i=1}^{m_t} \xi_i^t e_i^t + \sum_{t=1}^{s} \zeta_t x_t \in R(N_{m_1,\dots,m_s}, s),$$

there exists a $\frac{1}{2}$-derivation $D_{q,p}(p)$, such that

$$\nabla(q) = D_{q,p}(q), \quad \nabla(p) = D_{q,p}(p).$$

Hence,

$$0 = \nabla(q) = D_{q,p}(q) = \sum_{t=1}^{s} \alpha_t x_t,$$

which implies, $\alpha_t = 0, \ 1 \leq t \leq s$.

Consequently, from the description of the $\frac{1}{2}$-derivation $R(N_{m_1,\dots,m_s}, s)$, we conclude that $D_{q,p} = 0$. Thus, we obtain that if $\nabla(q) = 0$, then ∇ is a zero.

Let now ∇ be an arbitrary 2-local $\frac{1}{2}$-derivation of $R(N_{m_1,\dots,m_s}, s)$. Take a $\frac{1}{2}$-derivation $D_{q,p}$, such that

$$\nabla(q) = D_{q,p}(q) \text{ and } \nabla(p) = D_{q,p}(p).$$

Set $\nabla_1 = \nabla - D_{q,p}$. Then ∇_1 is a 2-local $\frac{1}{2}$-derivation, such that $\nabla_1(q) = 0$. Hence $\nabla_1(p) = 0$ for all $\xi \in R(N_{m_1,\dots,m_s}, s)$, which implies $\nabla = D_{q,p}$. Therefore, ∇ is a $\frac{1}{2}$-derivation.

□

5.2 2-local $\frac{1}{2}$-derivation of solvable Leibniz algebras with alebian nilradical

Now we shall give the result concerning of 2-local $\frac{1}{2}$-derivations of solvable Leibniz algebras with abelian nilradical.

Proposition 5.1. *Any 2-local $\frac{1}{2}$-derivation of the algebra R_1 is a derivation.*

Proof. Let ∇ be a 2-local $\frac{1}{2}$-derivation on R_1, such that $\nabla(f_1) = 0$. Then for any element $\xi = \sum_{i=1}^{n} \xi_i f_i + \xi_{n+1} x \in R_1$, there exists a $\frac{1}{2}$-derivation $D_{f_1,\xi}(\xi)$, such that

$$\nabla(f_1) = D_{f_1,\xi}(f_1), \quad \nabla(\xi) = D_{f_1,\xi}(\xi).$$

Hence,

$$0 = \nabla(f_1) = D_{f_1,\xi}(f_1) = \alpha_1 f_1,$$

which implies, $\alpha_1 = 0$.

Consequently, from the description of the $\frac{1}{2}$-derivation of R_1, we conclude that $D_{f_1,\xi} = 0$. Thus, we obtain that if $\nabla(f_1) = 0$, then ∇ is a zero.

Let now ∇ be an arbitrary 2-local $\frac{1}{2}$-derivation of R_1. Take a $\frac{1}{2}$-derivation $D_{f_1,\xi}$, such that

$$\nabla(f_1) = D_{f_1,\xi}(f_1) \text{ and } \nabla(\xi) = D_{f_1,\xi}(\xi).$$

Set $\nabla_1 = \nabla - D_{f_1,\xi}$. Then ∇_1 is a 2-local $\frac{1}{2}$-derivation, such that $\nabla_1(f_1) = 0$. Hence, $\nabla_1(\xi) = 0$ for all $\xi \in R_1$, which implies that $\nabla = D_{f_1,\xi}$. Therefore, ∇ is a $\frac{1}{2}$-derivation. □

Theorem 5.2. *The solvable Leibniz algebra R_2 admits a 2-local $\frac{1}{2}$-derivation which is not a $\frac{1}{2}$-derivation.*

Proof. Let us define a homogeneous non-additive function f on $\mathbb{C}^2$ as follows

$$f(z_1, z_2) = \begin{cases} \frac{z_1^2}{z_2}, & \text{if } z_2 \neq 0, \\ 0, & \text{if } z_2 = 0, \end{cases}$$

where $(z_1, z_2) \in \mathbb{C}^2$.

Define the operator ∇ on R_2, such that

$$\nabla(\xi) = f(\xi_1, \xi_{n+1}) f_1, \tag{5.1}$$

for any element $\xi = \sum_{i=1}^{n} \xi_i f_i + \sum_{i=1}^{n} \xi_{n+i} x_i$,

The operator ∇ is not a $\frac{1}{2}$-derivation, since it is not linear.

Let us show that ∇ is a 2-local $\frac{1}{2}$-derivation. For this purpose, define a $\frac{1}{2}$-derivation D on R_2 by

$$D(\xi) = (a\xi_1 + b\xi_2) f_n.$$

For each pair of elements ξ and η, we choose a and b, such that $\nabla(\xi) = D(\xi)$ and $\nabla(\eta) = D(\eta)$. Let us rewrite the above equalities as system of linear equations with respect to the unknowns a, b as follows

$$\begin{cases} \xi_1 a + \xi_2 b = f(\xi_1, \xi_2), \\ \eta_1 a + \eta_2 b = f(\eta_1, \eta_2). \end{cases} \tag{5.2}$$

Case 1. $\xi_1\eta_2 - \xi_2\eta_1 = 0$. In this case, since the right-hand side of system (5.2) is homogeneous, it has infinitely many solutions.

Case 2. $\xi_1\eta_2 - \xi_2\eta_1 \neq 0$. In this case, system (5.2) has a unique solution.

□

Theorem 5.3. *The algebra $\mathcal{L}_t$ admits a 2-local $\frac{1}{2}$-derivation which is not a $\frac{1}{2}$-derivation.*

Proof. The proof is similar to the proof of Theorem 5.2. □

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