Eurasian Mathematical Journal

2017, Volume 8, Number 4

© The Eurasian National University

Aims and Scope

The Eurasian Mathematical Journal (EMJ) publishes carefully selected original research papers in all areas of mathematics written by mathematicians, principally from Europe and Asia. However papers by mathematicians from other continents are also welcome.

From time to time the EMJ publishes survey papers.

The EMJ publishes 4 issues in a year.

The language of the paper must be English only.

The contents of EMJ are indexed in Scopus, Web of Science (ESCI), Mathematical Reviews, MathSciNet, Zentralblatt Math (ZMATH), Referativnyi Zhurnal – Matematika, Math-Net.Ru.

The EMJ is included in the list of journals recommended by the Committee for Control of Education and Science (Ministry of Education and Science of the Republic of Kazakhstan) and in the list of journals recommended by the Higher Attestation Commission (Ministry of Education and Science of the Russian Federation).

Information for the Authors

Submission. Manuscripts should be written in LaTeX and should be submitted electronically in DVI, PostScript or PDF format to the EMJ Editorial Office via e-mail (eurasianmj@yandex.kz).

When the paper is accepted, the authors will be asked to send the tex-file of the paper to the Editorial Office.

The author who submitted an article for publication will be considered as a corresponding author. Authors may nominate a member of the Editorial Board whom they consider appropriate for the article. However, assignment to that particular editor is not guaranteed.

Copyright. When the paper is accepted, the copyright is automatically transferred to the EMJ. Manuscripts are accepted for review on the understanding that the same work has not been already published (except in the form of an abstract), that it is not under consideration for publication elsewhere, and that it has been approved by all authors.

Title page. The title page should start with the title of the paper and authors' names (no degrees). It should contain the Keywords (no more than 10), the Subject Classification (AMS Mathematics Subject Classification (2010) with primary (and secondary) subject classification codes), and the Abstract (no more than 150 words with minimal use of mathematical symbols).

Figures. Figures should be prepared in a digital form which is suitable for direct reproduction.

References. Bibliographical references should be listed alphabetically at the end of the article. The authors should consult the Mathematical Reviews for the standard abbreviations of journals' names.

Authors' data. The authors' affiliations, addresses and e-mail addresses should be placed after the References.

Proofs. The authors will receive proofs only once. The late return of proofs may result in the paper being published in a later issue.

Offprints. The authors will receive offprints in electronic form.

Publication Ethics and Publication Malpractice

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the EMJ implies that the work described has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The EMJ follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http : //publicationethics.org/files/u2/New$_C$ode.pdf). To verify originality, your article may be checked by the originality detection service CrossCheck http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the EMJ.

The Editorial Board of the EMJ will monitor and safeguard publishing ethics.

The procedure of reviewing a manuscript, established by the Editorial Board of the Eurasian Mathematical Journal

1. Reviewing procedure

1.1. All research papers received by the Eurasian Mathematical Journal (EMJ) are subject to mandatory reviewing.

1.2. The Managing Editor of the journal determines whether a paper fits to the scope of the EMJ and satisfies the rules of writing papers for the EMJ, and directs it for a preliminary review to one of the Editors-in-chief who checks the scientific content of the manuscript and assigns a specialist for reviewing the manuscript.

1.3. Reviewers of manuscripts are selected from highly qualified scientists and specialists of the L.N. Gumilyov Eurasian National University (doctors of sciences, professors), other universities of the Republic of Kazakhstan and foreign countries. An author of a paper cannot be its reviewer.

1.4. Duration of reviewing in each case is determined by the Managing Editor aiming at creating conditions for the most rapid publication of the paper.

1.5. Reviewing is confidential. Information about a reviewer is anonymous to the authors and is available only for the Editorial Board and the Control Committee in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (CCFES). The author has the right to read the text of the review.

1.6. If required, the review is sent to the author by e-mail.

1.7. A positive review is not a sufficient basis for publication of the paper.

1.8. If a reviewer overall approves the paper, but has observations, the review is confidentially sent to the author. A revised version of the paper in which the comments of the reviewer are taken into account is sent to the same reviewer for additional reviewing.

1.9. In the case of a negative review the text of the review is confidentially sent to the author.

1.10. If the author sends a well reasoned response to the comments of the reviewer, the paper should be considered by a commission, consisting of three members of the Editorial Board.

1.11. The final decision on publication of the paper is made by the Editorial Board and is recorded in the minutes of the meeting of the Editorial Board.

1.12. After the paper is accepted for publication by the Editorial Board the Managing Editor informs the author about this and about the date of publication.

1.13. Originals reviews are stored in the Editorial Office for three years from the date of publication and are provided on request of the CCFES.

1.14. No fee for reviewing papers will be charged.

2. Requirements for the content of a review

2.1. In the title of a review there should be indicated the author(s) and the title of a paper.

2.2. A review should include a qualified analysis of the material of a paper, objective assessment and reasoned recommendations.

2.3. A review should cover the following topics:

- compliance of the paper with the scope of the EMJ;
- compliance of the title of the paper to its content;
- compliance of the paper to the rules of writing papers for the EMJ (abstract, key words and phrases, bibliography etc.);
- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);
- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

2.4. The final part of the review should contain an overall opinion of a reviewer on the paper and a clear recommendation on whether the paper can be published in the Eurasian Mathematical Journal, should be sent back to the author for revision or cannot be published.

Web-page

The web-page of EMJ is www.emj.enu.kz. One can enter the web-page by typing Eurasian Mathematical Journal in any search engine (Google, Yandex, etc.). The archive of the web-page contains all papers published in EMJ (free access).

Subscription

For Institutions

- US$ 200 (or equivalent) for one volume (4 issues)
- US$ 60 (or equivalent) for one issue

For Individuals

- US$ 160 (or equivalent) for one volume (4 issues)
- US$ 50 (or equivalent) for one issue.

The price includes handling and postage.

The Subscription Form for subscribers can be obtained by e-mail:

eurasianmj@yandex.kz

The Eurasian Mathematical Journal (EMJ)
The Editorial Office
The L.N. Gumilyov Eurasian National University
Building no. 3
Room 306a
Tel.: +7-7172-709500 extension 33312
13 Kazhymukan St
010008 Astana
Kazakhstan

MUKHTARBAY OTELBAEV

(to the 75th birthday)

On October 3, 2017 was the 75th birthday of Mukhtarbay Otelbaev, Doctor of Physical and Mathematical Sciences (1978), Professor (1983), academician of the National Academy of Sciences of the Republic of Kazakhstan (2004), Honored Worker of the Republic of Kazakhstan (2012), laureate of the State Prize of the Republic of Kazakhstan in the field of science and technology (2007), Director of the Eurasian Mathematical Institute (since 2001), Professor of the Department вЂњFundamental MathematicsвЂќ of the L.N. Gumilyov Eurasian National University, the editor-in-chief of the Eurasian Mathematical Journal (together with V.I. Burenkov and V.A. Sadovnichy).

M. Otelbaev was born in the village of Karakemer of the Kurdai district, Zhambyl region. He graduated from the M.V. Lomonosov Moscow State University (1969) and then completed his postgraduate studies at the same university (1972). There he defended his doctor of sciences thesis (1978).

Professor Otelbaev's scientific interests are related to functional analysis, differential equations, computational mathematics, and theoretical physics.

He introduced the q-averaging, which is now called the Otelbaev function; using it he obtained a number of fundamental results. For embedding of the Sobolev weighted spaces and the resolvent of the Schrödinger operator, he established criterions for the compactness and finiteness of the type, as well as estimates of the eigenvalues of the Schrödinger and Dirac operators that are exact in order. He was the first to establish that there is no universal asymptotic formula for the distribution function of the Sturm-Liouville operator. He obtained effective conditions for the separation of the differential operators with nonsmooth and oscillating coefficients, he developed an abstract theory of extension and contraction of operators which are not necessarily linear in linear topological spaces. M. Otelbaev proposed a new numerical method for solving boundary value problems, and a method for approximate calculation of eigenvalues and eigenvectors of compact operators. He obtained the fundamental results in the theory of nonlinear evolution equations and in theoretical physics.

He has published more than 70 scientific papers in leading international journals entering the rating lists of Thomson Reuters and Scopus. Under his supervision 70 postgraduate students have defended their candidate of sciences theses, 9 of them became doctors of sciences. In 2006 and 2011 he was awarded the state grant "The best university teacher".

The Editorial Board of the Eurasian Mathematical Journal congratulates Mukhtarbay Otelbaev on the occasion of his 75th birthday and wishes him good health and new achievements in mathematics and mathematical education.

Award for the Eurasian Mathematical Journal

Dear readers, authors, reviewers and members of the Editorial Board of the Eurasian Mathematical Journal,

we are happy to inform you that in November 2017 the Eurasian Mathematical Journal was awarded the title "Leader of Science 2017" by the National Center of State Scientific-Technical Expertise of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan in the nomination "Leader of Kazakhstan Scientific Publications" for the high level of publication activities and high level of citations in Web of Science Core Collection in 2014-2016.

Recall that the Eurasian Mathematical Journal was founded by the L.N. Gumilyov Eurasian National University in 2010 in co-operation with the M.V. Lomonosov Moscow State University, the Peoples' Friendship University of Russia and the University of Padua (see [1]).

The journal pulishes carefully selected original research papers in all areas of mathematics, survey papers, and short communications. It publishes 4 issues in a year. The language of the paper must be English only. Papers accepted for publication are edited from the point of view of English.

More than 280 papers were published written by mathematicians from more than 40 countries representing all continents.

In 2014 the journal was registered in Scopus and in September 2014 the Elsevier-Kazakhstan Research Excellence Forum was held at the L.N. Gumilyov Eurasian National University dedicated to this occasion in which the Elsevier Chairman Professor Y.S. Chi participated (see [3] for details).

In 2015 the Eurasian Mathematical Journal was included in the list of Scopus mathematical journals, quartile Q4, and it is on the way to entering quartile Q3 (see [3]).

Attached is the invitation letter to the Rector of the L.N. Gumilyov Eurasian National University Professor E.B. Sydykov to the ceremony of awarding, which took place in Almaty on November 8, 2017.

On behalf of the Editorial Board of the EMJ V.I. Burenkov, E.D. Nursultanov, T.Sh. Kalmenov, R. Oinarov, M. Otelbaev, T.V. Tararykova, A.M. Temirkhanova

References

[1] B. Abdrayim, *Opening address by the rector of the L.N. Gumilyov Eurasian National University,* Eurasian Math. J. 1 (2010), no. 1, p. 5.

[2] *Eurasian Mathematical Journal is indexed in Scopus,* Eurasian Math. J. 5 (2014), no. 3, 6-8.

[3] V.I. Burenkov, E.D. Nursultanov, T.Sh. Kalmenov, R. Oinarov, M. Otelbaev, T.V. Tararykova, A.M. Temirkhanova, *EMJ: from Scopus Q4 to Scopus Q3 in two years?!,* Eurasian Math. J. 7 (2016), no. 3, p. 6.

Исх №: 3367/12-04
«30». 10. 2017г.

Ректору
Евразийского национального
университета имени Л.Н.Гумилева
г-ну Сыдыкову Е.Б.

Уважаемый Ерлан Батташевич!

АО «Национальный центр государственной научно-технической экспертизы» (далее АО «НЦГНТЭ») и компания Clarivate Analytics имеют честь пригласить Вас на церемонию вручения независимой награды «Лидер науки-2017» за высокие показатели публикационной активности и цитируемости в Web of Science Core Collection в период 2014-2016 годы.

Ваш журнал *«Eurasian Mathematical Journal»* награждается в номинации **«Лидер казахстанских научных изданий»**.

Торжественное мероприятие состоится 8 ноября 2017 года по адресу: г. Алматы, ул. Богенбай батыра 221, Актовый зал, начало в 10.00, регистрация с 09.00 ч.

Для участия в мероприятии просим пройти онлайн регистрацию на сайте www.nauka.kz до 6 ноября 2017 года.

По всем вопросам обращаться по тел.:+7 727 378 05 78, e-mail: 3780544@bk.ru.

Президент   Ибраев А.Ж.

ESTIMATES FOR THE KOLMOGOROV WIDTHS OF WEIGHTED SOBOLEV CLASSES ON A DOMAIN WITH CUSP: CASE OF WEIGHTS THAT ARE FUNCTIONS OF THE DISTANCE FROM THE BOUNDARY

A.A. Vasil'eva

Communicated by V.D. Stepanov

Key words: Kolmogorov widths, domains with cusp.

AMS Mathematics Subject Classification: 41A46.

Abstract. In this article order estimates for the Kolmogorov widths of weighted Sobolev classes in weighted Lebesgue spaces on domains with cusp were obtained. Here weights are functions of distance from the boundary.

1 Introduction

It is well-known (see, e.g., [2, 1, 8, 9, 10, 11, 12]) that if a domain has zero angles, then conditions under which the embedding of the weighted Sobolev space into weighted Lebesgue space is compact differ from those in the case of domains with Lipschitz boundary. This suggests the following problem: how do such characteristics of a compact embedding like widths depend on the singularities on the boundaries and singularities of weights?

Given $x = (x_1, \ldots, x_{d-1}, x_d) \in \mathbb{R}^d$, we set $x' = (x_1, \ldots, x_{d-1}) \in \mathbb{R}^{d-1}$.

O.V. Besov [3] obtained order estimates for Kolmogorov widths of non-weighted Sobolev classes $W_p^r(\Omega)$ in $L_q(\Omega)$, where $p < q$, $r \in \mathbb{N}$, $d \geq 2$,

$$\Omega = \{(x', x_d) \in \mathbb{R}^d : \ 0 < x_d < 1, \ |x'| < x_d^\sigma\}, \quad \sigma > 1,$$

$r + (\sigma(d-1) + 1) \left(\frac{1}{q} - \frac{1}{p} \right) > 0$. It was proved that the orders of widths are the same as in the case of $\Omega = [0, 1]^d$. In [17] some more general domains with cusp and weights depending only on the coordinate x_d were considered. Here the analogue of embedding result of Maz'ya and Poborchii and order estimates of widths were obtained. The case $p = q$ was investigated by W.D. Evans, D.J. Harris and Y. Saito for generalized ridged domains [6, 7].

The case of widths that are powers of the distance from the Lipschitz boundary was considered by H. Triebel [16] and I.V. Boykov [4, 5]. This result was generalized for John domains and some other types of weights in [18].

In this paper we obtain order estimates for Kolmogorov widths of weighted Sobolev classes $W_{p,g}^r(\Omega)$ on a domain with cusp in weighted Lebesgue spaces $L_{q,v}(\Omega)$, where $1 < p < q < \infty$, $r \in \mathbb{N}$, $d \geq 2$, and weights are functions of distance from the boundary.

We give some notation.

Let $\Omega \subset \mathbb{R}^d$ be a bounded domain, let $g, v : \Omega \to (0, \infty)$ be measurable functions, and let $r \in \mathbb{N}$, $1 < p, q < \infty$. We denote by $l_{r,d}$ the number of components of the vector-valued distribution $\nabla^r f$ and define the weighted Sobolev class by

$$W_{p,g}^r(\Omega) = \left\{ f : \ \Omega \to \mathbb{R} \,\big|\, \exists \psi : \ \Omega \to \mathbb{R}^{l_{r,d}} : \ \|\psi\|_{L_p(\Omega)} \le 1, \ \nabla^r f = g \cdot \psi \right\},$$

and the weighted Lebesgue space by

$$\|f\|_{L_{q,v}(\Omega)} = \|f\|_{q,v} = \|fv\|_{L_q(\Omega)}, \qquad L_{q,v}(\Omega) = \{ f : \Omega \to \mathbb{R} \,|\, \ \|f\|_{q,v} < \infty \}.$$

Let

$$\psi(z) = z^\sigma |\log z|^\theta, \quad \sigma > 1, \quad \theta \in \mathbb{R}, \quad 0 < z < 1, \tag{1.1}$$

$$\Omega = \left\{ (x', x_d) \in \mathbb{R}^d : \ 0 < x_d < 1/2, \ |x'| < \psi(x_d) \right\}, \tag{1.2}$$

$$r \in \mathbb{N}, \quad 1 < p < q < \infty, \quad \delta := r + \frac{d}{q} - \frac{d}{p} > 0, \quad \beta_g, \ \beta_v, \ \alpha_g, \ \alpha_v \in \mathbb{R}, \tag{1.3}$$

$$g(x) = \varphi_g(\text{dist}\,(x, \ \partial\Omega)), \quad v(x) = \varphi_v(\text{dist}\,(x, \ \partial\Omega)), \tag{1.4}$$

$$\varphi_g(t) = t^{-\beta_g} |\log t|^{-\alpha_g}, \quad \varphi_v(t) = t^{-\beta_v} |\log t|^{-\alpha_v}. \tag{1.5}$$

We set $\beta = \beta_g + \beta_v$, $\alpha = \alpha_g + \alpha_v$.

Let $n \in \mathbb{Z}_+$. By the Kolmogorov n-width of a set $M \subset X$ in the normed space X, we mean the quantity

$$d_n(M, \ X) = \inf_{L \in \mathcal{L}_n(X)} \sup_{x \in M} \inf_{y \in L} \|x - y\|_X,$$

where $\mathcal{L}_n(X)$ is the family of subspaces of X of dimension at most n. The main properties of widths can be found in [14, 15, 13].

Let $\{x_n\}_{n \in \mathbb{N}}$ and $\{y_n\}_{n \in \mathbb{N}}$ be sequences. We write $x_n \asymp y_n$ if there exist $c \ge 1$ and $n_0 \in \mathbb{N}$ such that $c^{-1} x_n \le y_n \le c x_n$ for all $n \ge n_0$.

The main result of this paper is as follows.

Theorem 1.1. *Let (1.1) – (1.5) hold and $\beta_v < \frac{1}{q}$,*

$$\beta = \frac{1}{\sigma} \left(r + \frac{1}{q} - \frac{1}{p} \right) + (d - 1) \left(\frac{1}{q} - \frac{1}{p} \right),$$

$\alpha + \theta\beta + \theta(d - 1) \left(\frac{1}{q} - \frac{1}{p} \right) > 0.$ *Suppose that* $\alpha + \theta\beta + \theta(d - 1) \left(\frac{1}{q} - \frac{1}{p} \right) \ne \frac{\delta - \beta}{d - 1}.$

  1. Let $q \le 2$. We set

$$\theta_1 = \frac{\delta}{d}, \quad \theta_2 = \min \left\{ \frac{\delta - \beta}{d - 1}, \ \alpha + \theta\beta + \theta(d - 1) \left(\frac{1}{q} - \frac{1}{p} \right) \right\},$$

$$\nu_1 = 0, \quad \nu_2 = \begin{cases} \alpha, & \text{if} \quad \alpha + \theta\beta + \theta(d - 1) \left(\frac{1}{q} - \frac{1}{p} \right) > \frac{\delta - \beta}{d - 1}, \\ 0, & \text{if} \quad \alpha + \theta\beta + \theta(d - 1) \left(\frac{1}{q} - \frac{1}{p} \right) < \frac{\delta - \beta}{d - 1}. \end{cases}$$

  Suppose that $\theta_1 \ne \theta_2$, $j_* \in \{1, 2\}$ is chosen so that $\theta_{j_*} = \min\{\theta_1, \theta_2\}$. Then

$$d_n(W_{p,g}^r(\Omega), \ L_{q,v}(\Omega)) \asymp n^{-\theta_{j*}} (\log n)^{-\nu_{j*}}.$$

2. Let $q > 2$. We set

$$\theta_1 = \frac{\delta}{d} + \min\left\{\frac{1}{2} - \frac{1}{q}, \; \frac{1}{p} - \frac{1}{q}\right\}, \quad \theta_2 = \frac{q\delta}{2d},$$

$$\theta_3 = \min\left\{\frac{\delta - \beta}{d - 1}, \; \alpha + \theta\beta + \theta(d-1)\left(\frac{1}{q} - \frac{1}{p}\right)\right\} + \min\left\{\frac{1}{2} - \frac{1}{q}, \; \frac{1}{p} - \frac{1}{q}\right\},$$

$$\theta_4 = \frac{q}{2} \cdot \min\left\{\frac{\delta - \beta}{d - 1}, \; \alpha + \theta\beta + \theta(d-1)\left(\frac{1}{q} - \frac{1}{p}\right)\right\},$$

$$\nu_1 = \nu_2 = 0, \quad \nu_3 = \nu_4 = \begin{cases} \alpha, & \text{if} \quad \alpha + \theta\beta + \theta(d-1)\left(\frac{1}{q} - \frac{1}{p}\right) > \frac{\delta - \beta}{d-1}, \\ 0, & \text{if} \quad \alpha + \theta\beta + \theta(d-1)\left(\frac{1}{q} - \frac{1}{p}\right) < \frac{\delta - \beta}{d-1}. \end{cases}$$

Suppose that there exists $j_* \in \{1, 2, 3, 4\}$ such that $\theta_{j_*} < \min_{j \neq j_*} \theta_j$. Then

$$d_n(W^r_{p,g}(\Omega), \; L_{q,v}(\Omega)) \asymp n^{-\theta_{j*}}(\log n)^{-\nu_{j*}}.$$

Acknowledgments

References

[1] O.V. Besov, *Integral representations of functions, and imbedding theorems for a region with flexible horn condition.* Investigations in the theory of differentiable functions of several variables and its applications. Part 10, Trudy Mat. Inst Steklov, 170 (1984), 12–30; English transl. in Proc. Steklov Inst. Math. 170 (1987), 11–31.

[2] O.V. Besov, *Sobolev's embedding theorem for anisotropically irregular domains.* Eurasian Math. J. 2 (2011), no. 1, 32–51.

[3] O.V. Besov, *Kolmogorov widths of Sobolev classes on an irregular domain.* Orthogonal series, approximation theory and related questions, To the 60th birthday of academician Boris Sergeevich Kashin. Proc. Steklov Inst. Math. 280 (2013), no. 1, 34–45.

[4] I.V. Bojkov, *Approximation of some classes of functions by local splines.* Zh. Vychisl. Mat. Mat. Fiz. 38 (1998), no. 1, 25–33; English transl. in Comput. Math. Math. Phys. 38 (1998), no. 1, 21–29.

[5] I.V. Boykov, *Optimal approximation and Kolmogorov widths estimates for certain singular classes related to equations of mathematical physics.* arXiv:1303.0416v1.

[6] W.D. Evans, D.J. Harris, *Fractals, trees and the Neumann Laplacian.* Math. Ann., 296 (1993), no. 3, 493–527.

[7] W.D. Evans, D.J. Harris, Y. Saito, *On the approximation numbers of Sobolev embeddings on singular domains and trees.* Quart. J. Math. 55 (2004), 267–302.

[8] P. Hajlasz, P. Koskela, *Isoperimetric inequalities and imbedding theorems in irregular domains.* J. London Math. Soc. (2). 58 (1998), no. 2, 425–450.

[9] T. Kilpeläinen, J. Malý, *Sobolev inequalities on sets with irregular boundaries.* Z. Anal. Anwendungen, 19 (2000), no. 2, 369–380.

[10] D.A. Labutin, *Integral representations of functions and embeddings of Sobolev spaces on cuspidal domains.* Math. Notes, 61 (1997), no. 2, 164–179.

[11] D.A. Labutin, *Embedding of Sobolev spaces on Hölder domains.* Investigations in the theory of differentiable functions of several variables and its applications. Proc. Steklov Inst. Math. 227 (1999), 163–172.

[12] V.G. Maz'ya and S.V. Poborchi, *Imbedding theorems for Sobolev spaces on domains with peak and on Hölder domains.* St. Petersburg Math. J., 18 (2007), no. 4, 583–605.

[13] A. Pinkus, *n-widths in approximation theory.* Berlin: Springer, 1985. 291 pp.

[14] V.M. Tikhomirov, *Some questions in approximation theory.* Moscow State University Publishing House, Moscow 1976, 304 pp. (Russian)

[15] V.M. Tikhomirov, *Approximation theory.* Analysis-2, Sovrem. Probl. Mat. Fundam. Naprav., vol. 14, VINITI, Moscow 1987, pp. 103–260; English transl. in Analysis II, Encyclopaedia of Math. Sci., vol. 14, 1990.

[16] H. Triebel, *Interpolation theory, function spaces, differential operators.* VEB Deutscher Verlag der Wissenschaften, Berlin 1978, 528 pp.

[17] A.A. Vasil'eva, *Widths of Sobolev weight classes on a domain with cusp.* Matem. Sbornik, 206 (2015), no. 10, 37–70.

[18] A.A. Vasil'eva, *Widths of function classes on sets with tree-like structure.* J. Appr. Theory, 192 (2015), 19–59.

Anastasia Andreevna Vasil'eva
Department of Mechanics and Mathematics
Lomonosov Moscow State University
GSP-1, 1 Leninskiye Gory, Main Building
119991 Moscow, Russia
E-mails: vasilyeva_nastya@inbox.ru