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EMJ: from Scopus Q4 to Scopus Q3 in two years?!

Recently the list was published of all mathematical journals included in 2015 Scopus quartiles Q1 (334 journals), Q2 (318 journals), Q3 (315 journals), and Q4 (285 journals). Altogether 1252 journals.

With great pleasure we inform our readers that the Eurasian Mathematical Journal was included in this list, currently the only mathematical journal in the Republic of Kazakhstan and Central Asia.

It was included in Q4 with the SCImago Journal & Country Rank (SJR) indicator equal to 0,101, and is somewhere at the bottom of the Q4 list. With this indicator the journal shares places from 1240 to 1248 in the list of all 2015 Scopus mathematical journals. Nevertheless, this may be considered to be a good achievement, because Scopus uses information about journals for the three previous years, i. e. for years 2013-2015, and the EMJ is in Scopus only from the first quarter of year 2015.

The SJR indicator is calculated by using a sophisticated formula, taking into account various characteristics of journals and journals publications, in particular the average number of weighted citations received in the selected year by the documents published in the selected journal in the three previous years. This formula and related comments can be viewed on the web-page

$$http : //www.scimagojr.com/journalrank.php?category = 2601\&area = 2600\&page = 1\&total_size = 373$$

(Help/Journals/Understand tables and charts/Detailed description of SJR.)

In order to enter Q3 the SJR indicator should be greater than 0,250. It looks like the ambitious aim of entering Q3 in year 2017 is nevertheless realistic due to recognized high level of the EMJ.

We hope that all respected members of the international Editorial Board, reviewers, authors of our journal, representing more than 35 countries, and future authors will provide high quality publications in the EMJ which will allow to achieve this aim.

On behalf of the Editorial Board of the EMJ
V.I. Burenkov, E.D. Nursultanov, T.Sh. Kalmenov,
R. Oinarov, M. Otelbaev, T.V. Tararykova, A.M. Temirkhanova

VICTOR IVANOVICH BURENKOV

(to the 75th birthday)

On July 15, 2016 was the 75th birthday of Victor Ivanovich Burenkov, editor-in-chief of the Eurasian Mathematical Journal (together with V.A. Sadovnichy and M. Otelbaev), director of the S.M. Nikol'skii Institute of Mathematics, head of the Department of Mathematical Analysis and Theory of Functions, chairman of Dissertation Council at the RUDN University (Moscow), research fellow (part-time) at the Steklov Institute of Mathematics (Moscow), scientific supervisor of the Laboratory of Mathematical Analysis at the Russian-Armenian (Slavonic) University (Yerevan, Armenia), doctor of physical and mathematical sciences (1983), professor (1986), honorary professor of the L.N. Gumilyov Eurasian National University (Astana, Kazakhstan, 2006), honorary doctor of the Russian-Armenian (Slavonic) University (Yerevan, Armenia, 2007), honorary member of staff of the University of Padua (Italy, 2011), honorary distinguished professor of the Cardiff School of Mathematics (UK, 2014), honorary professor of the Aktobe Regional State University (Kazakhstan, 2015).

V.I. Burenkov graduated from the Moscow Institute of Physics and Technology (1963) and completed his postgraduate studies there in 1966 under supervision of the famous Russian mathematician academician S.M. Nikol'skii.

He worked at several universities, in particular for more than 10 years at the Moscow Institute of Electronics, Radio-engineering, and Automation, the RUDN University, and the Cardiff University. He also worked at the Moscow Institute of Physics and Technology, the University of Padua, and the L.N. Gumilyov Eurasian National University.

He obtained seminal scientific results in several areas of functional analysis and the theory of partial differential and integral equations. Some of his results and methods are named after him: Burenkov's theorem of composition of absolutely continuous functions, Burenkov's theorem on conditional hypoellipticity, Burenkov's method of mollifiers with variable step, Burenkov's method of extending functions, the Burenkov-Lamberti method of transition operators in the problem of spectral stability of differential operators, the Burenkov-Guliyevs conditions for boundedness of operators in Morrey-type spaces. On the whole, the results obtained by V.I. Burenkov have laid the groundwork for new perspective scientific directions in the theory of functions spaces and its applications to partial differential equations, the spectral theory in particular.

More than 30 postgraduate students from more than 10 countries gained candidate of sciences or PhD degrees under his supervision. He has published more than 170 scientific papers. The lists of his publications can be viewed on the portals MathSciNet and MathNet.Ru. His monograph "Sobolev spaces on domains" became a popular text for both experts in the theory of function spaces and a wide range of mathematicians interested in applying the theory of Sobolev spaces.

In 2011 the conference "Operators in Morrey-type Spaces and Applications", dedicated to his 70th birthday was held at the Ahi Evran University (Kirsehir, Turkey). Proceedings of that conference were published in the EMJ 3-3 and EMJ 4-1.

The Editorial Board of the Eurasian Mathematical Journal congratulates Victor Ivanovich Burenkov on the occasion of his 75th birthday and wishes him good health and new achievements in science and teaching!

Short communications

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ON RELATIONSHIP BETWEEN THE RESOLVENT CONVERGENCE AND THE SMOOTHNESS OF SOLUTIONS TO BOUNDARY VALUE PROBLEMS

I.V. Tsylin

Communicated by T.V. Tararykova

Key words: elliptic operators, resolvent convergence, regularity up to the boundary.

AMS Mathematics Subject Classification: 35J15, 35J25, 47B38.

Abstract. Relationship between the resolvent convergence property and the smoothness of solutions to boundary value problems are studied. The results use pointwise multipliers and B-spaces.

1 Introduction

Let $K \subset \mathbb{R}^d$, $d \geq 2$, be a compact set and K_0 be the interior of K. Consider an elliptic operator of order $2m$, $m \geq 1$, defined on subdomains of K:

$$\mathcal{A}u = \sum_{0 \leq |\alpha|, |\beta| \leq m} (-1)^{|\beta|} \partial^\beta (a^{\alpha\beta} \partial^\alpha u),$$

where α, β are multi-indices of length d, $|\alpha| = \sum_i \alpha_i$, $\partial^\alpha u = \frac{\partial^{\alpha_1 + \cdots \alpha_d} u}{\partial^{\alpha_1} x^1 \ldots \partial^{\alpha_d} x^d}$. The operator $\mathcal{A}$ is understood in the Friedrichs sense. For a fixed domain Ω we consider the first boundary value problem

$$\mathcal{A}u = f, \quad u \in \overset{\circ}{H}{}^m(\Omega), \tag{1.1}$$

where the right-hand side $f \in H^{-m}(\Omega)$.

There are two classic questions on problem (1.1), namely, the resolvent convergence problem (see [10, 2]) and the problem of smoothness (see [11, 9, 1]) of solutions to (1.1).

In more detail the first problem can be formulated as follows. Let $\mathcal{G}_\Omega$ be a solving operator to problem (1.1) with right-hand sides in $H^{-m}(K_0)$. For a family of subdomains $\{\Omega_\varepsilon\}$, $\overline{\Omega_\varepsilon} \subset K_0$, such that Ω_ε tends in a certain sence to $\Omega \subset K_0$, one can ask about the validity of the following statement

$$\|\mathcal{G}_\Omega - \mathcal{G}_{\Omega_\varepsilon}\|_{\mathcal{L}(B_1, B_2)} \leq \phi(\varepsilon) \to 0 \quad \text{as} \quad \varepsilon \to 0, \tag{1.2}$$

where B_1, B_2 are some Banach spaces, $\mathcal{L}(B_1, B_2)$ is the space of all bounded linear operators from B_1 to B_2. Due to Lemma in [3], from the rate of resolvent convergence one can obtain the rate of spectral convergence of $\mathcal{A}$ under domain variation (on this topic see [6, 5, 4]).

One can formulate the second problem as follows. Let X, Y be Banach spaces, such that there are continuous embeddings $X \hookrightarrow H^{-m}(K_0)$, $Y \hookrightarrow \overset{\circ}{H}{}^1(K_0)$. What requirements on the operator $\mathcal{A}$ and the spaces X, Y ensure that the operator

$$\mathcal{G}_\Omega : X \to Y. \tag{1.3}$$

is bounded?

Due to E. Feleqi paper [7], if the boundary of Ω is Lipschitz and for some spaces X, Y (1.3) holds then one can obtain (1.2) for a certain function ϕ. In this paper we formulate a theorem, which allows to obtain results of type (1.3) from (1.2), even in the case, of an arbitrary bounded domain Ω.

2 Main results

Definition 1. *Let X, Y be Banach spaces of functions, such that there are dense continuous embeddings $\mathcal{D}(\Omega) \hookrightarrow X$, $\mathcal{D}(\Omega) \hookrightarrow Y$. We say that a distribution f belongs to the space of pointwise multipliers (see [8]) $M(X \to Y^\star) = M(Y \to X^\star)$ if the following norm is finite*

$$\|f\|_{M(X\to Y^\star)} \overset{\text{def}}{=} \sup_{u,v\in\mathcal{D}(\Omega)} \frac{|(f, uv)|}{\|u\|_X \|v\|_Y} < \infty.$$

We assume that the operator $\mathcal{A}$ satisfies the following conditions:

A1 $\forall x \in K_0\ \forall \beta, \delta \in \mathbb{Z}_+^d : |\beta| = |\delta| = m \Rightarrow a^{\beta,\delta}(x) = \overline{a^{\delta,\beta}(x)}$;

A2 $\exists \alpha > 0 : \forall x \in K_0\ \forall \xi \in \mathbb{C}^{d^m} \Rightarrow \alpha \sum_{|\beta|=m} |\xi^\beta|^2 \le \sum_{|\beta|=|\delta|=m} a^{\beta,\delta}(x)\xi^\beta \bar{\xi}^\delta$;

A3 $\forall \beta, \delta \in \mathbb{Z}_+^d$, $0 \le |\beta|, |\delta| \le m \Rightarrow a^{\beta,\delta} \in M(\overset{\circ}{H}{}^{m-|\beta|}(K_0) \to H^{-m+|\delta|}(K_0))$, in particular, if $|\beta| = |\delta| = m$ then $a^{\beta,\delta} \in L_\infty(K_0) = M(L_2(K_0) \to L_2(K_0))$. We also suppose that the form associated with the operator $\mathcal{A}$ is positive definite.

Definition 2. *Let X be a Banach space of functions defined on $\mathbb{R}^d$. For a certain domain $\Omega \subset \mathbb{R}^d$ we intoduce the spaces $\tilde{X}(\Omega) = \{f \in X \mid \operatorname{supp} f \subset \overline{\Omega}\}$, $X^\star(\Omega) = \left[\tilde{X}(\Omega)\right]^\star$. Let*

$$N^\gamma(Y) = \left\{ f \in Y \;\middle|\; \|f\|_{N^\gamma(Y)} = \sup_{h\ne 0} \frac{\|f(x) - f(x+h)\|_Y}{|h|^\gamma} < \infty \right\},$$

where $\gamma \in (0, 1]$ and Y is one of the spaces X, $X^\star$, $\tilde{X}(\Omega)$. For $Y = X^\star(\Omega)$ we modify the definition

$$N^\gamma(X^\star(\Omega)) = \left\{ f \in X^\star(\Omega) \;\middle|\; \|f\|_{N^\gamma(X^\star(\Omega))} = \sup_{v\in\tilde{X}(\Omega),\, h\ne 0} \frac{|(f, v(\cdot) - v(\cdot + h))|}{|h|^\gamma \|v\|_{\tilde{X}(\Omega)}} < \infty \right\}.$$

Theorem 2.1. *Let $\mathcal{A}$ satisfy conditions **A1**–**A3**, and, for $\gamma \in (0, 1)$, satisfy the following condition*

A4 *$\forall \beta, \delta \in \mathbb{Z}_+^d$, $0 \le |\beta|, |\delta| \le m \Rightarrow a^{\beta,\delta} \in N^\gamma(M(\overset{\circ}{H}{}^{m-|\beta|}(K_0) \to H^{-m+|\delta|}(K_0)))$, in particular, if $|\beta| = |\delta| = m$ then $a^{\beta,\delta} \in C^{0,\gamma}(K_0)$.*

Also assume that $\overline{\Omega} \subset K_0$ *is a domain such that for any domain* $\Omega^* \Subset K_0$ *for some Banach space* $X \hookrightarrow H^{-1}(K_0)$ *we have*

$$\|\mathcal{G}_\Omega - \mathcal{G}_{\Omega^*}\|_{\mathcal{L}(X,\,\mathring{H}^1(K_0))} \leq C\left(d^{\mathcal{H}}(\Omega,\Omega^*)\right)^\gamma, \tag{2.1}$$

where $C > 0$ *is independent of* Ω^*.

Then the operator

$$\mathcal{G}_\Omega : X \cap B^{-1+\gamma}_{2,1}(K_0) \to \tilde{N}^{1+\gamma}_2(K_0)$$

is bounded.

Here $\tilde{N}^{1+T}_2(K_0)$, $B^{-1+T}_{2,1}(K_0)$ are Nikolskii and Besov spaces, $d^{\mathcal{H}}(\Omega,\Omega^*)$ is the Hausdorf distance.

Let $\mathcal{U}$ be a certain open covering of $\partial\Omega$, and let any element $U \in \mathcal{U}$ have a fixed coordinate system. We say, that the boundary of Ω belongs to the space $C^{0,1}_{\mathcal{U}}$ if for any $U \in \mathcal{U}$ one can represent the set $\partial\Omega \cap U$ as a graph of a certain function $g \in C^{0,1}$ (with respect to the coordinates in U). In the same way, a function f belongs to the anisotropic space $C^{(s_1,\ldots,s_d)}_{\mathcal{U}}$ if $f|_U \in C^{(s_1,\ldots,s_d)}(U)$ for any $U \in \mathcal{U}$.

As a corollary of Theorem 2.1 and arguments from [10] we obtain the following generalization of SavarΓ©'s theorem [9].

Theorem 2.2. *Let* $\Omega \Subset K_0$, $\partial\Omega \in C^{0,1}_{\mathcal{U}}$, $\mathcal{A} = -\sum_{i,j=1}^d \partial_j a^{i,j}\partial_j$, $a^{ij} \in C^{(1,1/2,\ldots,1/2)}_{\mathcal{U}}$, *then for any* $s \in (0,1/2)$ *the solving operator* $\mathcal{R}_\Omega : H^{-1+s}(\Omega) \to \tilde{H}^{1+s}(\Omega)$ *is bounded.*

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