ISSN (Print): 2077-9879
ISSN (Online): 2617-2658

Eurasian Mathematical Journal

2023, Volume 14, Number 1

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia (RUDN University)
the University of Padua

Starting with 2018 co-funded
by the L.N. Gumilyov Eurasian National University
and
the Peoples' Friendship University of Russia (RUDN University)

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Astana, Kazakhstan

EURASIAN MATHEMATICAL JOURNAL

Aims and Scope

The Eurasian Mathematical Journal (EMJ) publishes carefully selected original research papers in all areas of mathematics written by mathematicians, principally from Europe and Asia. However papers by mathematicians from other continents are also welcome.

From time to time the EMJ publishes survey papers.

The EMJ publishes 4 issues in a year.

The language of the paper must be English only.

The contents of the EMJ are indexed in Scopus, Web of Science (ESCI), Mathematical Reviews, MathSciNet, Zentralblatt Math (ZMATH), Referativnyi Zhurnal – Matematika, Math-Net.Ru.

The EMJ is included in the list of journals recommended by the Committee for Control of Education and Science (Ministry of Education and Science of the Republic of Kazakhstan) and in the list of journals recommended by the Higher Attestation Commission (Ministry of Education and Science of the Russian Federation).

Information for the Authors

Submission. Manuscripts should be written in LaTeX and should be submitted electronically in DVI, PostScript or PDF format to the EMJ Editorial Office through the provided web interface (www.enu.kz).

When the paper is accepted, the authors will be asked to send the tex-file of the paper to the Editorial Office.

The author who submitted an article for publication will be considered as a corresponding author. Authors may nominate a member of the Editorial Board whom they consider appropriate for the article. However, assignment to that particular editor is not guaranteed.

Copyright. When the paper is accepted, the copyright is automatically transferred to the EMJ. Manuscripts are accepted for review on the understanding that the same work has not been already published (except in the form of an abstract), that it is not under consideration for publication elsewhere, and that it has been approved by all authors.

Title page. The title page should start with the title of the paper and authors' names (no degrees). It should contain the Keywords (no more than 10), the Subject Classification (AMS Mathematics Subject Classification (2010) with primary (and secondary) subject classification codes), and the Abstract (no more than 150 words with minimal use of mathematical symbols).

Figures. Figures should be prepared in a digital form which is suitable for direct reproduction.

References. Bibliographical references should be listed alphabetically at the end of the article. The authors should consult the Mathematical Reviews for the standard abbreviations of journals' names.

Authors' data. The authors' affiliations, addresses and e-mail addresses should be placed after the References.

Proofs. The authors will receive proofs only once. The late return of proofs may result in the paper being published in a later issue.

Offprints. The authors will receive offprints in electronic form.

Publication Ethics and Publication Malpractice

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the EMJ implies that the work described has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The EMJ follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http://publicationethics.org/files/u2/NewCode.pdf). To verify originality, your article may be checked by the originality detection service CrossCheck http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the EMJ.

The Editorial Board of the EMJ will monitor and safeguard publishing ethics.

The procedure of reviewing a manuscript, established by the Editorial Board of the Eurasian Mathematical Journal

1. **Reviewing procedure**

1.1. All research papers received by the Eurasian Mathematical Journal (EMJ) are subject to mandatory reviewing.

1.2. The Managing Editor of the journal determines whether a paper fits to the scope of the EMJ and satisfies the rules of writing papers for the EMJ, and directs it for a preliminary review to one of the Editors-in-chief who checks the scientific content of the manuscript and assigns a specialist for reviewing the manuscript.

1.3. Reviewers of manuscripts are selected from highly qualified scientists and specialists of the L.N. Gumilyov Eurasian National University (doctors of sciences, professors), other universities of the Republic of Kazakhstan and foreign countries. An author of a paper cannot be its reviewer.

1.4. Duration of reviewing in each case is determined by the Managing Editor aiming at creating conditions for the most rapid publication of the paper.

1.5. Reviewing is confidential. Information about a reviewer is anonymous to the authors and is available only for the Editorial Board and the Control Committee in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (CCFES). The author has the right to read the text of the review.

1.6. If required, the review is sent to the author by e-mail.

1.7. A positive review is not a sufficient basis for publication of the paper.

1.8. If a reviewer overall approves the paper, but has observations, the review is confidentially sent to the author. A revised version of the paper in which the comments of the reviewer are taken into account is sent to the same reviewer for additional reviewing.

1.9. In the case of a negative review the text of the review is confidentially sent to the author.

1.10. If the author sends a well reasoned response to the comments of the reviewer, the paper should be considered by a commission, consisting of three members of the Editorial Board.

1.11. The final decision on publication of the paper is made by the Editorial Board and is recorded in the minutes of the meeting of the Editorial Board.

1.12. After the paper is accepted for publication by the Editorial Board the Managing Editor informs the author about this and about the date of publication.

1.13. Originals reviews are stored in the Editorial Office for three years from the date of publication and are provided on request of the CCFES.

1.14. No fee for reviewing papers will be charged.

2. **Requirements for the content of a review**

2.1. In the title of a review there should be indicated the author(s) and the title of a paper.

2.2. A review should include a qualified analysis of the material of a paper, objective assessment and reasoned recommendations.

2.3. A review should cover the following topics:

- compliance of the paper with the scope of the EMJ;

- compliance of the title of the paper to its content;

- compliance of the paper to the rules of writing papers for the EMJ (abstract, key words and phrases, bibliography etc.);

- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);

- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

2.4. The final part of the review should contain an overall opinion of a reviewer on the paper and a clear recommendation on whether the paper can be published in the Eurasian Mathematical Journal, should be sent back to the author for revision or cannot be published.

Web-page

The web-page of the EMJ is www.emj.enu.kz. One can enter the web-page by typing Eurasian Mathematical Journal in any search engine (Google, Yandex, etc.). The archive of the web-page contains all papers published in the EMJ (free access).

Subscription

Subscription index of the EMJ 76090 via KAZPOST.

E-mail

eurasianmj@yandex.kz

The Eurasian Mathematical Journal (EMJ)
The Astana Editorial Office
The L.N. Gumilyov Eurasian National University
Building no. 3
Room 306a
Tel.: +7-7172-709500 extension 33312
13 Kazhymukan St
010008 Astana, Kazakhstan

The Moscow Editorial Office
The Peoples' Friendship University of Russia
(RUDN University)
Room 473
3 Ordzonikidze St
117198 Moscow, Russia

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 14, Number 1 (2023), 71 – 80

APPROXIMATE SOLUTIONS OF THE SWIFT-HOHENBERG EQUATION WITH DISPERSION

H. Rouhparvar

Communicated by T.V. Tararykova

Key words: differential transform method, reduced differential transform method, Swift-Hohenberg equation.

AMS Mathematics Subject Classification: 35K35, 65D99.

Abstract. In this paper, the initial and boundary value problems for the Swift-Hohenberg equation as over the finite spatial interval $x \in [0, l]$ and finite time interval $t \in [0, t^*]$ are considered. Approximate solutions for the initial and boundary value problems are obtained via the differential transform method and reduced differential transform method. Finally, several numerical examples are presented in order to demonstrate the effectivity of the methods and clarify the influence of the parameters on the solution.

DOI: https://doi.org/10.32523/2077-9879-2023-14-1-71-80

1 Introduction

The Swift-Hohenberg equation is a model pattern-forming equation which was introduced by Jack Swift and Pierre Hohenberg as a model for a fluid which is thermally convecting [24]. The Swift-Hohenberg equation is one of important equations for describing localized structures in the modern physics. This equation occurs in fluid dynamics, optical physics and other fields [4, 11, 22]. The Swift-Hohenberg equation with dispersion has the form [9]

$$u_t + 2u_{xx} - \sigma u_{xxx} + u_{xxxx} = \alpha u + \beta u^2 - \gamma u^3, \tag{1.1}$$

where α, β, γ and σ are parameters of the equation. At $\sigma = 0$ equation (1.1) is reduced to the standard Swift-Hohenberg equation. We consider the problem with the boundary conditions

$$\begin{aligned} &u = 0, \quad u_{xx} = 0, \text{ at } x = 0, l, \ \forall t, \, t > 0, \\ &u(x, 0) = u_0(x), \ \forall x, \ \ 0 < x < l, \end{aligned} \tag{1.2}$$

so that solutions can be extended as periodic functions over the real line. For $\sigma = \beta = 0$ and $\alpha = 1 - a$, $a \in \mathbb{R}$, equation (1.1) and (1.2) were solved by the homotopy analysis method in [3] and the differential transform method as time-fractional derivative in [19].

The aim of this paper is to find an approximate analytical solution of (1.1) and (1.2) with the help of powerful analytic methods. We use the differential transform method (DTM) and reduced differential transform method (RDTM) to obtain the solutions and compare them with each other. We know that the DTM is based on the use of Taylor series in all variables, while RDTM does not require Taylor series in all variables and therefore it reduces significantly the numerical computation. For the standard cases, comparing the methodology with some known techniques, shows that these approaches are effective and powerful.

2 Methods

In this section, the techniques are explained for the two-dimensional differential transform.

2.1 The DTM

The DTM was first proposed by Zhou [25], who solved linear and nonlinear initial value problems in electric circuit analysis, then was widely used in the literature and was successfully applied to fractional differential equations [5], integro-differential equations [6], higher-order initial value problems [1], systems of differential equations [2, 7, 12], partial differential equation [10, 13, 21, 23], high index differential-algebraic equations [20].

In [8, 14] the basic definitions and fundamental operations are introduced for the two-dimensional differential transform as the following

$$U(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}u(x,t)\right]_{(0,0)}, \tag{2.1}$$

where $u(x,t)$ is the original function and $U(k,h)$ is the transformed function. The differential inverse transform of $U(k,h)$ is of the form

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^kt^h, \tag{2.2}$$

and from equations (2.1) and (2.2) can be concluded that

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}u(x,t)\right]_{(0,0)}x^kt^h. \tag{2.3}$$

In Table 2.1 the fundamental mathematical operations of the two-dimensional differential transform are listed. The proofs are available in [8].

Table 2.1. Two-dimensional differential transformation

Original Function	**Transformed Function**
$u(x,t)\pm v(x,t)$	$U(k,h)\pm V(k,h)$
$cu(x,t)$	$cU(k,h)$
$\frac{\partial u(x,t)}{\partial x}$	$(k+1)U(k+1,h)$
$\frac{\partial u(x,t)}{\partial t}$	$(h+1)U(k,h+1)$
$\frac{\partial^{r+s}u(x,t)}{\partial x^r\partial t^s}$	$\frac{(k+r)!}{k!}\frac{(h+s)!}{h!}U(k+r,h+s)$
$u(x,t)v(x,t)$	$\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)V(k-r,s)$
$u(x,t)v(x,t)w(x,t)$	$\sum_{r=0}^{k}\sum_{s=0}^{k-r}\sum_{q=0}^{h}\sum_{p=0}^{h-q}U(r,h-q-p)V(s,q)W(k-r-s,p)$

2.2 The RDTM

The basic definitions and operations of the RDTM [15, 16, 17, 18] are defined as follows.

Definition 1. If a function $u(x,t)$ is analytic with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}, \tag{2.4}$$

where the t-dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase $u(x,t)$ represent the original function while the uppercase $U_k(x)$ stands for the transformed function.

Definition 2. The reduced differential transform of the sequence $\{U_k(x)\}_{k=0}^{\infty}$ is introduced as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k. \tag{2.5}$$

By combining equation (2.4) and (2.5), we have

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} t^k. \tag{2.6}$$

Some basic properties of the reduced differential transformation obtained from definitions (2.4) and (2.6) are summarized in Table 2.2. The proofs and the basic definitions of the RDTM are available in [15].

Table 2.2. Basic operations of RDTM

Original Function	Transformed Function
$u(x,t)$	$U_h(x)$
$u(x,t) \pm v(x,t)$	$U_h(x) \pm V_h(x)$
$cu(x,t)$	$cU_h(x)$ c is a cons.
$x^m t^n$	$x^m \delta(h-n)$
$x^m t^n u(x,t)$	$x^m U_{h-n}(x)$
$\frac{\partial}{\partial x}u(x,t)$	$U_h'(x)$
$\frac{\partial^r}{\partial t^r}u(x,t)$	$\frac{(h+r)!}{h!}U_{h+r}(x)$
$u(x,t)v(x,t)$	$\sum_{r=0}^{h} U_r(x)V_{h-r}(x)$
$u(x,t)v(x,t)w(x,t)$	$\sum_{r=0}^{h}\sum_{s=0}^{h-r} U_r(x)V_s(x)W_{h-r}(x)$

3 The Swift-Hohenberg equation

In this section, we consider two methodologies DTM and RDTM for the Swift-Hohenberg equation. To illustrate the capability, reliability and simplicity of the methods, several different cases for parameters of the equation will be discussed here.

3.1 Solution of the problem by the DTM

We apply the DTM to equation (1.1), the resulting transformed version of equation (1.1) is

$$\begin{aligned}(h+1)U(k,h+1) &= -2\tfrac{(k+2)!}{k!}U(k+2,h) + \sigma\tfrac{(k+3)!}{k!}U(k+3,h) - \tfrac{(k+4)!}{k!}U(k+4,h) \\ &\quad +\alpha U(k,h) + \beta \textstyle\sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)U(k-r,s) \\ &\quad -\gamma \textstyle\sum_{r=0}^{k}\sum_{s=0}^{k-r}\sum_{q=0}^{h}\sum_{p=0}^{h-q} U(r,h-q-p)U(s,q)U(k-r-s,p).\end{aligned} \tag{3.1}$$

From the boundary conditions given by (1.2), we have

$$\begin{aligned} &U(k,0)=\tfrac{1}{k!}u_0^{(k)}(0), \quad k=0,1,2,\dots\\ &U(0,h)=0, \quad h=0,1,2,\dots\\ &U(2,h)=0, \quad h=0,1,2,\dots\\ &\textstyle\sum_{k=0}^{\infty}U(k,h)l^k=0, \quad h=0,1,2,\dots\\ &\textstyle\sum_{k=0}^{\infty}\frac{(k+2)!}{k!}U(k+2,h)l^k=0, \quad h=0,1,2,\dots. \end{aligned} \tag{3.2}$$

In real applications, the function $u(x,t)$ is given by a finite series of equations (3.1) and (3.2) can be written as follows

$$u(x,t)\approx\tilde{u}(x,t)=\sum_{k=0}^{n-2h}\sum_{h=0}^{m}U(k,h)x^kt^h,$$

where the value of the parameter m should not be greater than $\frac{n}{2}$.

By using equations (3.1) and (3.2), the corresponding $U(k,h)$ can be calculated for arbitrary different selections of n and m. In real applications, we seek obtain an excellent approximate solution of the differential equation. Therefore the selection n and m i.e. iterations continue until the absolute value of the error function defined as follows

$$E_{DTM}(x,t)=|\tilde{u}_t+2\tilde{u}_{xx}-\sigma\tilde{u}_{xxx}+\tilde{u}_{xxxx}-\alpha\tilde{u}-\beta\tilde{u}^2+\gamma\tilde{u}^3|, \tag{3.3}$$

becomes very small for each x,t in the domain, in other words $|E_{DTM}(x,t)|<\textit{tolerance}$ for all $x\in[0,l]$, $t\in[0,t^*]$.

Then the corresponding $U(k,h)$ can be obtained as follows

$$\begin{aligned} &U(0,0)=u_0(0),U(1,0)=u_0'(0),\dots,U(n,0)=\tfrac{1}{n!}u_0^{(n)}(0),\dots,\\ &U(0,0)=0,U(0,1)=0,\dots,U(0,m)=0,\dots,\\ &U(2,0)=0,\ U(2,1)=0,\dots,U(2,m)=0,\dots. \end{aligned}$$

If $h=0$, then from (3.1) for $k=1$ *and* $k=3,\dots,n-4$ we have

$$\begin{aligned} U(k,1)=&-2\tfrac{(k+2)!}{k!}U(k+2,0)+\sigma\tfrac{(k+3)!}{k!}U(k+3,0)-\tfrac{(k+4)!}{k!}U(k+4,0)\\ &+\alpha U(k,0)+\beta\textstyle\sum_{r=0}^{k}U(r,0)U(k-r,0)\\ &-\gamma\textstyle\sum_{r=0}^{k}\sum_{s=0}^{k-r}U(r,0)U(s,0)U(k-r-s,0), \end{aligned}$$

and by the final two relations of (3.2) also can obtain

$$\begin{aligned} U(n-3,1)&=\tfrac{1}{l^{(n-3)}}\textstyle\sum_{k=0}^{n-4}l^kU(k,1),\\ U(n-2,1)&=\tfrac{1}{(n-3)(n-2)l^{(n-4)}}\textstyle\sum_{k=0}^{n-5}(k+1)(k+2)l^kU(k+2,1). \end{aligned}$$

If $h=1$, then for $k=1$ *and* $k=3,\dots,n-6$ we have

$$\begin{aligned} U(k,2)=&\tfrac{1}{2}(-2\tfrac{(k+2)!}{k!}U(k+2,1)+\sigma\tfrac{(k+3)!}{k!}U(k+3,1)-\tfrac{(k+4)!}{k!}U(k+4,1)\\ &+\alpha U(k,1)+\beta\textstyle\sum_{r=0}^{k}\sum_{s=0}^{1}U(r,1-s)U(k-r,s)\\ &-\gamma\textstyle\sum_{r=0}^{k}\sum_{s=0}^{k-r}\sum_{q=0}^{1}\sum_{p=0}^{1-q}U(r,1-q-p)U(s,q)U(k-r-s,p)), \end{aligned}$$

and

$$U(n-5,2)=\frac{1}{l^{(n-5)}}\sum_{k=0}^{n-6}l^kU(k,2),$$
$$U(n-4,2)=\frac{1}{(n-5)(n-4)l^{(n-6)}}\sum_{k=0}^{n-7}(k+1)(k+2)l^kU(k+2,2).$$

If $h=2$, then for $k=1$ *and* $k=3,\dots,n-8$ we have

$$\begin{aligned}U(k,3)=\tfrac{1}{3}(&-2\tfrac{(k+2)!}{k!}U(k+2,2)+\sigma\tfrac{(k+3)!}{k!}U(k+3,2)-\tfrac{(k+4)!}{k!}U(k+4,2)\\&+\alpha U(k,2)+\beta\textstyle\sum_{r=0}^{k}\sum_{s=0}^{2}U(r,2-s)U(k-r,s)\\&-\gamma\textstyle\sum_{r=0}^{k}\sum_{s=0}^{k-r}\sum_{q=0}^{2}\sum_{p=0}^{2-q}U(r,2-q-p)U(s,q)U(k-r-s,p)),\end{aligned}$$

and

$$U(n-7,3)=\frac{1}{l^{(n-7)}}\sum_{k=0}^{n-8}l^kU(k,3),$$
$$U(n-6,3)=\frac{1}{(n-7)(n-6)l^{(n-8)}}\sum_{k=0}^{n-9}(k+1)(k+2)l^kU(k+2,3).$$

By using the recursive scheme of equation (3.1) and conditions (3.2), the rest values of $U(k,h)$ can be obtained.

3.2 Solution of the problem by the RDTM

To solve equation (1.1) by the RDTM, we consider differential transformation of Table 2 and have

$$\begin{aligned}(h+1)U_{h+1}(x)=&-2U_h''(x)+\sigma U_h^{(3)}(x)-U_h^{(4)}(x)+\alpha U_h(x)+\\&\beta\textstyle\sum_{r=0}^{h}U_r(x)U_{h-r}(x)-\gamma\sum_{r=0}^{h}\sum_{s=0}^{h-r}U_r(x)U_s(x)U_{h-r}(x).\end{aligned}\tag{3.4}$$

We can obtain the initial and boundary conditions as follows

$$\begin{aligned}&U_0(x)=u_0(x),\\&U_h(0)=0,\quad h=0,1,\dots\\&U_h(l)=0,\quad h=0,1,\dots\\&U_h''(0)=0,\quad h=0,1,\dots\\&U_h''(l)=0,\quad h=0,1,\dots.\end{aligned}\tag{3.5}$$

By substituting (3.5) into (3.4) and by a straight forward iterative calculations, we obtain the all required values of $U_h(x)$. Therefore, the inverse transformation of the set of values $\{U_h(x)\}_{h=0}^m$ gives the approximate solution as

$$u(x,t)\approx\hat{u}(x,t)=\sum_{h=0}^{m}U_h(x)t^h.$$

Similarly to the previous case, let us consider the error functional for approximate solution

$$E_{RDTM}(x,t)=|\hat{u}_t+2\hat{u}_{xx}-\sigma\hat{u}_{xxx}+\hat{u}_{xxxx}-\alpha\hat{u}-\beta\hat{u}^2+\gamma\hat{u}^3|,\tag{3.6}$$

and the iterations continue until $|E_{RDTM}(x,t)|<\mathit{tolerance}$ for all $x\in[0,l]$, $t\in[0,t^*]$.

4 Numerical results and discussion

The convergence of the proposed methods will depend on $\alpha, \beta, \gamma, \sigma, l$ and on the number of terms employed in a series approximation. These methods consist in building a sequence of numerical approximations of $u(x,t)$ via the generated sequence. To find the solution of equation (1.1), an error analysis is performed. Here, $E_{DTM}(x,t)$ and $E_{RDTM}(x,t)$ show the error functions of the proposed method for fixed $n, m, \alpha, \beta, \gamma, \sigma$ and l.

To see the effects of the parameters on the solutions, we fix $u_0(x) = \frac{1}{10}sin(\frac{\pi x}{l})$ and $l = 10$, consider solutions $u(x,t)$ for various values of parameters. To avoid a three-dimensional plot, we plot two-dimensional cross sections. The qualitative properties of such solutions are displayed in figures 1, 3 and 5. A comparison of the figures allows one to see the influence of the parameters on the solution profiles.

A clear conclusion from the numerical results is that the DTM and RDTM provide highly accurate numerical solutions without the need for spatial discretizations in solving the Swift-Hohenberg equation.

Because of memory problem, we only increase the number of iterations until we achieve that the modulus of the error function is less than 0.05 (tolerance). The results show that in the memory problem and boundary conditions the DTM acts better than the RDTM, and in the number of iterations and careful of solutions the RDTM is better than the DTM.

Here, we take different values of the parameters to compare the results of DTM and RDTM in the form of two dimensional figures for each case, we would see that DTM and RDTM solutions are in excellent agreement.

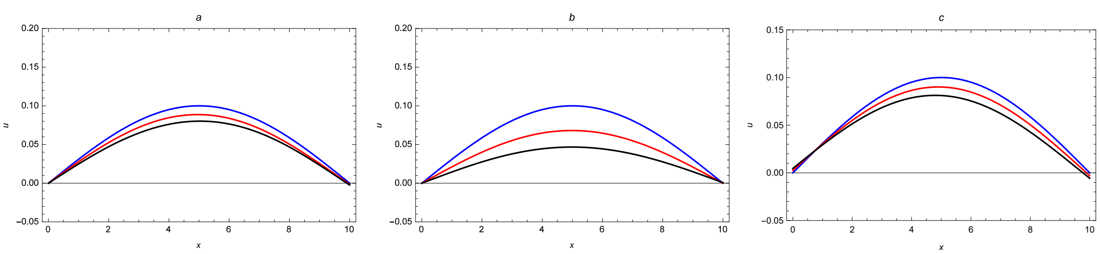

Fig. 1: (a) Profiles of $u(x,t)$ versus x at $\alpha = -0.3, \sigma = -1, \beta = 0.1$ and $\gamma = 0.2$ for $t = 0\,(Upper), 2\,(Middle), 4\,(Lower)$ with $n = 15$ and $m = 4$ by DTM. (b) Profiles of $u(x,t)$ versus x at $\alpha = -0.95, \sigma = -1, \beta = 0.1$ and $\gamma = 0.2$ for $t = 0\,(Upper), 2\,(Middle), 4\,(Lower)$ with $n = 20$ and $m = 5$ by DTM. (c) Profiles of $u(x,t)$ versus x at $\alpha = -0.3, \sigma = -1, \beta = 0.1$ and $\gamma = 0.2$ for $t = 0\,(Upper), 1\,(Middle), 2\,(Lower)$ with $m = 4$ by RDTM.

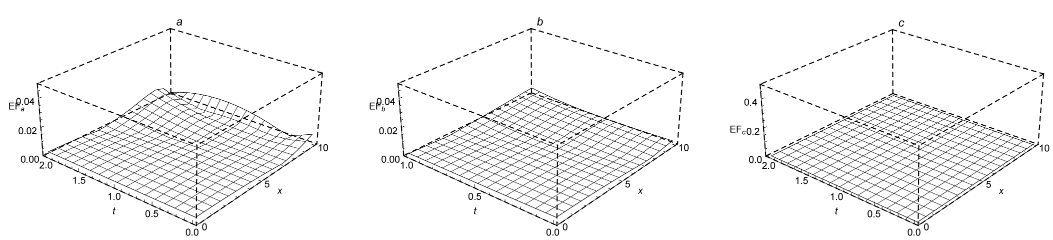

Fig. 2: (a) Profiles of $E_{DTM}(x,t)$ for Fig. 1 (a). (b) Profiles of $E_{DTM}(x,t)$ for Fig. 1 (b). (c) Profiles of $E_{RDTM}(x,t)$ for Fig. 1 (c).

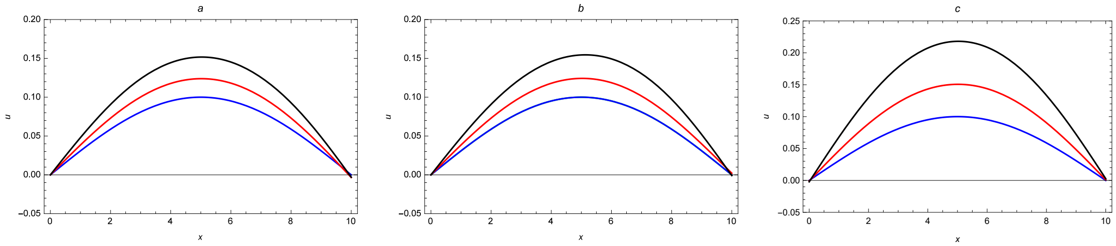

Fig. 3: (a) Profiles of $u(x,t)$ versus x at $\alpha = 0.25, \sigma = 0.2, \beta = -0.04$ and $\gamma = 1.1$ for $t = 0\,(Lower), 2\,(Middle), 4\,(Upper)$ with $n = 14$ and $m = 6$ by DTM.
(b) Profiles of $u(x,t)$ versus x at $\alpha = 0.25, \sigma = -0.15, \beta = -0.04$ and $\gamma = 1.1$ for $t = 0\,(Lower), 2\,(Middle), 4\,(Upper)$ with $n = 15$ and $m = 6$ by DTM.
(c) Profiles of $u(x,t)$ versus x at $\alpha = 0.25, \sigma = 0.2, \beta = -0.04$ and $\gamma = 1.1$ for $t = 0\,(Lower), 1\,(Middle), 2\,(Upper)$ with $m = 3$ by RDTM.

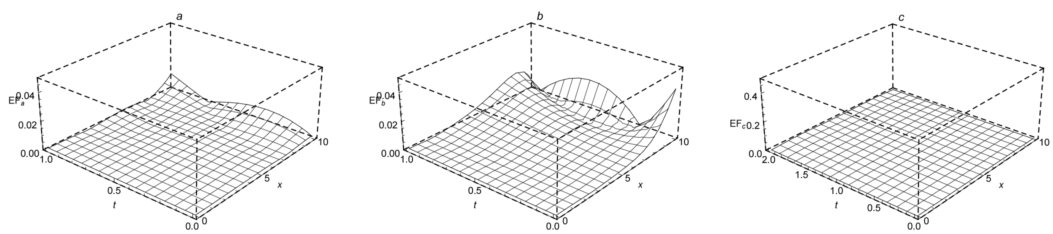

Fig. 4: (a) Profiles of $E_{DTM}(x,t)$ for Fig. 3 (a). (b) Profiles of $E_{DTM}(x,t)$ for Fig. 3 (b). (c) Profiles of $E_{RDTM}(x,t)$ for Fig. 3 (c).

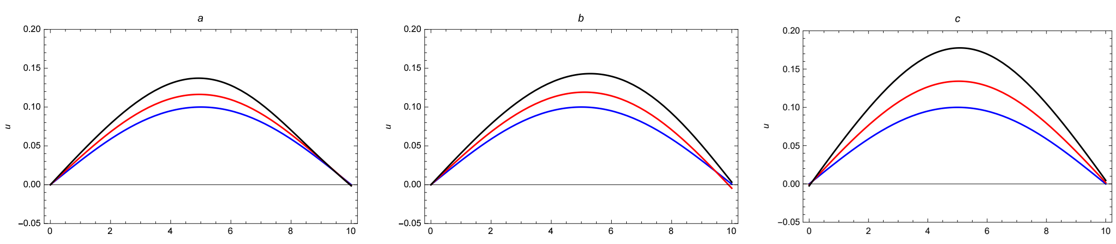

Fig. 5: (a) Profiles of $u(x,t)$ versus x at $\alpha = 0.1, \sigma = 0.4, \beta = 0.1$ and $\gamma = -2.3$ for $t = 0\,(Lower), 2\,(Middle), 4\,(Upper)$ with $n = 16$ and $m = 6$ by DTM.
(b) Profiles of $u(x,t)$ versus x at $\alpha = 0.1, \sigma = 0.4, \beta = -0.16$ and $\gamma = -2.3$ for $t = 0\,(Lower), 2\,(Middle), 4\,(Upper)$ with $n = 16$ and $m = 6$ by DTM.
(c) Profiles of $u(x,t)$ versus x at $\alpha = 0.1, \sigma = 0.4, \beta = -0.16$ and $\gamma = -2.3$ for $t = 0\,(Lower), 1\,(Middle), 2\,(Upper)$ with $m = 3$ by RDTM.

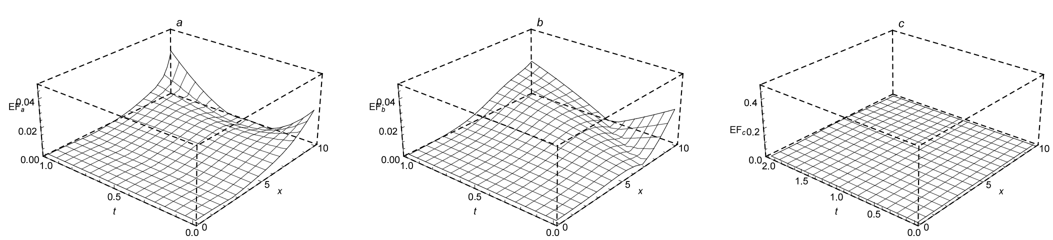

Fig. 6: (a) Profiles of $E_{DTM}(x,t)$ for Fig. 5 (a). (b) Profiles of $E_{DTM}(x,t)$ for Fig. 5 (b). (c) Profiles of $E_{RDTM}(x,t)$ for Fig. 5 (c).

5 Conclusion

Application of the DTM and RDTM to the Swift-Hohenberg equation with dispersion have been presented. The results show that the DTM and RDTM are powerful and efficient methods for finding analytic approximate solutions to the Swift-Hohenberg equation. Also, not many iterations are required to achieve fairly accurate solutions of the equation by the DTM and RDTM.

Acknowledgments

The author would like to express thanks to the referees for their valuable suggestions.

References

[1] I.H. Abdel-Halim Hassan, *Differential transformation technique for solving higher-order initial value problems.* Appl. Math. Comput. 154 (2004), 299-311.

[2] I.H. Abdel-Halim Hassan, *Application to differential transformation method for solving systems of differential equations.* Appl. Math. Modell. 32 (2008), no. 12, 2552-2559.

[3] F.T. Akyildiz, D.A. Siginer, K. Vajravelu, R.A. Van Gorder, *Analytical and numerical results for the SwiftвЂ"Hohenberg equation.* Appl. Math. Comput. 216 (2010), no. 1, 221-226.

[4] I.S. Aranson, K.A. Gorshkov, A.S. Lomov, M.I. Rabinovich, *Stable particle-like solutions of multidimensional nonlinear fields.* Phys. D 43 (1990), 435-453.

[5] A. Arikoglu, I. Ozkol, *Solution of fractional differential equations by using differential transform method.* Chaos, Solitons and Fractals 34 (2007), no. 5, 1473-1481.

[6] A. Arikoglu, I. Ozkol, *Solution of boundary value problems for integro-differential equations by using differential transform method.* Appl. Math. Comput. 168 (2005), 1145-1158.

[7] F. Ayaz, *Solutions of the system of differential equations by differential transform method.* Applied Mathematics and Computation 147 (2004), no. 2, 547-567.

[8] F. Ayaz, *On the two-dimensional differential transform method.* Appl. Math. Comput. 143 (2003), 361-374.

[9] J. Burke, S.M. Houghton, E. Knobloch, *Swift-Hohenberg equation with broken reflection symmetry.* Phys Rev E 80 (2009), 036202.

[10] S. Çatal, *Solution of free vibration equations of beamon elastic soil by using differential transform method.* Applied Mathematical Modelling 32 (2008), no. 9, 1744-1757.

[11] L.Yu, Glebsky, L.M. Lerman, *On small stationary localized solutions for the generalized 1-D Swift-Hohenberg equation.* CHAOS 5 (1995), 424-431.

[12] M.J. Jang, C.L. Chen, *Analysis of the response of a strongly nonlinear damped system using a differential transformation technique.* Appl. Math. Comput. 88 (1997), 137-151.

[13] M.J. Jang, C.L. Chen, Y.C. Liu, *Two-dimensional differential transform for partial differential equations.* Applied Mathematics and Computation 121 (2001), no. 2-3, 261-270.

[14] F. Kangalgil, F. Ayaz, *Solitary wave solutions for the KdV and mKdV equations by differential transform method.* Chaos Solitons & Fractals 41 (2009), 464-472.

[15] Y. Keskin, *Ph.D. Thesis.* Selcuk University, 2010 (in Turkish).

[16] Y. Keskin, G. Oturanç *Reduced differential transform method for partial differential equations.* Int. J. of Nonl. Sci. and Num. Simulation 10 (2009), no. 6, 741-749.

[17] Y. Keskin, G. Oturanç, *Reduced differential transform method for generalized KdV equations.* Math. & Computational Applications 15 (2010), no. 3, 382-393.

[18] Y. Keskin, G. Oturanç, *Reduced differential transform method for solving linear and nonlinear wave equations.* Iranian J. of Sci. & Tech. Transaction A 34 (2010), no. A2, 113-122.

[19] N.A. Khan, N.-U. Khan, M. Ayaz, A. Mahmood, *Analytical methods for solving the time-fractional Swift-Hohenberg (S-H) equation.* Computers and Math. with App. 61 (2011), 2182-2185.

[20] H. Liu, S. Yongzhong, *Differential transform method applied to high index differential algebraic equations.* Appl. Math. Comput. 184 (2007), 748-753.

[21] O.O. Ozdemir, M.O. Kaya, *Flapwise bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by differential transform method.* J. of Sound and Vibration 289 (2006), no. 1-2, 413-420.

[22] Y. Pomeau, S. Zaleski, *Dislocation motion in cellular structures.* J. Phys. 42 (1981), 2710-2726.

[23] H. Rouhparvar, *Computational technique of linear partial differential equations by reduced differential transform method.* Int. J. of Industrial Math. 8 (2016), no. 4, 339-346.

[24] J. Swift, P.C. Hohenberg, *Hydrodynamic fluctuations at the convective instability.* Phys. Rev. A 15 (1977), 319-328.

[25] J.K. Zhou, *Differential transformation and its applications for electrical circuIts.* Huazhong University Press, Wuhan, 1986 (in Chinese).

Hamid Rouhparvar
Department of Mathematics
College of Technical and Engineering
Saveh Branch, Islamic Azad University
Saveh, Iran
E-mail: rouhparvar59@gmail.com

Received: 09.12.2022