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SELECTION OF SLIGHTLY B^*-CONTINUOUS MULTIFUNCTIONS

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Key words: slight B^*-continuity, strongly lower slight B^*-continuous multifunction, categorically closed topological spaces, slightly B^*-minimal multifunction.

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Abstract. In the present paper, the existence of quasicontinuous selection for slightly B^*-continuous multifunctions is established.

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1 Introduction and preliminaries

In 1956, Michael [11] showed that a closed convex-valued continuous multifunction $F : X \to Y$ from a paracompact space X to a Banach space Y admits a continuous selection. After that several authors obtained continuous selections for nonconvex-valued multifunctions in Banach spaces, see e.g., [2] and references therein. Carbone [1] showed that not always continuous selection exists. In this direction, many authors have studied the problems of existence of various types of selections for generalized continuous multifunctions. Matejdes [9] proved the existence of a quasicontinuous selection for compact-valued multifunctions from a Baire space to a compact metric space. Later, Kupka [8] showed the existence of a quasicontinuous selection for finite-valued quasicontinuous multifunctions. In 2009, Ganguly and Mallick [4] proved the existence of a quasicontinuous selection for generlized continuous multifunctions defined on categorically closed topological spaces. Matejdes [10] also showed the existence of a quasicontinuous selection for compact-valued upper $\mathcal{E}$-continuous multifunctions. In 2014, Mitra and Ganguly [12] introduced the concept of a special type of a cluster system $\mathcal{E}_P$ and proved some important results related to lower and upper $\mathcal{E}_P$-continuous multifunctions which may fail to hold for $\mathcal{E}$-cluster system.

The aim, in this paper, is to obtain a quasicontinuous selection for slightly B^*-continuous multifunctions. The notions of slightly B^*-continuous functions and multifunctions are defined respectively, in [6] and [7].

Throughout the paper, X and Y will denote topological spaces, unless specified otherwise. By $\operatorname{int}(A)$ and $\operatorname{cl}(A)$, we shall denote the interior and closure of the set A.

By $F : X \to Y$, we shall mean that F is a multifunction with the domain X and the co-domain $P(Y) \setminus \emptyset$, the power set of Y excluding the empty set.

If $F : X \to Y$ is a multifunction then for $A \subset Y$, we denote

$$F^+(A) = \{x \in X : F(x) \subset A\}$$

and

$$F^-(A) = \{x \in X : F(x) \cap A \neq \emptyset\}.$$

A multifunction S is a sub-multifunction of F, if $S \subset F$, i.e., $S(x) \subset F(x)$ for all $x \in X$. By a selection of a multifunction $F : X \to Y$ we mean a single-valued function $f : X \to Y$ such that $f(x) \in F(x)$ for each $x \in X$.

A set B is said to be a B^*-set if it is not nowhere dense having the property of Baire [3].

A topological space is said to be categorically closed if every first category set is closed [5].

Proposition A. *[5] In a categorically closed topological space, the following hold:*

(*i*) *if G is open and P is of the first category, then $G \setminus P$ is open.*

(*ii*) *if E is a second category set with the Baire property, then E contains a nonempty open set.*

(*iii*) *if E is a B^*-set, then E contains a nonempty open set.*

A space X is said to be 0-dimensional, if each point of X has a neighborhood base of clopen sets, i.e., sets which are both open as well as closed. Equivalently, for each point $x \in X$ and each closed set G not containing x, there exists a clopen set containing x disjoint from G, see [13].

Definition 1. A multifunction $F : X \to Y$ is said to be

(a) upper slightly B^*-continuous at a point x, if for every open set $U \subset X$ containing x and for every clopen set V such that $F(x) \subset V$, there exists a B^*-set B such that

$$B \subset F^{+}(V) \cap U,$$

(b) lower slightly B^*-continuous at a point x, if for every open set $U \subset X$ containing x and for every clopen set V such that $F(x) \cap V \neq \emptyset$, there exists a B^*-set B such that

$$B \subset F^{-}(V) \cap U.$$

(c) slightly B^*-continuous if it is both upper and lower slightly B^*-continuous.

For $A \subseteq X$, the symbol $\mathcal{S}(F, A)$ will denote the set of all slightly B^*-continuous selections of the restricted multifunction $F|_A : A \to Y$.

2 Main results

We begin with the following:

Definition 2. A multifunction $F : X \to Y$ is said to be strongly lower slight B^*-continuous at a point x, if for every open set $U \subset X$ containing x and for any finite collection $\{V_1, V_2, ...V_n\}$ of clopen subsets of Y with $F(x) \cap V_i \neq \emptyset$, for all $i = 1, 2, \ldots n$, there exists a B^*-set $B \subset U$ containing x such that

$$B \subset F^{-}(V_1, V_2 \ldots V_n).$$

Here $F^{-}(V_1, V_2 \ldots V_n) = \{x \in X : F(x) \cap V_i \neq \emptyset, i = 1, 2, \ldots, n\}$.

In this section we provide the conditions when upper slightly B^*-continuous multifunction have a slightly B^*-continuous selection and also the conditions when strongly lower slight B^*-continuous multifunction will have slightly quasicontinuous or quasicontinuous selection.

First we prove the following:

Theorem 2.1. *Let X be a categorically closed space and $F : X \to Y$ be a mildly compact-valued upper slightly B^*-continuous multifunction. Let there exists an open dense subset U of X such that $\mathcal{S}(F, U) \neq \emptyset$. Then F has a slightly B^*-continuous selection on X.*

Proof. Let $g \in \mathcal{S}(F, U)$.
Claim: For each $z \in X \setminus U$ there exists an element $y_z \in F(z)$ such that for every open set G containing z in X and every clopen set V_y containing y_z, there exists a B^*-set B containing z such that $B \setminus \{z\} \subset G \cap U$ and $g(B) \subset V$.

Suppose that the above claim is not true. Then there exists some $z \in X \setminus U$ such that for all $y \in F(z)$ there exist an open set U_y containing z and a clopen set V_y containing y such that every B^*-set $B \subset U_y$ contains a point b with

$$g(b) \notin V_y,$$

which gives that

$$g(U_y \cap U) \cap V_y = \emptyset, \tag{2.1}$$

because, otherwise, there exists a B^*-set B_1 such that $B_1 \subset G \cap U$ and $g(B_1) \subset V_y$.

Now, the family $\{V_y : y \in F(z)\}$ forms a clopen cover for $F(z)$ and $F(z)$ being mildly compact can be covered by finitely many of them, say

$$F(z) \subset \bigcup_{i=1}^{m} V_{y_i}.$$

Since F is upper slightly B^*-continuous at z there exists a B^*-set $B'' \subset U_{y_1} \cap U_{y_2} \cap \ldots, \cap U_{y_m}$ such that

$$B'' \subseteq F^+\Big(\bigcup_{i=1}^{m} V_{y_i}\Big).$$

Let

$$S = U \cap U_1 \cap U_2 \cap \cdots \cap U_m \cap F^+\Big(\bigcup_{i=1}^{m} V_{y_i}\Big).$$

Since U is dense, in view of Proposition A, $S \neq \emptyset$ and $g(S) \subset \bigcup_{i=1}^{m} V_{y_i}$. This contradicts (2.1) so that the claim is established.

Now, the function $h : X \to Y$ defined by

$$h(z) = \begin{cases} g(z), & \text{if } z \in U \\ y_z, & z \in X \setminus U \text{ and } y_z \text{ is an element mentioned in the claim} \end{cases}$$

is a slightly B^*-continuous selection of F. □

Lemma 2.1. *Let Y be an ultra Hausdorff space and $F : X \to Y$ be a strongly lower slight B^*-continuous multifunction with* card$(F(x)) = n$ *for all $x \in X$. Then F is upper slightly B^*-continuous.*

Proof. Let $x \in X$, U be an open set containing x and V be a clopen set with $F(x) \subset V$. Let $F(x) = \{y_1, y_2, \ldots, y_n\}$. Since Y is ultra Hausdorff there exist pairwise disjoint clopen sets $V_1, V_2, \ldots, V_n$ such that $y_i \in V_i$ and $V_i \subset V$ for each $i = 1, 2, ..., n$. Since F is strongly lower slight B^*-continuous at x there exists a B^*-set $B \subset U$ containing x such that

$$B \subseteq F^-(V_1, V_2, \ldots, V_n).$$

Let $b \in B$. Then $F(b) \cap V_i \neq \emptyset$ for all $i = 1, 2, ..., n$. Since card$(F(b)) = n$ and $V_i \subseteq V$ for all $i = 1, 2, ..., n$, we have that $F(b) \subseteq V$. Hence F is upper slightly B^*-continuous at x. □

Lemma 2.2. *Let X be a categorically closed space and Y be ultra Hausdorff. Let $F : X \to Y$ be a strongly lower slight B^*-continuous multifunction with* card$(F(x)) = n$ *for all $x \in X$. Then for every nonempty open subset U of X, there exists a nonempty open subset W of U such that $\mathcal{S}(F, W) \neq \emptyset$.*

Proof. Let U be a nonempty open subset of X and $x \in U$. Let $F(x) = \{x_1, x_2, \ldots x_n\}$. Since Y is ultra Hausdorff, there exist clopen sets $V_1, V_2 \ldots V_n$ in Y which are pairwise disjoint and $x_i \in V_i$ for all $i = 1, 2, \ldots, n$. Since F is strongly lower slight B^*-continuous at $x \in X$, there exists a B^*-set $B \subset U$ such that

$$B \subset F^-(V_1, V_2, \ldots, V_n).$$

Then for each $i = 1, 2, \ldots, n$ and for all $b \in B$ we have

$$\text{card}(F(b) \cap V_i) = 1.$$

By Proposition A, the set B contains a nonempty open set W. Define $g : W \to Y$ such that

$$g(w) \in F(w) \cap V_1, \qquad w \in W.$$

Let $w \in W$, G be an open neighborhood of w in W and V be clopen in Y with $g(w) \in V$. Consider $V \cap V_1 = V_1'$, clopen in Y, containing $g(w)$, i.e., $F(w) \cap V_1' \neq \emptyset$. Since $F : X \to Y$ is strongly lower slight B^*-continuous at w, there exists a B^*-set $B' \subset G$ such that

$$B' \subset F^-(V_1', V_2, \ldots, V_n).$$

Since $\operatorname{card}(F(b)) = n$ for each $b \in B'$, $g(b) \in V_1' \subset V$ for each $b \in B'$. Hence $g \in \mathcal{S}(F, W)$ and we are done. □

Remark 1. Let $F : X \to Y$ be a multifunction such that for every nonempty open set W of X there exists an open subset D of W such that $\mathcal{S}(F, D) \neq \emptyset$. Then, as an immediate consequence of Lemma 2.2, we find that there exists a dense open subset U of X such that $\mathcal{S}(F, U) \neq \emptyset$.

In view of Theorem 2.1, Lemma 2.1, Lemma 2.2, Remark 1 and Proposition A, we find that a finite-valued, strongly lower slight B^*-continuous multifunction from a categorically closed space to an ultra Hausdorff space have a slightly B^*-continuous selection. In fact, it has a slightly quasi continuous selection. Precisely we have proved the following:

Theorem 2.2. *Let X be a categorically closed space, Y be ultra Hausdorff and n be a positive integer. Let $F : X \to Y$ be a strongly lower slight B^*-continuous multifunction such that $\operatorname{card}(F(x)) = n$ for each $x \in X$. Then F has a slightly quasicontinuous selection.*

A result similar to Theorem 2.2 was proved in [4] (see also [8]) where Y is Hausdorff and F is strongly lower $\mathcal{E}_p$ continuous. Those authors asserted that F admits a quasicontinuous selection on X. In our case, we obtain a weaker selection, namely, slightly quasicontinuous.

Remark 2. If Y is 0-dimensional, it is known (see [7]) that slight B^*-continuity is equivalent to B^*-continuity.

In view of Remark 2, we immediately have the following:

Corollary 2.1. *Let X be a categorically closed space, Y be ultra Hausdorff and 0-dimensional and n be a positive integer. Let $F : X \to Y$ be strongly lower slight B^*-continuous multifunction such that $\operatorname{card}(F(x)) = n$ for each $x \in X$. Then F has a quasicontinuous selection.*

Next, we prove the following:

Theorem 2.3. *Let X be a categorically closed space such that each open set is dense in X, Y be ultra Hausdorff and 0-dimensional. Let $F : X \to Y$ be strongly lower slight B^*-continuous such that for each $x \in X$, $F(x)$ is finite. Then F has a quasi continuous selection on X.*

Proof. For $i \in \mathbb{N}$, let B_i denote the set of all $x \in X$ such that there exists a B^*-set B containing x with $\operatorname{card}(F(b)) = i$ for all $b \in B$ and

$$L_i = \{x \in X : \operatorname{card}(F(x)) \leq i\}.$$

Note that B_i's are pairwise disjoint. Let $U = \bigcup_{i=1}^{\infty} B_i$ and $Z = X \setminus U$. For each positive integer i, define $Z_i = Z \cap L_i$. Then $Z = \bigcup_{i=1}^{\infty} Z_i$. We claim that Z is closed. Suppose, on the contrary, that Z is not closed. Then Z is not of first category. Let n be the first positive integer such that $\operatorname{int}(\operatorname{cl}(Z_n)) \neq \emptyset$. Let $H = \operatorname{int}(\operatorname{cl}(Z_n)) \setminus \operatorname{cl}(Z_{n-1})$. Then H is nonempty open and $H \subseteq \operatorname{int}(Z)$. It is clear that Z_n is dense in H. Since

$$H \cap B_n = \emptyset \qquad \text{and} \qquad H \cap Z_{n-1} = \emptyset,$$

there exists a point $h \in H$ such that $\text{card}(F(h)) = k$ and $k > n$. Let $F(h) = \{y_1, y_2, \ldots, y_k\}$. Since Y is ultra Hausdorff, there are pairwise disjoint clopen sets $V_1, V_2, ...V_k$ such that $y_i \in V_i$, $i = 1, 2, ...k$. Since F is strongly lower slight B^*-continuous at h, there exists a B^*-set $B \subseteq H$ such that

$$B \subset F^-(V_1, V_2, \ldots, V_k).$$

For any $b \in B$, $\text{card}(F(b)) \geq k > n$. Then $B \cap Z_n = \emptyset$. Since X is categorically closed, B contains a nonempty open set, say, W. Then $W \cap Z_n = \emptyset$. This contradicts the fact that Z_n is dense in H. Hence Z is closed and U is open. By Corollary 2.1, for every $i \in \mathbb{N}$ such that $B_i \neq \emptyset$, F has a quasicontinuous selection on B_i. Define $g : U \to Y$ by $g(t) = g_i(t)$ if and only if $t \in B_i$. Clearly, g is a selection of F on U. Let $x \in U$, U' be open subset of X containing x and V be clopen in Y such that $g(x) \in V$. Suppose $g(x) = g_i(x)$. Since g_i is quasi continuous at $x \in B_i$, there exists a nonempty open set G in X such that

$$G \subseteq U \cap U' \cap B_i$$

and $g_i(t) \in V$ for all $t \in G$. So g is quasicontinuous on U and by Theorem 2.1 and Remark 1 the assertion follows. □

For the final result of this paper, we require the following notion:

Definition 3. A multifunction $F : X \to Y$ is said to be slightly B^*-minimal at $x \in X$, if for every open set U containing x and for every clopen set V such that $V \cap F(x) \neq \emptyset$, there exists a B^*-set $B \subset U$ such that $F(b) \subset V$ for all $b \in B$.

Now we prove the following:

Theorem 2.4. *Let Y be Hausdorff and $F : X \to Y$ be compact valued upper slightly B^*-continuous multifunction. Then F has a slightly B^*-continuous selection.*

Proof. Let $\mathcal{M}$ be a family of all upper slightly B^*-continuous compact-valued sub-multifunction of F, which is partially ordered by inclusion. It is a nonempty family since $F \in \mathcal{M}$. For any linearly ordered subfamily $\mathcal{M}_0$ of $\mathcal{M}$, we define a multifunction

$$M_0(x) := \bigcap \Big\{ M(x) : M \in \mathcal{M}_0 \Big\}.$$

Then M_0 is a nonempty compact-valued sub-multifunction of F. Let U be an open set containing x and V be a clopen set such that $M_0(x) \subset V$. Then there exists some $M \in \mathcal{M}_0$ such that

$$M(x) \subset V.$$

From upper slight B^*-continuity of M, there exists a B^*-set B such that

$$x \in B \subset U \cap M^+(V).$$

Thus, for any $b \in B$ we have that $M_0(b) \subset M(b) \subset V$. This means M_0 is upper slightly B^*-continuous and using Zorn's lemma we find that $\mathcal{M}$ has a minimal element say M_m. We shall prove that M_m is slightly B^*-minimal. Let M_m be not slightly B^*-minimal at some $a \in X$. Then there exists an open set U containing a and a clopen set V with

$$M_m(a) \cap V \neq \emptyset,$$

such that every B^*-set B containing a in U has at least one point $b \in B$ such that $M_m(b) \not\subseteq V$. Define a multifunction

$$N(x) = \begin{cases} M_m(x) \cap (Y \setminus V), & \text{if } x \in U \\ M_m(x), & \text{otherwise.} \end{cases}$$

Then N is a compact-valued sub-multifunction of F. If we prove that N is upper slightly B^*-continuous, it would contradict the minimality of M_m and then M_m would be slightly B^*-minimal.

Now, if $x \notin U$ there is nothing to prove. Let $x \in U$, and $U_1 \subset U$ be an open set containing x and W be a clopen set such that $N(x) \subset W$. Then $M_m(x) \subset V \cup W$ and from the upper slight B^*-continuity of M_m it follows that there is a B^*-set $B \subset U_1$ containing x such that

$$M_m(b) \subset V \cup W \qquad \text{for all} \qquad b \in B.$$

This means $N(b) \subset W$ for all $b \in B$. Hence $N \in \mathcal{M}$, in particular, N is upper slightly B^*-continuous.

Now, we will show that M_m is single-valued. Otherwise, $M_m(x_1)$ contains at least two points for some $x_1 \in X$. Choose $y_1 \in M_m(x_1)$, and define a multifunction $G : X \to Y$ by

$$G(x) = \begin{cases} \{y_1\}, & \text{if } x = x_1 \\ M_m(x) & \text{otherwise.} \end{cases}$$

Clearly, G is compact-valued. Let U be an open set containing x_1 and W be clopen set such that $G(x_1) \subset W$. This means $M_m(x_1) \cap W \neq \emptyset$ and using the slight B^*-minimality of M_m, there exists a B^*-set $B \subset U$ such that

$$M_m(b) \subset W \qquad \text{for all} \quad b \in B \setminus \{x\}.$$

This implies that $G(b) \subset W$ for all $b \in B$. Hence $G \in \mathcal{M}$ which is a contradiction to the minimality of M_m and we are done. □

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References

[1] L. Carbone, *Selezioni continue in spazi non lineari e punti fissi*, Rend. Circ. Math. Palermo, 25 (1976), 101-115.

[2] A. Fryszkowski, *Continuous selection for a class of non-convex multivalued maps*, Sudia Mathematica, 76 (1983), 163-173.

[3] D.K. Ganguly, C. Mitra, *On some weaker forms of B^*-continuity for multifunctions*, Soochow J. Math., 3 (2006), 59–69.

[4] D.K. Ganguly, P. Mallick, *Quasi continuous selections for finite valued multifunctions*, Proceeding of IDOMS International Conference of Mathematics and its Applications, (2009), 113–118.

[5] D.K. Ganguly, P. Mallick, *Note on some classes of generalized continuous functions*, Bulletin of Pure and Applied Mathematics, 4 (2010), 184–188.

[6] P. Jain, C. Basu, V. Panwar, *On some generalized B^*-continuity, B^*-coverings and B^*-separations*, Eurasian Math. J., 10 (2019), 28–39.

[7] P. Jain, C. Basu, V. Panwar, *On B^*-clopen continuity, oscillation and convergence*, Tbilisi Math. J., 13 (2020), 129–140.

[8] I. Kupka, *Quasicontinuous selections for one-to-finite multifunctions*, Universities Comeniana Acta Mathematica Universitatis Comenianae, $LVIII - LIX$, (1992), 207–213.

[9] M. Matejdes, *Sur les selecteurs des multifonctions*, Math. Slovaca, 37 (1987), 111–124.

[10] M. Matejdes, *Selection theorems and mimimal mappings in a cluster setting*, Rocky Moun. J. Math., 41 (2011), 851–867.

[11] E. Michael, *Continuous selections I*, Annals of Mathematics, 63 (1956), 361–382.

[12] C. Mitra, D.K. Ganguly, *Some results on the $\mathcal{E}_P$-cluster system*, Bull. Cal. Math. Soc., 106 (2014), 453–458.

[13] S. Willard, *General topology*, Addison-Wesley, 1970.

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