ISSN (Print): 2077-9879
ISSN (Online): 2617-2658

Eurasian Mathematical Journal

2021, Volume 12, Number 2

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia (RUDN University)
the University of Padua

Starting with 2018 co-funded
by the L.N. Gumilyov Eurasian National University
and
the Peoples' Friendship University of Russia (RUDN University)

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Nur-Sultan, Kazakhstan

Aims and Scope

The Eurasian Mathematical Journal (EMJ) publishes carefully selected original research papers in all areas of mathematics written by mathematicians, principally from Europe and Asia. However papers by mathematicians from other continents are also welcome.

From time to time the EMJ publishes survey papers.

The EMJ publishes 4 issues in a year.

The language of the paper must be English only.

The contents of the EMJ are indexed in Scopus, Web of Science (ESCI), Mathematical Reviews, MathSciNet, Zentralblatt Math (ZMATH), Referativnyi Zhurnal – Matematika, Math-Net.Ru.

The EMJ is included in the list of journals recommended by the Committee for Control of Education and Science (Ministry of Education and Science of the Republic of Kazakhstan) and in the list of journals recommended by the Higher Attestation Commission (Ministry of Education and Science of the Russian Federation).

Information for the Authors

Submission. Manuscripts should be written in LaTeX and should be submitted electronically in DVI, PostScript or PDF format to the EMJ Editorial Office through the provided web interface (www.enu.kz).

When the paper is accepted, the authors will be asked to send the tex-file of the paper to the Editorial Office.

The author who submitted an article for publication will be considered as a corresponding author. Authors may nominate a member of the Editorial Board whom they consider appropriate for the article. However, assignment to that particular editor is not guaranteed.

Copyright. When the paper is accepted, the copyright is automatically transferred to the EMJ. Manuscripts are accepted for review on the understanding that the same work has not been already published (except in the form of an abstract), that it is not under consideration for publication elsewhere, and that it has been approved by all authors.

Title page. The title page should start with the title of the paper and authors' names (no degrees). It should contain the Keywords (no more than 10), the Subject Classification (AMS Mathematics Subject Classification (2010) with primary (and secondary) subject classification codes), and the Abstract (no more than 150 words with minimal use of mathematical symbols).

Figures. Figures should be prepared in a digital form which is suitable for direct reproduction.

References. Bibliographical references should be listed alphabetically at the end of the article. The authors should consult the Mathematical Reviews for the standard abbreviations of journals' names.

Authors' data. The authors' affiliations, addresses and e-mail addresses should be placed after the References.

Proofs. The authors will receive proofs only once. The late return of proofs may result in the paper being published in a later issue.

Offprints. The authors will receive offprints in electronic form.

Publication Ethics and Publication Malpractice

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the EMJ implies that the work described has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The EMJ follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http://publicationethics.org/files/u2/NewCode.pdf). To verify originality, your article may be checked by the originality detection service CrossCheck http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the EMJ.

The Editorial Board of the EMJ will monitor and safeguard publishing ethics.

The procedure of reviewing a manuscript, established by the Editorial Board of the Eurasian Mathematical Journal

1. **Reviewing procedure**

1.1. All research papers received by the Eurasian Mathematical Journal (EMJ) are subject to mandatory reviewing.

1.2. The Managing Editor of the journal determines whether a paper fits to the scope of the EMJ and satisfies the rules of writing papers for the EMJ, and directs it for a preliminary review to one of the Editors-in-chief who checks the scientific content of the manuscript and assigns a specialist for reviewing the manuscript.

1.3. Reviewers of manuscripts are selected from highly qualified scientists and specialists of the L.N. Gumilyov Eurasian National University (doctors of sciences, professors), other universities of the Republic of Kazakhstan and foreign countries. An author of a paper cannot be its reviewer.

1.4. Duration of reviewing in each case is determined by the Managing Editor aiming at creating conditions for the most rapid publication of the paper.

1.5. Reviewing is confidential. Information about a reviewer is anonymous to the authors and is available only for the Editorial Board and the Control Committee in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (CCFES). The author has the right to read the text of the review.

1.6. If required, the review is sent to the author by e-mail.

1.7. A positive review is not a sufficient basis for publication of the paper.

1.8. If a reviewer overall approves the paper, but has observations, the review is confidentially sent to the author. A revised version of the paper in which the comments of the reviewer are taken into account is sent to the same reviewer for additional reviewing.

1.9. In the case of a negative review the text of the review is confidentially sent to the author.

1.10. If the author sends a well reasoned response to the comments of the reviewer, the paper should be considered by a commission, consisting of three members of the Editorial Board.

1.11. The final decision on publication of the paper is made by the Editorial Board and is recorded in the minutes of the meeting of the Editorial Board.

1.12. After the paper is accepted for publication by the Editorial Board the Managing Editor informs the author about this and about the date of publication.

1.13. Originals reviews are stored in the Editorial Office for three years from the date of publication and are provided on request of the CCFES.

1.14. No fee for reviewing papers will be charged.

2. **Requirements for the content of a review**

2.1. In the title of a review there should be indicated the author(s) and the title of a paper.

2.2. A review should include a qualified analysis of the material of a paper, objective assessment and reasoned recommendations.

2.3. A review should cover the following topics:

- compliance of the paper with the scope of the EMJ;

- compliance of the title of the paper to its content;

- compliance of the paper to the rules of writing papers for the EMJ (abstract, key words and phrases, bibliography etc.);

- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);

- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

2.4. The final part of the review should contain an overall opinion of a reviewer on the paper and a clear recommendation on whether the paper can be published in the Eurasian Mathematical Journal, should be sent back to the author for revision or cannot be published.

Web-page

The web-page of the EMJ is www.emj.enu.kz. One can enter the web-page by typing Eurasian Mathematical Journal in any search engine (Google, Yandex, etc.). The archive of the web-page contains all papers published in the EMJ (free access).

Subscription

Subscription index of the EMJ 76090 via KAZPOST.

E-mail

eurasianmj@yandex.kz

The Eurasian Mathematical Journal (EMJ)
The Nur-Sultan Editorial Office
The L.N. Gumilyov Eurasian National University
Building no. 3
Room 306a
Tel.: +7-7172-709500 extension 33312
13 Kazhymukan St
010008 Nur-Sultan, Kazakhstan

The Moscow Editorial Office
The Peoples' Friendship University of Russia
(RUDN University)
Room 562
Tel.: +7-495-9550968
3 Ordzonikidze St
117198 Moscow, Russia

VLADIMIR MIKHAILOVICH FILIPPOV

(to the 70th birthday)

Vladimir Mikhailovich Filippov was born on 15 April 1951 in the city of Uryupinsk, Stalingrad Region of the USSR. In 1973 he graduated with honors from the Faculty of Physics and Mathematics and Natural Sciences of the Patrice Lumumba University of Peoples' Friendship in the specialty "Mathematics". In 1973-1975 he is a postgraduate student of the University; in 1976-1979 - Chairman of the Young Scientists' Council; in 1980-1987 - Head of the Research Department and the Scientific Department; in 1983-1984 - scientific work at the Free University of Brussels (Belgium); in 1985-2000 - Head of the Mathematical Analysis Department; from 2000 to the present - Head of the Comparative Educational Policy Department; in 1989–1993 - Dean of the Faculty of Physics, Mathematics and Natural Sciences; in 1993–1998 - Rector of the Peoples' Friendship University of Russia; in 1998-2004 - Minister of General and Professional Education, Minister of Education of the Russian Federation; in 2004-2005 - Assistant to the Chairman of the Government of the Russian Federation (in the field of education and culture); from 2005 to May 2020- Rector of the Peoples' Friendship University of Russia, since May 2020 - President of the Peoples' Friendship University of Russia, since 2013 - Chairman of the Higher Attestation Commission of the Ministry of Science and Higher Education of the Russian Federation.

In 1980, he defended his PhD thesis in the V.A. Steklov Mathematical Institute of Academy of Sciences of the USSR on specialty 01.01.01 - mathematical analysis (supervisor - a corresponding member of the Academy of Sciences of the USSR, Professor L.D. Kudryavtsev), and in 1986 in the same Institute he defended his doctoral thesis "Quasi-classical solutions of inverse problems of the calculus of variations in non-Eulerian classes of functionals and function spaces". In 1987, he was awarded the academic title of a professor.

V.M. Filippov is an academician of the Russian Academy of Education; a foreign member of the Ukrainian Academy of Pedagogical Sciences; President of the UNESCO International Organizing Committee for the World Conference on Higher Education (2007-2009); Vice-President of the Eurasian Association of Universities; a member of the Presidium of the Rectors' Council of Moscow and Moscow Region Universities, of the Governing Board of the Institute of Information Technologies in Education (UNESCO), of the Supervisory Board of the European Higher Education Center of UNESCO (Bucharest, Romania),

Research interests: variational methods; non-potential operators; inverse problems of the calculus of variations; function spaces.

In his Ph.D thesis, V.M. Filippov solved a long standing problem of constructing an integral extremal variational principle for the heat equation. In his further research he developed a general theory of constructing extremal variational principles for broad classes of differential equations with non-potential (in classical understanding) operators. He showed that all previous attempts to construct variational principles for non-potential operators "failed" because mathematicians and mechanics from the time of L. Euler and J. Lagrange were limited in their research by functionals of the type Euler - Lagrange. Extending the classes of functionals, V.M. Filippov introduced a new scale of function spaces that generalize the Sobolev spaces, and thus significantly expanded the scope of the variational methods. In 1984, famous physicist, a Nobel Prize winner I.R. Prigogine presented the report of V.M. Filippov to the Royal Academy of Sciences of Belgium. Results of V.M. Filippov's variational principles for non-potential operators are quite fully represented in some of his and his colleagues' monographs.

Honors: Honorary Legion (France), "Commander" (Belgium), Crown of the King (Belgium); in Russia - orders "Friendship", "Honor", "For Service to the Fatherland" III and IV degrees; Prize of the President of the Russian Federation in the field of education; Prize of the Governement of the Russian Federation in the field of education; Gratitude of the President of the Russian Federation; "For Merits in the Social and Labor Sphere of the Russian Federation", "For Merits in the Development of the Olympic Movement in Russia", "For Strengthening the Combat Commonwealth; and a number of other medals, prizes and awards.

He is an author of more than 270 scientific and scientific-methodical works, including 32 monographs, 2 of which were translated and published in the United States by the American Mathematical Society.

V.M. Filippov meets his 70th birthday in the prime of his life, and the Editorial Board of the Eurasian Mathematical Journal heartily congratulates him on his jubilee and wishes him good health, new successes in scientific and pedagogical activity, family well-being and long years of fruitful life.

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 12, Number 2 (2021), 82 – 89

ON THE CLOSURE OF STOCHASTIC DIFFERENTIAL EQUATIONS OF MOTION

M.I. Tleubergenov, G.T. Ibraeva

Communicated by K.N. Ospanov

Key words: inverse problems, stochastic differential equations, integral manifolds.

AMS Mathematics Subject Classification: 34K29, 60H10.

Abstract. The quasi-inversion method is used to obtain necessary and sufficient conditions for the solvability of the inverse closure problem in the class of stochastic differential Itô systems of the first-order with random perturbations from the class of processes with independent increments, with degeneration with respect to a part of variables and with given properties depending only on a part of variables.

DOI: https://doi.org/10.32523/2077-9879-2021-12-2-82-89

1 Introduction

The theory of inverse problems for differential systems goes back to works [3, 4] and has been further developed in [1, 8-10, 23-25] for ordinary differential equations (ODE). Thus, the set of ODE with a given integral curve was constructed in [3]. The cited paper is fundamental in the formation and development of the theory of inverse problems in the dynamics of systems described by ODE. The various statements of the inverse problems and their solving in the class of ODE are discussed in [1, 4, 8-10, 23-25].

In solving of inverse problems of construction of a set of differential equations corresponding to a given integral manifold, the following methods are used: the Erugin method and the quasi-inversion method. First, the Erugin method of introduction of an auxiliary function provides necessary and sufficient conditions ensuring for a given set to be an integral manifold [3, 4]. Secondly, the quasi-inversion method allows to write out the general solution of the functional-algebraic equation, to which the problem of construction of a set of the differential equations for a given integral manifold is reduced [9, 10].

Methods for solving inverse problems of differential systems are generalized to a wide class of partial differential equations in [2, 14, 15].

However, the increasing requirements to the accuracy of description and workability of material systems lead to the situation when numerous observed phenomena cannot be explained on the basis of the analysis of deterministic processes. This circumstance requires, in particular, the attraction of probability laws for the modeling of the behaviour of real systems.

Thus, the problem of generalization of the methods used for the solution of inverse problems for ordinary differential equations to the class of stochastic differential equations seems to be quite actual [7, 13].

The problem of the simultaneous construction of a set of stochastic differential equations with a given integral manifold and a set of comparison functions is investigated in [21]. And the problem of constructing a force field along a given integral manifold in the presence of random perturbing forces is solved in [22].

Stochastic differential equations of the Itô-type describe various models of mechanical systems taking into account the action of external random forces, for example: the motion of artificial satellites under the action of gravity and aerodynamic forces [12], the fluctuation drift of a heavy gyroscope in the gimbal suspension [16], and many others. In [5, 6, 17-20], the inverse problems of dynamics are studied under the additional assumptions of presence of random perturbations. In particular, the following problems are solved by the method of quasi-inversion:

(a) the main inverse problem of dynamics, in which it is necessary to construct the set of Itô-type second-order stochastic differential equations with a given integral manifold [5, 6];

(b) the problem of reconstruction of the equations of motion, in which it is necessary to construct the set of control parameters contained in a given system of Itô-type second-order stochastic differential equations according to a given integral manifold [17, 18, 20], and

(c) the problem of closure of the equations of motion, in which it is necessary to construct the set of closing Itô-type second-order stochastic differential equations for a given system of equations and a given integral manifold [19].

This paper is adjacent to [19]. In [19], the problem of construction the set of closing stochastic differential Itô-type equations according to the given system of equations and the given integral manifold is considered under the assumption that a given integral manifold depends on all variables. In the present paper, in contrast to [19], we suppose that a given integral manifold depends only on a part of variables.

For solving of inverse problems, we will use the method of quasi-inversion which is based on the following statement.

Lemma 1 [9, p. 12-13]. The set of all solutions of the linear system

$$Hv = g, \; H = (h_{\mu k}), \; v = (v_k) \; g = (g_\mu), \tag{1.1}$$

$\mu = \overline{1, m}$, $k = \overline{1, n}$, $m \leq n$, where the matrix H has rank m, is defined by the expression

$$v = sv^\tau + v^\nu, \tag{1.2}$$

here s is an arbitrary scalar value,

$$v^\tau = [HC] = [h_1 \dots h_m c_{m+1} \dots c_{n-1}] =$$

$$= \begin{vmatrix} e_1 & \dots & e_n \\ h_{11} & \dots & h_{n-1} \\ \dots & \dots & \dots \\ h_{m1} & \dots & h_{mn} \\ c_{m+1,1} & \dots & c_{m+1,n} \\ \dots & \dots & \dots \\ c_{n-1,1} & \dots & c_{n-1,n} \end{vmatrix}$$

is the vector product of vectors $h_\mu = (h_{\mu k})$ and arbitrary vectors $c_\rho = (c_{\rho k})$, $\rho = \overline{m+1, n-1}$; e_k are the unit vectors of the space R^n, $v^\tau = (v_k^\tau)$, where

$$v_k^\tau = \begin{vmatrix} 0 & \dots & 1 & \dots & 0 \\ h_{11} & \dots & h_{k,1} & \dots & h_{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ h_{m1} & \dots & h_{mk} & \dots & h_{mn} \\ c_{m+1,1} & \dots & c_{m+1,n} & \dots & c_{m+1,n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{n-1,1} & \dots & c_{n-1,k} & \dots & c_{n-1,n} \end{vmatrix},$$

$v^\nu = H^+g$, $H^+ = H^T(HH^T)^{-1}$, H^T is the transpose of H.

2 Statement of the general problem of constructing of closing stochastic differential equations and its solving

We assume that the set

$$\Lambda(t): \quad \lambda(y,v,t) = 0, \quad \lambda \in R^m, \tag{2.1}$$

$\lambda = \lambda(y,v,t) \in C^{222}_{y\,v\,t}$ and the system of the first-order Itô stochastic differential equations of the form

$$\begin{cases} \dot{y} = g_1(y,z,v,w,t), \\ \dot{z} = g_2(y,z,v,w,t) + \sigma_1(y,z,v,w,t)\dot{\xi}, \end{cases} \tag{2.2}$$

are given. It is necessary to construct a system of closing equations of the form

$$\begin{cases} \dot{v} = g_3(y,z,v,w,t), \\ \dot{w} = g_4(y,z,v,w,t) + \sigma_2(y,z,v,w,t)\dot{\xi} \end{cases} \tag{2.3}$$

such that set (2.1) is an integral manifold of system of equations (2.2), (2.3). Here $y \in R^{l_1}$, $z \in R^{l_2}$, $v \in R^{p_1}$, $w \in R^{p_2}$, $l_1 + l_2 + p_1 + p_2 = n$, σ_1 is a $(l_2 \times k)$ matrix, σ_2 is a $(p_2 \times k)$ matrix. The system of random processes with independent increments $\{\xi_1(t,\omega), ..., \xi_k(t,\omega)\}$, as in [11], can be represented as the sum of the processes: $\xi = \xi_0 + \int c(x)P^0(t,dx)$, where $\xi = (\xi_1^T, \ldots, \xi_k^T)^T$ is a vector process with independent increments, $\xi_0 = (\xi_{10}^T, \ldots, \xi_{k0}^T)^T$ is the vector Wiener process; P^0 is the Poisson process; $P^0(t,dx)$ is the number of jumps of the process P^0 in the interval $[0,t]$ into the set dx; $c(x)$ is a vector function mapping the space of $R^n \ni x$ into the space of values of R^k of the process $\xi(t)$ for any t.

We say that a function $g(x,t)$ belongs to the class K, $g \in K$, if g is continuous in t, $t \in [0,\infty]$, satisfies the Lipschitz condition with respect to x in the entire space $x = (y^T, z^T, \nu^T, w^T)^T \in R^n$, i.e.,

$$\|g(x,t) - g(\widetilde{x},t)\| \le M\|x - \widetilde{x}\|$$

and satisfies the condition of a linear growth

$$\|g(x,t)\| \le M(1 + \|x\|)$$

with a certain constant M.

It is assumed that the given vector functions g_1, g_2, the given matrix σ_1, and as well as the unknown vector functions g_3, g_4, the unknown matrix σ_2 belong to the class K, which guarantees the existence and uniqueness (up to the stochastic equivalence) of the solution $x(t)$ of system of equations (2.2), (2.3) with the initial condition $x(t_0) = x_0$ in any neighborhood of set (2.1).

In addition, we assume for solving this problem that $g_1 \in C^{12111}_{y\,z\,v\,w\,t}$, $g_3 \in C^{11211}_{y\,z\,v\,w\,t}$.

Previously, we sequentially calculate $\dot{\lambda}$, then $\ddot{\lambda}$ according to the rule of stochastic differentiation Itô [11, p. 204] of a complex function in the case of the process with independent increments

$$\dot{\lambda} = \frac{\partial \lambda}{\partial t} + \frac{\partial \lambda}{\partial y}\dot{y} + \frac{\partial \lambda}{\partial v}\dot{v}, \qquad \ddot{\lambda} = \frac{\partial^2 \lambda}{\partial t^2} + \frac{\partial^2 \lambda}{\partial t \partial y}\dot{y} + \frac{\partial^2 \lambda}{\partial t \partial v}\dot{v}.$$

$$+ \left(\frac{\partial^2 \lambda}{\partial t \partial y} + \dot{y}^T \frac{\partial^2 \lambda}{\partial y \partial y} + \dot{v}^T \frac{\partial^2 \lambda}{\partial y \partial v} \right) \dot{y} + \frac{\partial \lambda}{\partial y}\ddot{y}$$

$$+\left(\frac{\partial^2\lambda}{\partial t\partial v}+\dot{y}^T\frac{\partial^2\lambda}{\partial v\partial y}+\dot{v}^T\frac{\partial^2\lambda}{\partial v\partial v}\right)\dot{v}+\frac{\partial\lambda}{\partial v}\ddot{v}. \tag{2.4}$$

Let us write in more detail the expressions $\frac{\partial\lambda}{\partial y}\ddot{y}$ and $\frac{\partial\lambda}{\partial v}\ddot{v}$:

$$\frac{\partial\lambda}{\partial y}\ddot{y}=\frac{\partial\lambda}{\partial y}[\frac{\partial g_1}{\partial t}+\frac{\partial g_1}{\partial y}\dot{y}+\frac{\partial g_1}{\partial z}\dot{z}+\frac{\partial g_1}{\partial v}\dot{v}+\frac{\partial g_1}{\partial w}\dot{w}]=\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial t}+\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial y}g_1$$

$$+\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial z}(g_2+\sigma_1\dot{\xi})+\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial v}g_3+\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial w}(g_4+\sigma_2\dot{\xi})+\frac{\partial\lambda}{\partial y}(S^y_{1z}+S^y_{2z}+S^y_{3z}),$$

$$\frac{\partial\lambda}{\partial v}\ddot{v}=\frac{\partial\lambda}{\partial v}[\frac{\partial g_3}{\partial t}+\frac{\partial g_3}{\partial y}\dot{y}+\frac{\partial g_3}{\partial z}\dot{z}+\frac{\partial g_3}{\partial v}\dot{v}+\frac{\partial g_3}{\partial w}\dot{w}]=\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial t}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial y}g_1$$

$$+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial z}(g_2+\sigma_1\dot{\xi})+\frac{\partial\lambda}{\partial y}\frac{\partial g_3}{\partial v}g_3+\frac{\partial\lambda}{\partial y}\frac{\partial g_3}{\partial w}(g_4+\sigma_2\dot{\xi})+\frac{\partial\lambda}{\partial v}(S^v_{1w}+S^v_{2w}+S^v_{3w}),$$

where

$$S^y_{2z}=\int[g_1(y,z+\sigma_1c(x),v,w,t)-g_1(y,z,v,w,t)-\frac{\partial g_1(y,z,v,w,t)}{\partial z}\sigma_1c(x)]\nu_p(t,x)dx;$$

$$S^y_{3z}=\int[g_1(y,z-\sigma_1c(x),v,w,t)-g_1(y,z,v,w,t)]d\dot{P}^0(t,dx);$$

$$S^y_{1z}=\frac{1}{2}\frac{\partial^2 g_1}{\partial z\partial z}:\sigma_1\nu_0\sigma_1^T;\qquad S^v_{1w}=\frac{1}{2}\frac{\partial^2 g_3}{\partial v\partial v}:\sigma_2\nu_0\sigma_2^T;$$

$$S^v_{2w}=\int[g_3(y,z,v,w+\sigma_2c(x),t)-g_1(y,z,v,w,t)-\frac{\partial g_3(y,z,v,w,t)}{\partial w}\sigma_2c(x)]\nu_p(t,x)dx;$$

$$S^v_{3w}=\int[g_3(y,z,v,w-\sigma_2c(x),t)-g_3(y,z,v,w,t)]d\dot{P}^0(t,dx).$$

Here $\frac{1}{2}\frac{\partial^2 g_1}{\partial z\partial z}:D$, as in [11], stands for a vector whose elements are traces of the products of matrices of the second derivatives of the corresponding elements $g_{1\mu}(y,v,w,t)$ of the vector of $g_1(y,v,w,t)$ in the components of z for a matrix of D, where $D=\sigma_1\nu_0\sigma_1^T$.

Let us introduce the following notations:

$$G=\frac{\partial^2\lambda}{\partial t^2}+\frac{\partial^2\lambda}{\partial t\partial y}\dot{y}+\frac{\partial^2\lambda}{\partial t\partial v}\dot{v}+\left(\frac{\partial^2\lambda}{\partial t\partial y}+\dot{y}^T\frac{\partial^2\lambda}{\partial y\partial y}+\dot{v}^T\frac{\partial^2\lambda}{\partial y\partial v}\right)\dot{y}$$

$$+\left(\frac{\partial^2\lambda}{\partial t\partial v}+\dot{y}^T\frac{\partial^2\lambda}{\partial v\partial y}+\dot{v}^T\frac{\partial^2\lambda}{\partial v\partial v}\right)\dot{v}+\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial t}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial t}$$

$$+\left(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial y}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial y}\right)g_1+\left(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial z}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial z}\right)g_2+(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial v}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial v})g_3$$

$$+\frac{\partial\lambda}{\partial y}(S^y_{1z}+S^y_{2z}+S^y_{3z})+\frac{\partial\lambda}{\partial v}(S^v_{1w}+S^v_{2w}+S^v_{3w}),\qquad g=(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial w}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial w})g_4, \tag{2.5}$$

$$F=\left(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial z}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial z}\right)\sigma_1,\qquad f=\left(\frac{\partial\lambda}{\partial y}\frac{\partial g_1}{\partial w}+\frac{\partial\lambda}{\partial v}\frac{\partial g_3}{\partial w}\right)\sigma_2. \tag{2.6}$$

Then the equation of perturbed motion (2.4) we can rewrite using notation (2.5) and (2.6) in the form

$$\ddot{\lambda}=G+g+(F+f)\dot{\xi}. \tag{2.7}$$

Further, we will introduce the functions of Erugin [3]: m-dimensional vector function A and $(m \times k)-$matrix B having the property $A(0,0,y,z,v,w,t) \equiv 0$, $B(0,0,y,z,v,w,t) \equiv 0$, such that the following relation holds

$$\ddot{\lambda} = A(\lambda, \dot{\lambda}, y, z, v, w, t) + B(\lambda, \dot{\lambda}, y, z, v, w, t)\dot{\xi}, \tag{2.8}$$

where ξ is the same process with independent increments entering equation (2.7).

On the basis of equations (2.7) and (2.8) we come to the relations

$$\begin{cases} g = A - G, \\ f = B - F \end{cases}$$

or

$$\begin{cases} (\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial g_3}{\partial w})g_4 = A - G, \\ (\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial g_3}{\partial w})\sigma_2 = B - F, \end{cases} \tag{2.9}$$

from which we need to determine the control's vector functions g_3, g_4 and the matrix σ_2.

Let the vector function g_3 be any function $\varphi = \varphi(y,z,v,w,t)$ in the class $C^{1\,1\,1\,2\,1}_{y\,z\,v\,w\,t}$. Then we rewrite (2.9) in the form

$$\begin{gathered} \left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right) g_4 = A - G, \\ \left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right) \sigma_2 = B - F. \end{gathered} \tag{2.10}$$

According to formula (1.2) of Lemma 1 from relations (2.10), we define the unknown vector function g_4 and columns σ_{2i} of the matrix σ_2 in the form

$$g_4 = s_1 \left[\left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right) C \right] + \left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right)^+ (A - G), \tag{2.11}$$

$$\sigma_{2i} = s_2 \left[\left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right) C \right] + \left(\frac{\partial \lambda}{\partial y}\frac{\partial g_1}{\partial w} + \frac{\partial \lambda}{\partial v}\frac{\partial \varphi}{\partial w}\right)^+ B_i, \tag{2.12}$$

where B_i - i-th column of the matrix $B, i = \overline{1,k}$.

Consequently, we have the following statement.

Theorem. The system of Itô-type first-order stochastic differential equations (2.2) - (2.3) has the given integral manifold (2.1) if and only if the coefficients of the closing stochastic differential equations g_4 have form (2.11) for an arbitrary vector-function $g_3 = \varphi(y,z,v,w,t)$ of class $C^{1\,1\,1\,2\,1}_{y\,z\,v\,w\,t}$, and the columns σ_{2i} of the diffusion matrix σ_2 satisfy condition (2.12).

Conclusion

The posed inverse problem of closure of stochastic differential first order Itô equations by given properties of the motion is solved by the quasi-inversion method. The necessary and sufficient conditions for the solvability of this problem are obtained in the terms of coefficients of the closing equations. It is assumed that random perturbations are perturbations from the class of processes with independent increments.

Acknowledgments

The authors thank the unknown referee for comments and suggestions, which contributed to a better and correct presentation of the material of this paper. This publication is financially supported by a grant of the Ministry of Education and Science of Kazakhstan (grant number AP09258966).

References

[1] N.V. Abramov, R.G. Mukharlyamov, Zh.K. Kirgizbaev, *Control the dynamics of systems with program communications.* Monograph. - Nizhnevartovsk: Publishing House of Nizhnevartovsk. State. Univ. (2013)(in Russian).

[2] S.A. Budochkina, V.M. Savchin, *An operator equation with the second time derivative and Hamilton-admissible equations*, Doklady Mathematics 94 (2016), no. 2, 487-489.

[3] N.P. Erugin, *Construction of the entire set of system of differential equations with integral curve*, Prikl. mat. mech. 10 (1952), no. 6, 659-670.

[4] A.S. Galiullin, *Methods for the solution of inverse problems of dynamics*, Nauka, Moscow (1986) (in Russian).

[5] G.T. Ibraeva, M.I. Tleubergenov, *Main inverse problem for differential systems with degenerate diffusion*, Ukrainian Mathematical Journal 65 (2013), no. 5, 787-792.

[6] G.T. Ibraeva, M.I. Tleubergenov, *On the solvability of the main inverse problem for stochastic differential systems*, Ukrainian mathematical journal 71 (2019), no. 1, 157-165.

[7] R.Z. Khas'minskii, *Stability of systems of differential equations under random perturbations of their parameters*, Nauka, Moscow (1969)(in Russian).

[8] J. Llibre, R. Ramirez, *Inverse problems in ordinary differential equations and applications.* Springer International publishing switzerland, (2016).

[9] I.A. Mukhamedzyanov, R.G. Mukharlyamov, *Equations of program motions*, Moscow (1986)(in Russian).

[10] R.G. Mukharlyamov, *Simulation of control processes, stability and stabilization of systems with program contraints*, Journal of computer and systems sciences international 54 (2015), no.1, 13-26.

[11] V.S. Pugachev, I.N. Sinitsyn, *Stochastic differential systems, analysis and filtration*, Nauka, Moscow (1990) (in Russian).

[12] P. Sagirov, *Stochastic methods in the dynamics of satellites*, Mekhanika. Sbornik Perevodov, (1974), no. 5, 28-47, (1974), no. 6, 3-38.

[13] A.M. Samoilenko, O. Stanzhytskyi, *Qualitative and asymptotic analysis of differential equations with random perturbations*, World scientific, Singapore (2011).

[14] V.M. Savchin V.M., S.A. Budochkina, *Invariance of functionals and related Euler-Lagrange equations.* Russian Mathematics 61 (2017), no. 2, 49-54.

[15] V.M. Savchin, S.A. Budochkina, Y. Gondo, A.V. Slavko, *On the connection between first integrals, integral invariants and potentiality of evolutionary equations.* Eurasian Math. J. 9 (2018), no. 4, 82-90.

[16] I.N. Sinitsyn, *On the fluctuations of a gyroscope in gimbal suspension*, Izv. Akad. Nauk USSR, Mekh. Tverd. Tela (1976), no. 3, 23-31.

[17] M.I. Tleubergenov, G.T. Ibraeva, *Stochastic inverse problem with indirect control*, Differential equations 53 (2017), no. 10, 1418-1422.

[18] M.I. Tleubergenov, G.T. Ibraeva, *On the restoration problem with degenerated diffusion*, Turkic world mathematical society journal of pure and applied mathematics 6 (2015), no. 1, 93-99.

[19] M.I. Tleubergenov, G.T. Ibraeva, *On inverse problem of closure of differential systems with degenerate diffusion*, Eurasian mathematical journal 10 (2019), no. 2, 146-155.

[20] M.I. Tleubergenov, *On the inverse stochastic reconstruction problem*, Differential equations 50 (2014), no. 2, 274-278.

[21] M.I. Tleubergenov, G.K. Vassilina, *On stochastic inverse problem of construction of stable program motion*, Open Mathematics 19 (2021), no. 1, 157-162.

[22] M.I. Tleubergenov, G.K. Vassilina, G.A. Tuzelbaeva, *On construction of a field of forces along given trajectories in the presence of random perturbations*, Bulletin of Karaganda University. Mathematics series (2021), no. 1(101), 98-103.

[23] S.S. Zhumatov, *Asymptotic stability of impliciit differential systems in the vicinity of program manifold*, Ukrainian mathematical journal 66 (2014), no. 4, 625-632.

[24] S.S. Zhumatov, *Exponential stability of a program manifold of indirect control systems*, Ukrainian mathematical journal 62 (2010), no. 6, 907-915.

[25] S.S. Zhumatov, *Stability of a program manifold of control systems with locally quadratic relations*, Ukrainian mathematical journal 61 (2009), no. 3, 500-509.

Marat Idrisovich Tleubergenov
Department of Differential Equations
Institute of Mathematics and Mathematical Modelling
125 Pushkin St,
050010 Almaty, Kazakhstan
E-mails: marat207@mail.ru

Gulmira Temirgalikyzy Ibraeva
T. Begeldinov Aktobe Military Institute of Air Defense Forces
16 A. Moldagulova St,
030000 Aktobe, Kazakhstan
E-mail: gulmira_ibraeva@mail.ru

Received: 29.12.2019