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SHAVKAT ARIFJANOVICH ALIMOV

(to the 75th birthday)



Shavkat Arifjanovich Alimov was born on March 2, 1945 in the city of Nukus, Uzbekistan. In 1968, he graduated from the Department of Mathematics of Physical Faculty of the M.V. Lomonosov Moscow State University (MSU), receiving a diploma with honors. From 1968 to 1970, he was a post-graduate student in the same department under the supervision of Professor V.A. Il'in. He defended his PhD thesis in 1970. In May 1973, at the age of 28, he defended his doctoral thesis devoted to equations of mathematical physics. In 1973, for research on the spectral theory, he was awarded the highest youth prize of the USSR.

From 1974 to 1984, he worked as a professor in the Department of General Mathematics at the Faculty of Computational Mathematics and Cybernetics. In 1984, Sh.A. Alimov joined the Tashkent State University (TSU) as a professor. From 1985 to 1987 he worked as the Rector of the Samarkand State University, from 1987 to 1990 - the Rector of the TSU, from 1990 to 1992 - the Minister of Higher and Secondary Special Education of the Republic of Uzbekistan. From 1992 to 1994, he headed the Department of Mathematical Physics of the TSU.

After some years of diplomatic work, he continued his academic career as a professor of the Department of Mathematical Physics at the National University of Uzbekistan (NUU). From the first days of the opening of the Tashkent branch of the MSU in 2006, he worked as a professor in the Department of Applied Mathematics. From 2012 to 2017, he headed the Laboratory of Mathematical Modeling of the Malaysian Institute of Microelectronic Systems in Kuala Lumpur. From 2017 to 2019, he worked as a professor at the Department of Differential Equations and Mathematical Physics of the NUU. From 2019 to the present, Sh.A. Alimov is a Scientific Consultant at the Center for Intelligent Software Systems, and an adviser to the Rector of the NUU.

The main scientific activity of Sh.A. Alimov is connected with the spectral theory of partial differential equations and the theory of boundary value problems for equations of mathematical physics. He obtained series of remarkable results in these fields. They cover many important problems of the theory of Schrodinger equations with singular potentials, the theory of boundary control of the heat transfer process, the mathematical problems of peridynamics related to the theory of hypersingular integrals.

In 1984, Sh.A. Alimov was elected a corresponding member and in 2000 an academician of the Academy of Sciences of Uzbekistan. He was awarded several prestigious state prizes.

Sh.A. Alimov has over 150 published scientific and a large number of educational works. Among his pupils there are 10 doctors of sciences and more than 20 candidates of sciences (PhD) working at universities of Uzbekistan, Russia, USA, Finland, and Malaysia.

For about thirty years, Sh.A. Alimov has been actively involved in the reform of mathematical school education.

Sh.A. Alimov meets his 75th birthday in the prime of his life, and the Editorial Board of the Eurasian Mathematical Journal heartily congratulates him on his jubilee and wishes him good health, new successes in scientific and pedagogical activity, family well-being and long years of fruitful life.

ON TRIGONOMETRIC FOURIER SERIES
MULTIPLIERS IN $\lambda_{p,q}$ SPACES

N.T. Tleukhanova, A. Bakhyt

Communicated by V.I. Burenkov

Key words: trigonometric Fourier coefficients, Lorentz and Besov spaces, $M_{p_0, q_0}^{p_1, q_1}$ spaces.

AMS Mathematics Subject Classification: 42A16, 46E30, 47B37, 28A05.

Abstract. In this paper, we investigate the problem of trigonometric multipliers in the sequence spaces $\lambda_{p,q}$. Embedding theorems of the Lorentz and Besov spaces into the space of trigonometric Fourier series multipliers are formulated.

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1 Introduction

This work deals with the problem of trigonometric multipliers in the weight sequence spaces $\lambda_{p,q}$.

The classical definition of the problem of trigonometric multipliers can be formulated as follows: let $f \in L_1[0, 2\pi]$ and $a(f) = \{a_k(f)\}$ be its sequence of the Fourier coefficients via the trigonometric system. It is assumed that f is such that $a(f) = \{a_k(f)\} \in l_p, 1 \leq p \leq \infty$.

It is required to determine the smoothness and metric characteristics of a function ϕ so that $a(f \cdot g) = \{a_k(f \cdot \phi)\} \in l_p$, that is, to determine the conditions on the function ϕ that guarantee the following inequality holds

$$\|a(f \cdot g)\|_{l_p} \leq C_p \|a(f)\|_{l_p}, \tag{1.1}$$

where the constant C_p depends only on the p . The space of such functions is denoted by M_p . The norm in the space M_p is the smallest constant in inequality (1.1).

This problem was considered in the early works by S. Stechkin [11] and I. Hirschman [3]. The results were obtained in terms of the Hölder spaces C^α and bounded β -variation spaces V^β , namely: S. Stechkin proved the following embedding $V_1 \hookrightarrow M_p$, where V_1 is the space of all functions of bounded variation, $1 < p < \infty$ and I. Hirschman established that $C^\alpha \hookrightarrow M_p$ for $0 < \alpha \leq \frac{1}{2}$, $\left| \frac{1}{p} - \frac{1}{2} \right| < \alpha$ and $V_\beta \cap C^\alpha \hookrightarrow M_p$ for $\beta \geq 2$, $\alpha > 0$, $\left| \frac{1}{p} - \frac{1}{2} \right| < \frac{1}{\beta}$, where the symbol \hookrightarrow means continuous embedding.

These statements were extended by S. Edelstein [2] to multidimensional case in 1977. Further results related to trigonometric multipliers were given by M. Birman and M. Solomyak [1] in terms of the Sobolev spaces: if $1 < p < 2$, then

$$W_r^\alpha[0, 2\pi] \hookrightarrow M_p, \quad \alpha > \frac{1}{p} - \frac{1}{2}, \quad \frac{1}{r} = \frac{1}{p} - \frac{1}{2}.$$

G. Karadzhov [5] strengthened these results by using the Besov spaces: let $1 \leq p < 2$, then

$$B_{r,1}^{\frac{1}{p}} \hookrightarrow M_p, \quad \frac{1}{r} = \frac{1}{p} - \frac{1}{2}.$$

Our aim is to study the classes of multipliers in weighted spaces. Let $0 < p, q \leq \infty$. Let us introduce the weighted space of sequences

$$\lambda_{p,q} = \left\{ \xi = \{\xi_k\}_{k=-\infty}^{\infty} : \|\xi\|_{\lambda_{p,q}} = \left(\sum_{k \in \mathbb{Z}} |\xi_k|^q \bar{k}^{\frac{q}{p}-1} \right)^{\frac{1}{q}} < \infty \right.$$

if $q < \infty$ and

$$\left. \|\xi\|_{\lambda_{p,\infty}} = \sup_{k \in \mathbb{Z}} |\xi_k| \bar{k}^{\frac{1}{p}} < \infty \right\},$$

where $\bar{k} = \max(|k|, 1)$.

Let $f \in L_1[0, 1]$ and $a(f) = \{a_k(f)\}_{k=-\infty}^{\infty}$ be its sequence of the Fourier coefficients for the trigonometric system. We say that a function ϕ belongs to the space $M_{p_0, q_0}^{p_1, q_1}$, if the following inequality holds: for some $C > 0$

$$\|a(f \cdot \phi)\|_{\lambda_{p_1, q_1}} \leq C \|a(f)\|_{\lambda_{p_0, q_0}},$$

for all sequences $a(f)$ of the Fourier coefficients of functions $f \in L_1[0, 1]$.

This problem was studied in [4] for the scale of the Lorentz sequence spaces $l_{p,q}$.

The research methods are based on the methods developed in papers [6]-[8], which are devoted to the dual problem of Fourier series multipliers.

2 The space $M_{p_0, q_0}^{p_1, q_1}$

The introduced space coincides with the known space M_p in the case when $p_0 = q_0 = p_1 = q_1 = p$.

Let the parameters p_0, q_0, p_1, q_1 be such that,

$$\text{for } q_0 \leq q_1, \quad \frac{1}{p_0} - \frac{1}{p_1} \geq \frac{1}{q_0} - \frac{1}{q_1}$$

and

$$\text{for } q_0 > q_1, \quad \frac{1}{p_0} - \frac{1}{p_1} > 0,$$

since otherwise the space $M_{p_0, q_0}^{p_1, q_1}$ degenerates. Indeed, in order for a constant to belong to the space $M_{p_0, q_0}^{p_1, q_1}$, the following embedding $\lambda_{p_0, q_0} \hookrightarrow \lambda_{p_1, q_1}$ should hold, which is valid only under these conditions.

Let us formulate some properties the spaces $M_{p_0, q_0}^{p_1, q_1}$.

Proposition 2.1. *Let $1 < p_i, q_i \leq \infty, i = 0, 1, \frac{1}{p_i} + \frac{1}{p'_i} = \frac{1}{q_i} + \frac{1}{q'_i} = 1$, then*

$$M_{p_0, q_0}^{p_1, q_1} = M_{p'_1, q'_1}^{p'_0, q'_0}.$$

Proposition 2.2. *For the spaces $M_{p_0, q_0}^{p_1, q_1}$ the following scale of embeddings holds*

$$M_{p_0, q_0}^{p_1, q_1} \hookrightarrow M_{p_0(\theta), q_0(\theta)}^{p_1(\theta), q_1(\theta)},$$

where

$$\frac{1}{p_i(\theta)} = \frac{1}{p_i} + \theta \left(1 - \frac{1}{p_1} - \frac{1}{p_0} \right), \quad \frac{1}{q_i(\theta)} = \frac{1}{q_i} + \theta \left(1 - \frac{1}{q_1} - \frac{1}{q_0} \right).$$

3 Main result

In this section, we study embeddings of the Besov spaces $B_{r,s}^\alpha[0, 2\pi]$ and Lorentz spaces $L_{r,s}[0, 1]$ into the spaces $M_{p_0, q_0}^{p_1, q_1}$.

Let f be a measurable function on $[0, 1]$. A function

$$f^*(t) = \inf\{\sigma : \mu\{x \in \Omega : |f(x)| > \sigma\} \leq t\}$$

is called the non-increasing rearrangement of the function f .

The Lorentz space $L_{p,q}[0, 1]$, $0 < p < \infty$, is the set of all measurable functions f such that

$$\|f\|_{L_{p,q}} = \left(\int_0^1 (t^{\frac{1}{p}} f^*(t))^q \frac{dt}{t} \right)^{\frac{1}{q}} < \infty$$

for $1 \leq q < \infty$, and

$$\|f\|_{L_{p,\infty}} = \sup_{t>0} t^{\frac{1}{p}} f^*(t) < \infty$$

for $q = \infty$.

Let $\alpha > 0$, $1 < p < \infty$, $0 < q \leq \infty$. We define the space $B_{p,q}^\alpha[0, 1]$ as the set of all measurable functions f for which

$$\|f\|_{B_{p,q}^\alpha[0,1]} = \left(\sum_{k=0}^{\infty} (2^{\alpha k} \|\Delta_k f\|_{L_p[0,1]})^q \right)^{\frac{1}{q}} < \infty,$$

where $\Delta_k f = \sum_{2^{k-1} \leq |m| < 2^k} a_m(f) e^{2\pi i m x}$ if $k = 1, 2, \dots$, and $\Delta_0 f = f$.

For $q = \infty$ the values of $\left(\sum_{k \in \mathbb{Z}} |b_k|^q \right)^{\frac{1}{q}}$ are understood as $\sup_{k \in \mathbb{Z}} |b_k|$.

Theorem 3.1. *Let $1 < p_0 < 2 < p_1 < \infty$, $\frac{1}{r} = \frac{1}{p_0} - \frac{1}{p_1}$, $0 < q_1 \leq q_0 \leq \infty$, $\frac{1}{s} = \frac{1}{q_1} - \frac{1}{q_0}$. Then the embedding*

$$L_{r,s}[0, 1] \hookrightarrow M_{p_0,q_0}^{p_1,q_1}$$

holds if and only if $q_1 \leq p_1$, $p_0 \leq q_0$.

Theorem 3.2. *Let $0 < q_1 < q_0 < \infty$, $\frac{1}{s} = \frac{1}{q_1} - \frac{1}{q_0}$. Let one of the following conditions be fulfilled: either*

$$0 < q_1 \leq 1, \quad q_1 < p_0 < p_1 < \infty, \quad \frac{1}{q_1} + \frac{1}{p_1} = \frac{1}{t} + \frac{1}{p_0}$$

or

$$1 < q_1 \leq \infty, \quad 1 < p_0 < p_1 < \infty, \quad 1 + \frac{1}{p_1} = \frac{1}{p_0} + \frac{1}{t}.$$

If $s \leq 2$, then

$$B_{2,s}^\alpha[0, 1] \hookrightarrow M_{p_0,q_0}^{p_1,q_1},$$

where $\alpha = \frac{1}{t} - \frac{1}{2} > 0$.

If $s > 2$, then

$$B_{s',s}^\alpha \hookrightarrow M_{p_0,q_0}^{p_1,q_1},$$

where $\alpha = \frac{1}{t} - \frac{1}{s} > 0$, $s' = \frac{s}{s-1}$.

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References

- [1] M.Sh. Birman, M.Z. Solomjak, *Quantitative analysis in Sobolev's imbedding theorems and applications to spectral theory*. Proceedings of the Tenth Mathematical School (Summer School, Kaciveli/Nalchik, 1972), 5–189. Izдание Inst. Mat. Akad. Nauk Ukrain. SSR, Kiev, 1974 (in Russian).
- [2] S.L. Edel'shtein, *Bounded convolutions in $L_p(Z_m)$ and the smoothness of the symbol of the operator*. Math. Notes, 22(1977), no. 2, 978–984.
- [3] I.I. Hirshman, *On multiplier transformations*. Duke Math. J. 26 (1959), 221–242.
- [4] A. Jumabayeva, E. Smailov, N. Tleukhanova, *On spectral properties of the modified convolution operator*. J. Inequal. Appl. 2012 (2012), no. 146, 1–15.
- [5] G.E. Karadzhov, *Trigonometrical problems of multipliers*. In: Constructive Function Theory'81 (Varna, 1981). Publ. House Bulgar. Acad. Sci., Sofia, (1983), 82–86.
- [6] E.D. Nursultanov, N.T. Tleukhanova, *On multipliers of multiple Fourier series*. (in Russian) Tr. Mat. Inst. Steklova 227 (1999), 237–242 (in Russian); English translation in Proc. Steklov Inst. Math. 1999, no. 231–236.
- [7] E.D. Nursultanov, N.T. Tleukhanova, *Lower and upper bounds for the norm of multipliers of multiple trigonometric Fourier series in Lebesgue spaces*. Funktsional. Anal. i Prilozhen. 34 (2000), no. 2, 86–88 (in Russian); English translation in Funct. Anal. Appl. 34 (2000), no. 2, 151–153.
- [8] E. Nursultanov, L. Sarybekova, N. Tleukhanova, *Some new Fourier multiplier results of Lizorkin and Hörmander types*. Functional analysis in interdisciplinary applications, 58–82, Springer Proc. Math. Stat., 216, Springer, Cham, 2017.
- [9] L-E. Persson, L. Sarybekova, N. Tleukhanova, *A Lizorkin theorem on Fourier series multipliers for strong regular systems*. Analysis for science, engineering and beyond, 305–317, Springer Proc. Math., 6, Springer, Heidelberg, 2012.
- [10] L.O. Sarybekova, T.V. Tararykova, N.T. Tleukhanova, *On a generalization of the Lizorkin theorem on Fourier multipliers*. Math. Inequal. Appl. 13 (2010), no. 3, 613–624.
- [11] S.B. Stechkin, *About bilinear form*. Dokl. Akad. Nauk SSSR, 71 (1950), no. 3, 237–240 (in Russian).

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