ISSN (Print): 2077-9879
ISSN (Online): 2617-2658

Eurasian Mathematical Journal

2021, Volume 12, Number 1

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia (RUDN University)
the University of Padua

Starting with 2018 co-funded
by the L.N. Gumilyov Eurasian National University
and
the Peoples' Friendship University of Russia (RUDN University)

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Nur-Sultan, Kazakhstan

EURASIAN MATHEMATICAL JOURNAL

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SHAVKAT ARIFJANOVICH ALIMOV

(to the 75th birthday)

Shavkat Arifjanovich Alimov was born on March 2, 1945 in the city of Nukus, Uzbekistan. In 1968, he graduated from the Department of Mathematics of Physical Faculty of the M.V. Lomonosov Moscow State University (MSU), receiving a diploma with honors. From 1968 to 1970, he was a post-graduate student in the same department under the supervision of Professor V.A. Il'in. He defended his PhD thesis in 1970. In May 1973, at the age of 28, he defended his doctoral thesis devoted to equations of mathematical physics. In 1973, for research on the spectral theory, he was awarded the highest youth prize of the USSR.

From 1974 to 1984, he worked as a professor in the Department of General Mathematics at the Faculty of Computational Mathematics and Cybernetics. In 1984, Sh.A. Alimov joined the Tashkent State University (TSU) as a professor. From 1985 to 1987 he worked as the Rector of the Samarkand State University, from 1987 to 1990 - the Rector of the TSU, from 1990 to 1992 - the Minister of Higher and Secondary Special Education of the Republic of Uzbekistan. From 1992 to 1994, he headed the Department of Mathematical Physics of the TSU.

After some years of diplomatic work, he continued his academic career as a professor of the Department of Mathematical Physics at the National University of Uzbekistan (NUU). From the first days of the opening of the Tashkent branch of the MSU in 2006, he worked as a professor in the Department of Applied Mathematics. From 2012 to 2017, he headed the Laboratory of Mathematical Modeling of the Malaysian Institute of Microelectronic Systems in Kuala Lumpur. From 2017 to 2019, he worked as a professor at the Department of Differential Equations and Mathematical Physics of the NUU. From 2019 to the present, Sh.A. Alimov is a Scientific Consultant at the Center for Intelligent Software Systems, and an adviser to the Rector of the NUU.

The main scientific activity of Sh.A. Alimov is connected with the spectral theory of partial differential equations and the theory of boundary value problems for equations of mathematical physics. He obtained series of remarkable results in these fields. They cover many important problems of the theory of Schrodinger equations with singular potentials, the theory of boundary control of the heat transfer process, the mathematical problems of peridynamics related to the theory of hypersingular integrals.

In 1984, Sh.A. Alimov was elected a corresponding member and in 2000 an academician of the Academy of Sciences of Uzbekistan. He was awarded several prestigious state prizes.

Sh.A. Alimov has over 150 published scientific and a large number of educational works. Among his pupils there are 10 doctors of sciences and more than 20 candidates of sciences (PhD) working at universities of Uzbekistan, Russia, USA, Finland, and Malaysia.

For about thirty years, Sh.A. Alimov has been actively involved in the reform of mathematical school education.

Sh.A. Alimov meets his 75th birthday in the prime of his life, and the Editorial Board of the Eurasian Mathematical Journal heartily congratulates him on his jubilee and wishes him good health, new successes in scientific and pedagogical activity, family well-being and long years of fruitful life.

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 12, Number 1 (2021), 97 – 102

GLOBAL EXISTENCE THEOREMS OF A SOLUTION OF THE CAUCHY PROBLEM FOR SYSTEMS OF THE KINETIC CARLEMAN AND GODUNOV-SULTANGAZIN EQUATIONS

S.A. Dukhnovskii

Communicated by N.A. Bokayev

Key words: kinetic Carleman and Godunov-Sultangazin equations, Cauchy problem, rarefied gas.

AMS Mathematics Subject Classification: 35Q20, 35L45, 35L60.

Abstract. We consider the Cauchy problem for the one-dimensional systems of the kinetic Carleman and Godunov-Sultangazin equations with bounded energy and periodic initial data. We present theorems on the global existence of a solution of the Cauchy problem for the Carleman and Godunov-Sultangazin systems.

DOI: https://doi.org/10.32523/2077-9879-2021-12-1-97-102

1 Introduction

Kinetic theory considers gas as a combination of a huge number of chaotically moving particles in one way or another interacting with each other. As a result of such interactions, the particles exchange impulses and energy. Interaction can be carried out through direct collision of particles or with the help of certain forces. In the general case, discrete systems of kinetic equations have the form

$$\frac{\partial n_i}{\partial t} + (\vec{c}_i, \nabla n_i) = \sum_{k,l,j} \sigma_{kl}^{ij}(n_k n_l - n_i n_j), i = 1, ..., N, \tag{1.1}$$

where $\sigma_{kl}^{ij} = \sigma_{ij}^{kl}$ is a law of detailed balance which usually takes place for systems. Here the right-hand side is the collision integral in the discrete case, $n_i(t, x, y, z)$ is the density of gas particles with velocity $\vec{c}_i = (c_{x_i}, c_{y_i}, c_{z_i})$, constants σ_{kl}^{ij} are proportional to the probability that, as a result of the collision of two particles with velocities $\vec{c}_i$, $\vec{c}_j$, these particles acquire velocities $\vec{c}_k$, $\vec{c}_l$. From equation (1.1) systems of kinetic equations with a finite number of different groups of particles are obtained.

We consider the Cauchy problem for the kinetic Carleman [1–3, 6, 8, 10–12] and Godunov-Sultangazin [2, 4, 10, 11] systems:

$$\begin{gathered} \partial_t u + \partial_x u = -\frac{1}{\varepsilon}(u^2 - w^2), \quad x \in \mathbb{R},\ t > 0, \\ \partial_t w - \partial_x w = \frac{1}{\varepsilon}(u^2 - w^2), \\ u|_{t=0} = u^0,\ w|_{t=0} = w^0 \end{gathered} \tag{1.2}$$

and

$$\begin{aligned} \partial_t u + \partial_x u &= \frac{1}{\varepsilon}(v^2 - uw), \quad x \in \mathbb{R},\ t > 0, \\ \partial_t v &= -\frac{2}{\varepsilon}(v^2 - uw), \\ \partial_t w - \partial_x w &= \frac{1}{\varepsilon}(v^2 - uw), \\ u|_{t=0} = u^0,\ v|_{t=0} &= v^0,\ w|_{t=0} = w^0. \end{aligned} \tag{1.3}$$

Here initial data is periodic with the period 2π. The Carleman model is a system of two nonlinear partial differential equations. It describes a monatomic rarefied gas consisting of two groups of particles. Two groups of particles move along x-axis in the opposite directions with a unit velocity. It is worth noting that only particles within one group interact, that is, only with themselves, changing the direction of motion. For the Carleman model a proof of global solutions was conducted by O.V. Ilyin [3], T. Platkowski and R. Illner [6], V.V. Vedenyapin [12] and many other authors.

For any other discrete models close to the equilibrium results are known as the results of S. Kawashima [4], N.B. Maslova for the Boltzmann equation [5], E.V. Radkevich [7]. Unlike the Carleman model, the Godunov-Sultangazin model describes a gas consisting of three groups of particles moving along x-axis. One group consists of particles that are at rest, having zero velocity. Two other groups, like the Carleman system, consist of particles moving at a unit speed in the opposite directions.

2 Main result for Carleman system

We study Cauchy problem (1.2) for small perturbations of the equilibrium state $u_e^2 = w_e^2$, $u_e = w_e > 0$. We set

$$u = u_e + w_e^{1/2}\varepsilon^2\widehat{u},\ w = w_e + w_e^{1/2}\varepsilon^2\widehat{w}. \tag{2.1}$$

Then

$$\begin{aligned} \partial_t\widehat{u} + \partial_x\widehat{u} - 2w_e\frac{1}{\varepsilon}(\widehat{w} - \widehat{u}) &= -\varepsilon w_e^{1/2}(\widehat{u} + \widehat{w})(\widehat{u} - \widehat{w}), \quad x \in \mathbb{R},\ t > 0, \\ \partial_t\widehat{w} - \partial_x\widehat{w} + 2w_e\frac{1}{\varepsilon}(\widehat{w} - \widehat{u}) &= \varepsilon w_e^{1/2}(\widehat{u} + \widehat{w})(\widehat{u} - \widehat{w}), \\ \widehat{u}|_{t=0} = \widehat{u}^0, &\quad \widehat{w}|_{t=0} = \widehat{w}^0. \end{aligned} \tag{2.2}$$

For periodic perturbations

$$\widehat{u}(t,x) = u_0(t) + \sum_{k\in Z_0} u_k(t)e^{ikx}, \quad \widehat{w}(t,x) = w_0(t) + \sum_{k\in Z_0} w_k(t)e^{ikx},$$

$$Z_0 = \{k \in Z, k \neq 0\}$$

with zero averages

$$u_0^0 = \frac{1}{2\pi}\int_0^{2\pi} \widehat{u}^0(x)dx = w_0^0 = \frac{1}{2\pi}\int_0^{2\pi} \widehat{w}^0(x)dx = 0,$$

we introduce the weighted spaces $W^1_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)$, $L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)$, $\mathcal{H}_\sigma$ equipped with the norms:

$$\|\widehat{u}\|_{W^1_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)} = \|\frac{d}{dt}\widehat{u}\|_{L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)} + \|\widehat{u}\|_{L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)},$$

$$\|\widehat{u}\|^2_{L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)} = \int_0^\infty e^{2\gamma t}|u_0(t)|^2\,dt + \int_0^\infty e^{2\gamma t}\sum_{k\in Z_0}|k|^{2\sigma}|u_k(t)|^2\,dt,$$

$$|||\widehat{u}|_{t=0}|||^2_{\mathcal{H}_\sigma} = |u_0^0|^2 + \sum_{k\in Z_0}|k|^{2\sigma}|u_k^0|^2.$$

Let us formulate the main result for the kinetic Carleman system.

Theorem 2.1. *For any $\sigma > 2$ and $u_e = w_e > 0$ there exist $\mu_1, q \in (0,1)$ such that for periodic initial data $(\widehat{u}^0, \widehat{w}^0)$ with zero averages satisfying the inequality*

$$|||\widehat{u}^0|||_{\mathcal{H}_\sigma} + |||\widehat{w}^0|||_{\mathcal{H}_\sigma} \le \varepsilon^2 q,$$

there exists a global solution $(\widehat{u}, \widehat{w}) \in W^1_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)$ to Cauchy problem (2.2), where $\gamma = \varepsilon\mu_0 > 0, 0 < \mu_0 \le \frac{1-\mu_1^2}{8w_e}$. Hence, the local equilibrium principle with an exponential stabilization to the equilibrium state holds.

3 Main result for Godunov-Sultangazin system

We study Cauchy problem (1.3). We consider a neighbourhood of the equilibrium state $v_e^2 = w_e u_e$, $u_e, v_e, w_e > 0$. We set

$$u = u_e + u_e^{1/2}\varepsilon^2\widehat{u},\ v = v_e + v_e^{1/2}\varepsilon^2\widehat{v},\ w = w_e + w_e^{1/2}\varepsilon^2\widehat{w}, \tag{3.1}$$
$$\widehat{u}(t,x) = u_0(t) + \sum_{k\in Z_0} u_k(t)e^{ikx},\quad \widehat{w}(t,x) = w_0(t) + \sum_{k\in Z_0} w_k(t)e^{ikx},$$
$$\widehat{v}(t,x) = v_0(t) + \sum_{k\in Z_0} v_k(t)e^{ikx}.$$

Then we have

$$\begin{gathered}\partial_t\widehat{u} + \partial_x\widehat{u} - \frac{1}{\varepsilon}w_e^{1/2}\Big(2v_e^{1/2}\widehat{v} - u_e^{1/2}\widehat{w} - w_e^{1/2}\widehat{u}\Big) = \varepsilon\ w_e^{1/2}\Big(\widehat{v}^2 - \widehat{u}\widehat{w}\Big),\quad x\in\mathbb{R},\ t>0,\\ \partial_t\widehat{v} + \frac{2}{\varepsilon}v_e^{1/2}\Big(2v_e^{1/2}\widehat{v} - u_e^{1/2}\widehat{w} - w_e^{1/2}\widehat{u}\Big) = -2\varepsilon v_e^{1/2}\Big(\widehat{v}^2 - \widehat{u}\widehat{w}\Big),\\ \partial_t\widehat{w} - \partial_x\widehat{w} - \frac{1}{\varepsilon}u_e^{1/2}\Big(2v_e^{1/2}\widehat{v} - u_e^{1/2}\widehat{w} - w_e^{1/2}\widehat{u}\Big) = \varepsilon u_e^{1/2}\Big(\widehat{v}^2 - \widehat{u}\widehat{w}\Big),\\ \widehat{u}|_{t=0} = \widehat{u}^0,\ \widehat{v}|_{t=0} = \widehat{v}^0,\ \widehat{w}|_{t=0} = \widehat{w}^0.\end{gathered} \tag{3.2}$$

Theorem 3.1. *For any $\sigma > 2$ and $v_e^2 = u_e w_e > 0$ there exist $\mu_0, q \in (0,1)$ such that for periodic initial data $(\widehat{u}^0, \widehat{v}^0, \widehat{w}^0)$ with zero averages satisfying the inequality*

$$|||\widehat{u}^0|||_{\mathcal{H}_\sigma} + |||\widehat{v}^0|||_{\mathcal{H}_\sigma} + |||\widehat{w}^0|||_{\mathcal{H}_\sigma} \le \varepsilon^{3/4} q,$$

there exists a global solution $(\widehat{u}, \widehat{v}, \widehat{w}) \in W^1_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)$ to Cauchy problem (3.2), where $\gamma = \varepsilon\mu_0$. Hence, the local equilibrium principle with exponential stabilization to the equilibrium state holds.

It is proved (see [10]) that from (3.2) it follows that

$$\begin{gathered}u_k = (u_k^0 + \frac{1}{2}\frac{w_e^{1/2}}{v_e^{1/2}}v_k^0)e^{-ikt} - \frac{1}{2}\frac{w_e^{1/2}}{v_e^{1/2}}v_k + ik\frac{1}{2}\frac{w_e^{1/2}}{v_e^{1/2}}\int_0^t e^{ik(s-t)}v_k ds,\\ w_k = (w_k^0 + \frac{1}{2}\frac{u_e^{1/2}}{v_e^{1/2}}v_k^0)e^{ikt} - \frac{1}{2}\frac{u_e^{1/2}}{v_e^{1/2}}v_k - ik\frac{1}{2}\frac{u_e^{1/2}}{v_e^{1/2}}\int_0^t e^{-ik(s-t)}v_k ds,\\ v_k = v_k^0 e^{-\frac{1}{\varepsilon}L_e t} + y_k, L_e = 4v_e + u_e + w_e > 0, y_k \in L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)\end{gathered}$$

and there is the following finite-dimensional system of ordinary differential equations

$$\frac{d}{dt}y_k^{(m)}+\frac{1}{\varepsilon}L_e\ y_k^{(m)}+\frac{1}{\varepsilon}ik\int_0^t\Big(u_e e^{-ik(s-t)}-w_e e^{ik(s-t)}\Big)y_k^{(m)}ds \tag{3.3}$$
$$=2\frac{1}{\varepsilon}v_e^{1/2}\mathcal{D}^+_{k,m}e^{ikt}+2\frac{1}{\varepsilon}v_e^{1/2}\mathcal{D}^-_{k,m}e^{-ikt}+e^{-\frac{1}{\varepsilon}L_e t}(f_{k,L}^{(m)}(t)+f_{k,B}^{(m)}(t))$$
$$-\varepsilon v_e^{1/2}T_k^{add}(y^{(m)})-\varepsilon v_e^{1/2}\Big(\mathcal{L}_k^{(m)}(y^{(m)})+B_k^{(m)}(y^{(m)},y^{(m)})\Big),$$
$$y_k^{(m)}|_{t=0}=0,|k|\le m,k\in Z_0.$$

We look for a solution of (3.3) in the form

$$y_k^{(m)}=Q_k^{+,(m)}T_k^{-1}(e^{ikt})+Q_k^{-,(m)}T_k^{-1}(e^{-ikt})+T_k^{-1}(z_k^{(m)}),\quad z_k^{(m)}|_{t=0}=0,k\in Z_0,$$
$$Q^{-,(m)},Q^{+,(m)}\in\mathcal{H}_\sigma^{(m)},\quad z^{(m)}\in L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma),z^{(m)}=\{z_k^{(m)},|k|\le m\}.$$

If the conditions of secularity are satisfied

$$Q_k^{+,(m)}=2\frac{1}{\varepsilon}v_e^{1/2}\mathcal{D}^+_{k,m}-\varepsilon v_e^{1/2}S^+(Q^{-,(m)},Q^{+,(m)}),$$
$$Q_k^{-,(m)}=2\frac{1}{\varepsilon}v_e^{1/2}\mathcal{D}^-_{k,m}-\varepsilon v_e^{1/2}S^-(Q^{-,(m)},Q^{+,(m)}),\quad |k|=1,\dots,m,$$

we can write that

$$z_k^{(m)}=e^{-\frac{1}{\varepsilon}L_e t}(F_{k,L}^{(m)}(t)+F_{k,B}^{(m)}(t))-\varepsilon v_e^{1/2}\Big(H_{k,L}^{(m)}(t)+H_{k,B}^{(m)}(t)\Big)$$
$$-\varepsilon v_e^{1/2}\Big(\mathcal{U}_k^{(m)}(T^{-1}(z^{(m)}))+B_k^{(m)}(T^{-1}(z^{(m)}),T^{-1}(z^{(m)}))\Big)$$
$$-\varepsilon v_e^{1/2}T_k^{add}(y^{(m)}(z^{(m)},Q^{-,(m)},Q^{+,(m)})).$$

The equation for the zero mode in the Hilbert space $L_{2,\gamma}(\mathbb{R}_+)$, when

$$\mathcal{D}_0(Q^{+,(m)},Q^{-,(m)})=D_0-D_0^{(1)}=0,$$

has the form

$$z_0^{(m)}=-\frac{3}{2}\varepsilon v_e^{1/2}\int_0^t e^{\frac{1}{\varepsilon}L_e(s-t)}\Big[z_0^{(m)}\overline{z_0^{(m)}}$$
$$+\frac{2}{3}\Big(\mathcal{L}_0^{(1)}(T^{-1}(z^{(m)}))+B_0(T^{-1}(z^{(m)}),T^{-1}(z^{(m)}))+f_0^{(m)}(s)e^{-\frac{1}{\varepsilon}L_e s}+h_0^{(m)}(s)\Big)\Big]ds,$$

where $f_0^{(m)}(t)$ is a bounded function, $h_0^{(m)}\in L_{2,\gamma}(\mathbb{R})$,

$$\mathcal{L}_0^{(1)}(T^{-1}(z^{(m)}))=B_0(Z_Q^{+,(m)}+Z_Q^{-,(m)},T^{-1}(z^{(m)}))$$
$$+B_0(T^{-1}(z^{(m)}),Z_Q^{+,(m)}+Z_Q^{-,(m)})+\mathcal{L}_0(T^{-1}(z^{(m)})),$$

$$Z_Q^{-,(m)}=\{Q_k^{-,(m)}T_k^{-1}(e^{-ikt}),|k|\le m\},Z_Q^{+,(m)}=\{Q_k^{+,(m)}T_k^{-1}(e^{ikt}),|k|\le m\},$$

$$\mathcal{L}_0^{(m)}(T^{-1}(z^{(m)}))=L_0^{(m)}(T^{-1}(z^{(m)}))+B_0^{(m)}(v^0e^{-\frac{1}{\varepsilon}L_e t},T^{-1}(z^{(m)}))$$
$$+B_0^{(m)}(T^{-1}(z^{(m)}),v^0e^{-\frac{1}{\varepsilon}L_e t}),$$

$$B_0^{(m)}(T^{-1}(z^{(m)}), T^{-1}(z^{(m)})) =$$

$$= \sum_{k_1+k_2=0, |k_1|, |k_2|=1,\dots,m} \Big\{ T_{k_1}^{-1}(z_{k_1}^{(m)}) \overline{T_{k_2}^{-1}(z_{k_2}^{(m)})} + T_{k_2}^{-1}(z_{k_2}^{(m)}) \overline{T_{k_1}^{-1}(z_{k_1}^{(m)})}$$

$$-\frac{1}{4}\Big(-\frac{w_e^{1/2}}{v_e^{1/2}} T_{k_1}^{-1}(z_{k_1}^{(m)}) + ik_1 \frac{w_e^{1/2}}{v_e^{1/2}} \int_0^t e^{ik_1(s-t)} T_{k_1}^{-1}(z_{k_1}^{(m)}) ds \Big)$$

$$\times \overline{\Big(-\frac{u_e^{1/2}}{v_e^{1/2}} T_{k_2}^{-1}(z_{k_2}^{(m)}) - ik_2 \frac{u_e^{1/2}}{v_e^{1/2}} \int_0^t e^{-ik_2(s-t)} T_{k_2}^{-1}(z_{k_2}^{(m)}) ds \Big)}$$

$$-\frac{1}{4}\Big(-\frac{u_e^{1/2}}{v_e^{1/2}} T_{k_2}^{-1}(z_{k_2}^{(m)}) - ik_2 \frac{u_e^{1/2}}{v_e^{1/2}} \int_0^t e^{-ik_2(s-t)} T_{k_2}^{-1}(z_{k_2}^{(m)}) ds \Big)$$

$$\times \overline{\Big(-\frac{w_e^{1/2}}{v_e^{1/2}} T_{k_1}^{-1}(z_{k_1}^{(m)}) + ik_1 \frac{w_e^{1/2}}{v_e^{1/2}} \int_0^t e^{ik_1(s-t)} T_{k_1}^{-1}(z_{k_1}^{(m)}) ds \Big)} \Big\},$$

$$L_0^{(m)}(T^{-1}(z^{(m)})) =$$

$$= -\frac{1}{2} \sum_{k_1+k_2=0, |k_1|, |k_2|=1,\dots,m} \Big\{ \Big(-\frac{w_e^{1/2}}{v_e^{1/2}} T_{k_1}^{-1}(z_{k_1}^{(m)}) + ik_1 \frac{w_e^{1/2}}{v_e^{1/2}} \int_0^t e^{ik_1(s-t)} T_{k_1}^{-1}(z_{k_1}^{(m)}) ds \Big)$$

$$\times (\overline{w_{k_2}^0} + \frac{1}{2} \frac{u_e^{1/2}}{v_e^{1/2}} \overline{v_{k_2}^0}) e^{-ik_2 t}$$

$$+ (u_{k_1}^0 + \frac{1}{2} \frac{w_e^{1/2}}{v_e^{1/2}} v_{k_1}^0) e^{-ik_1 t} \overline{\Big(-\frac{u_e^{1/2}}{v_e^{1/2}} T_{k_2}^{-1}(z_{k_2}^{(m)}) - ik_2 \frac{u_e^{1/2}}{v_e^{1/2}} \int_0^t e^{-ik_2(s-t)} T_{k_2}^{-1}(z_{k_2}^{(m)}) ds \Big)}$$

$$+ \Big(-\frac{u_e^{1/2}}{v_e^{1/2}} T_{k_2}^{-1}(z_{k_2}^{(m)}) - ik_2 \frac{u_e^{1/2}}{v_e^{1/2}} \int_0^t e^{-ik_2(s-t)} T_{k_2}^{-1}(z_{k_2}^{(m)}) ds \Big) (\overline{u_{k_1}^0} + \frac{1}{2} \frac{w_e^{1/2}}{v_e^{1/2}} \overline{v_{k_1}^0}) e^{ik_1 t}$$

$$+ (w_{k_2}^0 + \frac{1}{2} \frac{u_e^{1/2}}{v_e^{1/2}} v_{k_2}^0) e^{ik_2 t} \overline{\Big(-\frac{w_e^{1/2}}{v_e^{1/2}} T_{k_1}^{-1}(z_{k_1}^{(m)}) + ik_1 \frac{w_e^{1/2}}{v_e^{1/2}} \int_0^t e^{ik_1(s-t)} T_{k_1}^{-1}(z_{k_1}^{(m)}) ds \Big)} \Big\}.$$

4 Conclusions

In general, there is no analytic solution for the Carleman and Godunov-Sultangazin systems. Numerical analysis of the Carleman and Godunov-Sultangazin systems was carried out by O.A. Vasil'eva [10, 9].

Acknowledgements

The author would like to thank Professor E.V. Radkevich and Assistant Professor O.A. Vasil'eva for a formulation of the problem and helpful discussions.

References

[1] S.A. Dukhnovskii, *On a speed of solutions stabilization of the Cauchy problem for the Carleman equation with periodic initial data*, Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.-Mat. Nauki. 21 (2017), no. 1, 7–41 (In Russian).

[2] S.K. Godunov, U.M. Sultangazin, *On discrete models of the kinetic Boltzmann equation*, Russ. Math. Surv. 26 (1971), no. 3, 1–56.

[3] O.V. Ilyin, *Investigation of the existence of solutions and of the stability of the Carleman kinetic system*, Comput. Math. Math. Phys. 47 (2007), no. 12, 1990–2001.

[4] S. Kawashima, *The asymptotic equivalence of the Broadwell model equation and its Navier–Stokes model equation*, Japan. J. Math. 7 (1981), no. 1, 1–43.

[5] N.B. Maslova, *Mathematical methods for studying the Boltzmann equation*, St. Petersburg Math. J. 3 (1992), no. 1, 1–43.

[6] T. Platkowski, R. Illner, *Discrete velocity models of the Boltzmann equation: A survey on the mathematical aspects of the theory*, SIAM Review 30 (1988), no. 2, 213–255.

[7] E.V. Radkevich, *On the large-time behavior of solutions to the Cauchy problem for a 2–dimensional discrete kinetic equation*, J. Math. Sci. 202 (2014), no. 5, 735–768.

[8] E.V. Radkevich, O.A. Vasil'eva, S.A. Dukhnovskii, *Local equilibrium of the Carleman equation*, J. Math. Sci. 207 (2015), no. 2, 296–323.

[9] E.V. Radkevich, O.A. Vasil'eva, *Generation of chaotic dynamics and local equilibrium for the Carleman equation*, J. Math. Sci. 224 (2017), no. 5, 763–795.

[10] O.A. Vasil'eva, S.A. Dukhnovskii, E.V. Radkevich, *On the nature of local equilibrium in the Carleman and Godunov-Sultangazin equations*, J. Math. Sci. 235 (2018), no. 4, 393–453.

[11] V. Vedenyapin, A. Sinitsyn, E. Dulov, *Kinetic Boltzmann, Vlasov and related equations*, Amsterdam, Elsevier, 2011.

[12] V.V. Vedenyapin, *On global solvability of the Cauchy problem for some discrete models of Boltzmann's equation*, Sov. Math., Dokl. 15 (1974), 398–401.

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Received: 03.06.2018
Revised: 12.12.2020