ISSN (Print): 2077-9879
ISSN (Online): 2617-2658

Eurasian Mathematical Journal

2021, Volume 12, Number 1

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia (RUDN University)
the University of Padua

Starting with 2018 co-funded
by the L.N. Gumilyov Eurasian National University
and
the Peoples' Friendship University of Russia (RUDN University)

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Nur-Sultan, Kazakhstan

EURASIAN MATHEMATICAL JOURNAL

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The Nur-Sultan Editorial Office
The L.N. Gumilyov Eurasian National University
Building no. 3
Room 306a
Tel.: +7-7172-709500 extension 33312
13 Kazhymukan St
010008 Nur-Sultan, Kazakhstan

The Moscow Editorial Office
The Peoples' Friendship University of Russia
(RUDN University)
Room 562
Tel.: +7-495-9550968
3 Ordzonikidze St
117198 Moscow, Russia

SHAVKAT ARIFJANOVICH ALIMOV

(to the 75th birthday)

Shavkat Arifjanovich Alimov was born on March 2, 1945 in the city of Nukus, Uzbekistan. In 1968, he graduated from the Department of Mathematics of Physical Faculty of the M.V. Lomonosov Moscow State University (MSU), receiving a diploma with honors. From 1968 to 1970, he was a post-graduate student in the same department under the supervision of Professor V.A. Il'in. He defended his PhD thesis in 1970. In May 1973, at the age of 28, he defended his doctoral thesis devoted to equations of mathematical physics. In 1973, for research on the spectral theory, he was awarded the highest youth prize of the USSR.

From 1974 to 1984, he worked as a professor in the Department of General Mathematics at the Faculty of Computational Mathematics and Cybernetics. In 1984, Sh.A. Alimov joined the Tashkent State University (TSU) as a professor. From 1985 to 1987 he worked as the Rector of the Samarkand State University, from 1987 to 1990 - the Rector of the TSU, from 1990 to 1992 - the Minister of Higher and Secondary Special Education of the Republic of Uzbekistan. From 1992 to 1994, he headed the Department of Mathematical Physics of the TSU.

After some years of diplomatic work, he continued his academic career as a professor of the Department of Mathematical Physics at the National University of Uzbekistan (NUU). From the first days of the opening of the Tashkent branch of the MSU in 2006, he worked as a professor in the Department of Applied Mathematics. From 2012 to 2017, he headed the Laboratory of Mathematical Modeling of the Malaysian Institute of Microelectronic Systems in Kuala Lumpur. From 2017 to 2019, he worked as a professor at the Department of Differential Equations and Mathematical Physics of the NUU. From 2019 to the present, Sh.A. Alimov is a Scientific Consultant at the Center for Intelligent Software Systems, and an adviser to the Rector of the NUU.

The main scientific activity of Sh.A. Alimov is connected with the spectral theory of partial differential equations and the theory of boundary value problems for equations of mathematical physics. He obtained series of remarkable results in these fields. They cover many important problems of the theory of Schrodinger equations with singular potentials, the theory of boundary control of the heat transfer process, the mathematical problems of peridynamics related to the theory of hypersingular integrals.

In 1984, Sh.A. Alimov was elected a corresponding member and in 2000 an academician of the Academy of Sciences of Uzbekistan. He was awarded several prestigious state prizes.

Sh.A. Alimov has over 150 published scientific and a large number of educational works. Among his pupils there are 10 doctors of sciences and more than 20 candidates of sciences (PhD) working at universities of Uzbekistan, Russia, USA, Finland, and Malaysia.

For about thirty years, Sh.A. Alimov has been actively involved in the reform of mathematical school education.

Sh.A. Alimov meets his 75th birthday in the prime of his life, and the Editorial Board of the Eurasian Mathematical Journal heartily congratulates him on his jubilee and wishes him good health, new successes in scientific and pedagogical activity, family well-being and long years of fruitful life.

Short communications

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 12, Number 1 (2021), 92 – 96

ON INTEGRAL OPERATORS WITH HOMOGENEOUS KERNELS IN MORREY SPACES

O.G. Avsyankin

Communicated by E.D. Nursultanov

Key words: Morrey space, integral operator, homogeneous kernel, boundedness, symbol, invertibility.

AMS Mathematics Subject Classification: 47G10.

Abstract. We consider integral operators with homogeneous kernels in Morrey spaces. For such operators we obtain sufficient conditions of their boundedness. Moreover, for an operator, which is the sum of the identity operator and an operator with a homogeneous kernel, we prove the invertibility criterion.

DOI: https://doi.org/10.32523/2077-9879-2021-12-1-92-96

1 Introduction

At the present time, there are quite a few papers dealing with integral operators with homogeneous kernels (e. g., see [1]–[3], [5], [11], [12] and the bibliography therein). For such operators, criteria of invertibility and Fredholm property were obtained, Banach algebra generated by these operators were studied, the necessary and sufficient conditions for the projection method to apply were found and spectral characteristics were described. However, all of the above results relate to operators acting in L_p-spaces. In other spaces the operators with homogeneous kernels were considered only in some papers. For example, in [14] operators with homogeneous kernels were studied in grand Lebesgue spaces.

The present paper is devoted to integral operators with homogeneous kernels in Morrey spaces. The study of Morrey spaces and operators in these spaces goes back to [13]. During the last three decades there was a lot of activity in this area (e. g., see [6], [7] and the bibliography therein). At the same time, much attention was paid to operators of classical analysis, such as maximal operator, Riesz potential, singular integral operator, Hardy operator. In recent years, there is a tendency to study convolution operators (see [8], [9], [4]).

In this paper we consider an integral operator K with a homogeneous kernel of degree -1. For such an operator we establish sufficient conditions for the boundedness in Morrey spaces on the half-axis. Moreover, for the operator $I + K$ we construct a symbol and in terms of this symbol we obtain the invertibility criterion of this operator.

2 Preliminaries

Let $\mathbb{R}_+ = (0, \infty)$, $1 \leqslant p \leqslant \infty$, and $D \subseteq \mathbb{R}_+$ be a measurable set. Then $L_p(D)$ is the space (of classes) of measurable complex–valued functions with the norm

$$\|f\|_{L_p(D)} = \left(\int\limits_D |f(x)|^p \, dx \right)^{1/p}, \ 1 \leqslant p < \infty; \quad \|f\|_{L_\infty(D)} = \operatorname*{ess\,sup}_{x \in D} |f(x)|.$$

A function $f \in L_p^{loc}(\mathbb{R}_+)$ if $f \in L_p(K)$ for any compact $K \subset \mathbb{R}_+$. For any $x, r \in \mathbb{R}_+$ put $J(x, r) = (x - r, x + r) \cap \mathbb{R}_+$.

Definition 1. Let $1 \leqslant p \leqslant \infty$ and $\lambda \in \mathbb{R}$. It is said that a function f belongs to $L_{p,\lambda}(\mathbb{R}_+)$ if $f \in L_p^{loc}(\mathbb{R}_+)$ and

$$\|f\|_{L_{p,\lambda}(\mathbb{R}_+)} \equiv \|f\|_{p,\lambda} = \sup_{x, r \in \mathbb{R}_+} \frac{\|f\|_{L_p(J(x,r))}}{r^\lambda} < \infty. \tag{2.1}$$

With respect to the usual linear operations and norm (2.1) the set $L_{p,\lambda}(\mathbb{R}_+)$ forms a Banach space, which is called a Morrey space.

The spaces $L_{p,\lambda}(\mathbb{R}_+)$ are nontrivial, i. e. they consist not only of functions equivalent to zero on $\mathbb{R}_+$ if and only if $0 \leqslant \lambda \leqslant 1/p$. For $\lambda = 0$ and $\lambda = 1/p$ Morrey spaces coincide with L_p–spaces:

$$L_{p,0}(\mathbb{R}_+) = L_p(\mathbb{R}_+), \quad L_{p,\frac{1}{p}}(\mathbb{R}_+) = L_\infty(\mathbb{R}_+). \tag{2.2}$$

Since the integral operators with homogeneous kernels in L_p–spaces are well studied, taking into account equalities (2.2), we will not consider the cases $\lambda = 0$ и $\lambda = 1/p$.

Denote by $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ the set of all bounded linear operators acting in $L_{p,\lambda}(\mathbb{R}_+)$. Since $L_{p,\lambda}(\mathbb{R}_+)$ is a Banach space, the set $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ with the usual operations of addition and multiplication and with the usual operator norm forms a Banach algebra.

3 Main results

In the space $L_{p,\lambda}(\mathbb{R}_+)$ we consider the operator

$$(K\varphi)(x) = \int\limits_0^\infty k(x, y)\varphi(y)dy, \quad x \in \mathbb{R}_+, \tag{3.1}$$

where the function $k(x, y)$ is defined on $\mathbb{R}_+ \times \mathbb{R}_+$ and satisfies the following conditions:

1° homogeneity of degree -1, i. e.

$$k(\alpha x, \alpha y) = \alpha^{-1} k(x, y), \quad \forall\, \alpha > 0;$$

2° integrability, i. e.

$$\kappa := \int\limits_0^\infty |k(1, y)| y^{-1/p+\lambda} dy < \infty.$$

Theorem 3.1. *Let $1 \leqslant p < \infty$, $0 < \lambda < 1/p$, and a function k satisfy conditions 1° and 2°. Then the operator K is bounded in the space $L_{p,\lambda}(\mathbb{R}_+)$ and the inequality*

$$\|K\varphi\|_{p,\lambda} \leqslant \kappa \|\varphi\|_{p,\lambda}$$

holds for every function $\varphi \in L_{p,\lambda}(\mathbb{R}_+)$.

Let $1 \leqslant p < \infty$, $0 < \lambda < 1/p$. Denote by $\mathfrak{A}_0$ the set consisting of all operators $cI + K$, where $c \in \mathbb{C}$, I is an identity operator, K is an operator of form (3.1). Let K_1 and K_2 be two operators with kernels $k_1(x,y)$ and $k_2(x,y)$ respectively. Their composition $K = K_1K_2$ is an integral operator of the same form with kernel

$$k(x,y) = \int\limits_0^\infty k_1(x,t)k_2(t,y)\,dt.$$

It is easy to see that the function $k(x,y)$ satisfies conditions 1° and 2°. Therefore the operation of multiplication is closed on the set $\mathfrak{A}_0$. It is easy to verify that the set $\mathfrak{A}_0$ equipped with usual operations of addition, multiplication, multiplication of operators by complex numbers and the following norm

$$\|cI + K\|_{\mathfrak{A}_0} = |c| + \int\limits_0^\infty |k(1,y)|y^{-1/p+\lambda}dy, \tag{3.2}$$

is a Banach algebra. This algebra is commutative because for any functions $k_1(x,y)$ and $k_2(x,y)$, which satisfy condition 1°, the equality

$$\int\limits_0^\infty k_1(x,t)k_2(t,y)\,dt = \int\limits_0^\infty k_2(x,t)k_1(t,y)\,dt$$

holds (see, e. g., [12, p. 381]).

Let us consider the function

$$\sigma_{cI+K}(\xi) = c + \int\limits_0^\infty k(1,y)y^{-1/p+\lambda+i\xi}dy, \quad \xi \in \mathbb{R}.$$

We call this function the symbol of the operator $cI + K$. The symbol $\sigma_{cI+K}(\xi)$ can be written in another form. Put

$$h(t) = k(1,e^t)e^{t/p'+t\lambda}.$$

From condition 2° it follows that $h \in L_1(\mathbb{R})$. It is easy to establish that

$$\sigma_{cI+K}(\xi) = c + \widehat{h}(\xi), \tag{3.3}$$

where $\widehat{h}(\xi) = \int\limits_{-\infty}^{\infty} h(t)e^{i\xi t}\,dt$ is the Fourier transform of the function h.

Further we shall denote by $\mathcal{W}(\mathbb{R})$ the Wiener algebra, i. e. the set of all functions of the form $c + \widehat{f}(\xi)$, where $c \in \mathbb{C}$ and $f \in L_1(\mathbb{R})$. It is known that with pointwise algebraic operations and the norm

$$\|c + \widehat{f}\|_{\mathcal{W}} = |c| + \|f\|_1 \tag{3.4}$$

the set $\mathcal{W}(\mathbb{R})$ is a commutative Banach algebra. Taking into account formulas (3.2)–(3.4), we get the following proposition.

Lemma 3.1. *The mapping*

$$\mathcal{S}\colon \mathfrak{A}_0 \to \mathcal{W}(\mathbb{R}), \quad cI + K \to \sigma_{cI+K}(\xi)$$

is an isometric isomorphism.

Denote by $\mathcal{M}(\mathfrak{A}_0)$ and $\mathcal{M}(\mathcal{W}(\mathbb{R}))$ the spaces of maximal ideals of Banach algebras $\mathfrak{A}_0$ and $\mathcal{W}(\mathbb{R})$ respectively. Since the algebra $\mathfrak{A}_0$ is isometrically isomorphic to the algebra $\mathcal{W}(\mathbb{R})$ then the space $\mathcal{M}(\mathfrak{A}_0)$ is homeomorphic to the space $\mathcal{M}(\mathcal{W}(\mathbb{R}))$. Denote by $\dot{\mathbb{R}}$ the one–point compactification of the locally compact space $\mathbb{R}$. It is known ([10, p. 20]) that the space $\mathcal{M}(\mathcal{W}(\mathbb{R}))$ is homeomorphic to the compact $\dot{\mathbb{R}}$. Therefore the space $\mathcal{M}(\mathfrak{A}_0)$ is also homeomorphic to the compact $\dot{\mathbb{R}}$. Under this homeomorphism each point $\xi_0 \in \dot{\mathbb{R}}$ corresponds to the ideal $\mathcal{M}_{\xi_0} \in \mathcal{M}(\mathfrak{A}_0)$ consisting of all operators $cI + K$ such that $\sigma_{cI+K}(\xi_0) = 0$. As a result, we obtain the invertibility criterion of the operator $cI + K$ in the algebra $\mathfrak{A}_0$.

Lemma 3.2. *The operator $cI + K$ is invertible in the algebra $\mathfrak{A}_0$ if and only if*

$$\sigma_{cI+K}(\xi) \neq 0, \quad \forall \xi \in \dot{\mathbb{R}}. \tag{3.5}$$

Lemma 3.2 implies that condition (3.5) is sufficient for invertibility of the operator $cI + K$ in $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$. It is much more difficult to obtain the necessary condition, because if the operator $cI + K$ is invertible in $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ this does not mean that $(cI + K)^{-1} \in \mathfrak{A}_0$.

Denote by $\mathfrak{A}$ the least closed subalgebra of the Banach algebra $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ containing all operators $cI + K$, where $c \in \mathbb{C}$ and K is an operator of form (3.1). This algebra is the closure of the set $\mathfrak{A}_0$ in the uniform operator topology. Note that the algebra $\mathfrak{A}$ is commutative.

Lemma 3.3. *Let K be an operator of form (3.1). If the operator $cI + K$ is invertible in $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ then the operator $(cI + K)^{-1}$ belongs to the algebra $\mathfrak{A}$.*

Taking into account that the algebra $\mathfrak{A}_0$ is densely embedded in the algebra $\mathfrak{A}$, we get the following lemma.

Lemma 3.4. *Let K be an operator of form (3.1). If the operator $cI + K$ is invertible in $\mathfrak{A}$ then the operator $(cI + K)^{-1}$ belongs to the algebra $\mathfrak{A}_0$.*

Lemma 3.2, Lemma 3.3 and Lemma 3.4 imply the following main result.

Theorem 3.2. *Let K be an operator of form (3.1). The operator $cI + K$ is invertible in $\mathcal{L}(L_{p,\lambda}(\mathbb{R}_+))$ if and only if condition (3.5) holds.*

Acknowledgments

The author thanks the referee for careful reading of the manuscript and useful suggestions which allowed us to improve the present paper. This work is supported by the Russian Foundation for Basic Research, project no. 20–51–56015.

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Oleg Gennadievich Avsyankin
Institute of Mathematics, Mechanics and Computer Sciences
Regional Mathematical Center
Southern Federal University
8a Mil'chakova St,
344090 Rostov-on-Don, Russia
E-mail: ogavsyankin@sfedu.ru

Received: 25.10.2019