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SHAVKAT ARIFJANOVICH ALIMOV

(to the 75th birthday)

Shavkat Arifjanovich Alimov was born on March 2, 1945 in the city of Nukus, Uzbekistan. In 1968, he graduated from the Department of Mathematics of Physical Faculty of the M.V. Lomonosov Moscow State University (MSU), receiving a diploma with honors. From 1968 to 1970, he was a post-graduate student in the same department under the supervision of Professor V.A. Il'in. He defended his PhD thesis in 1970. In May 1973, at the age of 28, he defended his doctoral thesis devoted to equations of mathematical physics. In 1973, for research on the spectral theory, he was awarded the highest youth prize of the USSR.

From 1974 to 1984, he worked as a professor in the Department of General Mathematics at the Faculty of Computational Mathematics and Cybernetics. In 1984, Sh.A. Alimov joined the Tashkent State University (TSU) as a professor. From 1985 to 1987 he worked as the Rector of the Samarkand State University, from 1987 to 1990 - the Rector of the TSU, from 1990 to 1992 - the Minister of Higher and Secondary Special Education of the Republic of Uzbekistan. From 1992 to 1994, he headed the Department of Mathematical Physics of the TSU.

After some years of diplomatic work, he continued his academic career as a professor of the Department of Mathematical Physics at the National University of Uzbekistan (NUU). From the first days of the opening of the Tashkent branch of the MSU in 2006, he worked as a professor in the Department of Applied Mathematics. From 2012 to 2017, he headed the Laboratory of Mathematical Modeling of the Malaysian Institute of Microelectronic Systems in Kuala Lumpur. From 2017 to 2019, he worked as a professor at the Department of Differential Equations and Mathematical Physics of the NUU. From 2019 to the present, Sh.A. Alimov is a Scientific Consultant at the Center for Intelligent Software Systems, and an adviser to the Rector of the NUU.

The main scientific activity of Sh.A. Alimov is connected with the spectral theory of partial differential equations and the theory of boundary value problems for equations of mathematical physics. He obtained series of remarkable results in these fields. They cover many important problems of the theory of Schrodinger equations with singular potentials, the theory of boundary control of the heat transfer process, the mathematical problems of peridynamics related to the theory of hypersingular integrals.

In 1984, Sh.A. Alimov was elected a corresponding member and in 2000 an academician of the Academy of Sciences of Uzbekistan. He was awarded several prestigious state prizes.

Sh.A. Alimov has over 150 published scientific and a large number of educational works. Among his pupils there are 10 doctors of sciences and more than 20 candidates of sciences (PhD) working at universities of Uzbekistan, Russia, USA, Finland, and Malaysia.

For about thirty years, Sh.A. Alimov has been actively involved in the reform of mathematical school education.

Sh.A. Alimov meets his 75th birthday in the prime of his life, and the Editorial Board of the Eurasian Mathematical Journal heartily congratulates him on his jubilee and wishes him good health, new successes in scientific and pedagogical activity, family well-being and long years of fruitful life.

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BOUNDEDNESS OF RIEMANN-LIOUVILLE FRACTIONAL INTEGRAL OPERATOR IN MORREY SPACES

M.A. Senouci

Communicated by T.V. Tararykova

Key words: Riemann-Liouville operator, Morrey spaces, boundedness.

AMS Mathematics Subject Classification: 35J20, 35J25.

Abstract. The aim of the present paper is to prove the boundedness of the multidimensional Riemann-Liouville operator from the quasi-normed Morrey space $M_p^\lambda(\Omega)$ to another quasi-normed Morrey space $M_q^\mu(\Omega)$ and to estimate the dependence of the norm of this operator on Ω.

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1 Introduction

The Morrey spaces M_p^λ were introduced by C. Morrey in 1938 (see [4]). They are used in the theory of partial differential equations, in functional analysis, and in other areas of mathematics. They are defined as follows.

Definition 1. Let $\Omega \subset \mathbb{R}^n$ be a Lebesgue measurable set, $0 < p \leq \infty, 0 \leq \lambda \leq \dfrac{n}{p}$. The Morrey space $M_p^\lambda(\Omega)$, is the space of all functions f Lebesgue measurable on Ω for which

$$\|f\|_{M_p^\lambda(\Omega)} = \sup_{x\in\Omega, r>0} r^{-\lambda}\|f\|_{L_p(B(x,r)\bigcap\Omega)} < \infty. \tag{1.1}$$

If $\lambda = 0$, then

$$M_p^0(\Omega) = L_p(\Omega). \tag{1.2}$$

If $\lambda = \dfrac{n}{p}$, then

$$M_p^{\frac{n}{p}}(\Omega) = L_\infty(\Omega). \tag{1.3}$$

If $p = \infty$, then (1.2) coincides with (1.3). If $\lambda < 0$ or $\lambda > \frac{n}{p}$, then the space $M_p^\lambda(\Omega)$ consists only of functions f equivalent to 0 on Ω.

Definition 2. The left multidimensional fractional Riemann-Liouville integral operator I_{a+}^α of order $\alpha = (\alpha_1, ..., \alpha_n)$, $0 < \alpha_i < 1$, $i = 1, ..., n, a = (a_1, a_2, ..., a_n) \in \mathbb{R}^n$, is defined as follows

$$\left(I_{a+}^\alpha f\right)(x) = \frac{1}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_{a_n}^{x_n} ... \int_{a_1}^{x_1} \left(\prod_{i=1}^n (x_i - t_i)^{\alpha_i - 1}\right) f(t_1, ..., t_n) dt_1 ... dt_n$$

for all $x = (x_1, ..., x_n) \in \mathbb{R}^n$ such that $x_i > a_i$, $i = 1, .., n$, where Γ is the Euler Gamma function.

The right multidimensional fractional Riemann-Liouville integral operator of order $\alpha=(\alpha_1,...,\alpha_n), \alpha_i>0$, is defined similarly:

$$\left(I^{\alpha}_{b_-}f\right)(x)=\frac{1}{\prod_{i=1}^n\Gamma(\alpha_i)}\int_{x_n}^{b_n}...\int_{x_1}^{b_1}\prod_{i=1}^n(t_i-x_i)^{\alpha_i-1}f(t_1...t_n)dt_1...dt_n$$

for all $x\in\mathbb{R}^n$ such that $x_i<b_i,\ i=1,...,n$.

In [6] the following theorem was proved.

Theorem 1.1. *Let $n=1$, $1<p,q<\infty$, $\frac{1}{p}<\alpha<1$ and $0\le\lambda,\ \mu\le 1$, $0\le\lambda\le\frac{1}{p}$, $0\le\mu\le\frac{1}{q}$, $0<T<\infty$. Then I^{α}_{0+} is bounded from $M^{\lambda}_p(0,T)$ to $M^{\mu}_q(0,T)$.*

The objective of the present paper is to generalize the results for the multidimensional Riemann-Liouville integral operator for $M^{\lambda}_p(\Omega)$, where $\Omega=Q(a;b)=\{x\in\mathbb{R}^n,\ a_i<x_i<b_i,\ i=1,...,n\}$ with $1<p<\infty$, $0<q<\infty$ and to enlarge the range of the parameter α. Moreover, sharp estimates of the norm of this operator via the diameter of $Q(a,b)$ are obtained.

2 Main results

Let $a\in\mathbb{R}^n$, $b\in\overline{\mathbb{R}^n}$, $-\infty<a_i<b_i\le\infty$, $i=1,...,n$ and $Q(a,b)=\{x\in\mathbb{R}^n,\ a_i<x_i<b_i,\ i=1,...,n\}$.

Lemma 2.1. *Let $0<p\le\infty$, $0\le\lambda\le\frac{n}{p}$. Then*

$$\parallel f\parallel_{L_p(Q(a,y))}\le|y-a|^{\lambda}\parallel f\parallel_{M^{\lambda}_p(Q(a,b))} \tag{2.1}$$

for any parallelepiped $Q(a,b)$ and for any $y\in Q(a,b)$.

Proof. Let $y\in Q(a,b)$ and $0<\epsilon<min\{y_1-a_1,...y_n-a_n\}$, then $a+\epsilon=(a_1+\epsilon,...,a_n+\epsilon)\in Q(a,y)$ and for any $z\in Q(a,y)$

$$|(a+\epsilon)-z|<diam\ Q(a,y)=|y-a|.$$

Therefore, $Q(a,y)\subset B(a+\epsilon,|y-a|)$ and for any $y\in Q(a,b)$

$$\parallel f\parallel_{M^{\lambda}_p(Q(a,b))}=\sup_{x\in Q(a,b),r>0} r^{-\lambda}\parallel f\parallel_{L_p(B(x,r)\bigcap Q(a,b))}$$

$$\ge r^{-\lambda}\parallel f\parallel_{L_p\left(B(x,r)\bigcap Q(a,b)\right)}\Big|_{x=a+\epsilon,r=|y-a|}$$

$$=|y-a|^{-\lambda}\parallel f\parallel_{L_p(Q(a,y)\bigcap Q(a,b)}$$

$$=|y-a|^{-\lambda}\parallel f\parallel_{L_p(Q(a,y))}.$$

Consequently, (2.1) follows. □

Theorem 2.1. *Let $1<p\le\infty$, $0<q\le\infty$, $0\le\lambda\le\frac{n}{p}$, $0\le\mu\le\frac{n}{q}$, $\frac{1}{p}<\alpha_i<1$, $i=1,...,n$. Then there exists $C_1>0$ such that*

$$\parallel I^{\alpha}_{a+}f\parallel_{M^{\mu}_q(Q(a,b))}\le C_1\mid b-a\mid^{\nu}\parallel f\parallel_{M^{\lambda}_p(Q(a,b))}, \tag{2.2}$$

where

$$\nu=\lambda+\alpha_1+...+\alpha_n-\frac{n}{p}+\frac{n}{q}-\mu, \tag{2.3}$$

for all finite parallelepipeds $Q(a,b)$ and for all $f\in M^{\lambda}_p(Q(a,b))$ and the exponent ν cannot be replaced by any other one.

Proof. Step 1. We suppose that $f \in M_p^{\lambda}(Q(a,b))$ and $y \in B(x,r)\bigcap Q(a,b) \neq \emptyset, i = 1, ..., n.$

By Hölder's inequality we get

$$\left|I_{a+}^{\alpha} f(y)\right| = \frac{1}{\prod\limits_{i=1}^{n}\Gamma(\alpha_i)} \left|\int_{a_n}^{y_n} ... \int_{a_1}^{y_1} (y_1 - t_1)^{\alpha_1 - 1} ... (y_n - t_n)^{\alpha_n - 1} f(t_1, ..., t_n) dt_1 ... dt_n\right|$$

$$\leq \frac{1}{\prod\limits_{i=1}^{n}\Gamma(\alpha_i)} \parallel f \parallel_{L_p(Q(a,y))} \left(\int\limits_{a_n}^{y_n} ... \int\limits_{a_1}^{y_1} (y_1 - t_1)^{(\alpha_1 - 1)\frac{p}{p-1}} ... (y_n - t_n)^{(\alpha_n - 1)\frac{p}{p-1}} dt_1 ... dt_n\right)^{\frac{p-1}{p}}$$

$$= \frac{1}{\prod\limits_{i=1}^{n}\Gamma(\alpha_i)} \parallel f \parallel_{L_p(Q(a,y))} \prod_{i=1}^{n}\left(\frac{\alpha_i p - 1}{p-1}\right)^{\frac{1}{p}-1} (y_i - a_i)^{\alpha_i - \frac{1}{p}}.$$

Thus

$$\left|\left(I_{a+}^{\alpha}\right)(y)\right| \leq C_2 \parallel f \parallel_{L_p(Q(a,y))} \prod_{i=1}^{n}(y_i - a_i)^{\alpha_i - \frac{1}{p}},$$

where

$$C_2 = \frac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i)} \prod_{i=1}^{n}\left(\frac{\alpha_i p - 1}{p-1}\right)^{\frac{1}{p}-1}.$$

By (2.1), we obtain

$$\left|\left(I_{a+}^{\alpha} f\right)(y)\right| \leq C_2 \mid y - a \mid^{\tilde{\lambda}} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))} \leq C_2 \mid b - a \mid^{\tilde{\lambda}} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))},$$

where $\tilde{\lambda} = \lambda + \alpha_1 + ... + \alpha_n - \frac{n}{p}$.

Thus

$$\left\|I_{a+}^{\alpha} f\right\|_{L_q(B(x,r)\bigcap Q(a,b))} \leq C_2 \mid b - a \mid^{\tilde{\lambda}} \parallel 1 \parallel_{L_q(B(x,r)\bigcap Q(a,b))} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}.$$

We have

$$\parallel 1 \parallel_{L_q(B(x,r)\bigcap Q(a,b))} \leq \begin{cases} v_n^{\frac{1}{q}} r^{\frac{n}{q}} & \text{if } 0 < r < |b-a|, \\ |b-a|^{\frac{n}{q}} & \text{if } r \geq |b-a|, \end{cases}$$

where v_n is the volume of the unit ball in $\mathbb{R}^n$.

Next, we distinguish two cases:

1) If $r < |b - a|$, then

$$\begin{aligned} r^{-\mu} \parallel I_{a+}^{\alpha} f \parallel_{L_q(B(x,r)\bigcap Q(a,b)} &\leq C_2 C_3 \mid b - a \mid^{\tilde{\lambda}} r^{-\mu + \frac{n}{q}} \parallel f \parallel_{M_p^{\lambda} Q(a,b)} \\ &\leq C_2 C_3 \mid b - a \mid^{\tilde{\lambda}} |b - a|^{\frac{n}{q} - \mu} \parallel f \parallel_{M_p^{\lambda} Q(a,b)} \\ &= C_2 C_3 |b - a|^{\tilde{\lambda} + \frac{n}{q} - \mu} \parallel f \parallel_{M_p^{\lambda} Q(a,b)}, \end{aligned}$$

where $C_3 = max(v_n, 1)^{\frac{1}{q}}$.

Hence

$$\sup_{x \in Q(a,b),\ 0 < r < |b-a|} r^{-\mu} \left\|I_{a+}^{\alpha} f\right\|_{L_q(B(x,r)\bigcap(Q(a,b))} \leq C_1 |b - a|^{\tilde{\lambda} + \frac{n}{q} - \mu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}, \quad (2.4)$$

where $C_1 = C_2 C_3$.

2) If $r \geq |b-a|$, then

$$r^{-\mu} \left\| I^{\alpha}_{a+} f \right\|_{L_q(B(x,r)\bigcap Q(a,b))} \leq C_2 |b-a|^{\tilde{\lambda}} r^{-\mu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}$$
$$\leq C_2 \mid b-a \mid^{\tilde{\lambda}+\frac{n}{q}-\mu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))} .$$

Thus,

$$\sup_{x\in Q(a,b),\ |b-a|\leq r} r^{-\mu} \left\| I^{\alpha}_{a+} f \right\|_{L_q(B(x,r)\bigcap Q(a,b))}$$
$$\leq C_2 |b-a|^{\tilde{\lambda}+\frac{n}{q}-\mu} \parallel f \parallel_{M_p^{\lambda} Q(a,b)} . \tag{2.5}$$

Consequently, by (2.4) and (2.5), we obtain inequality (2.2) because

$$\left\| I^{\alpha}_{a+} f \right\|_{M_q^{\mu}(Q(a,b))}$$
$$= max \left\{ \sup_{x\in Q(a,b), 0<r<|b-a|} r^{-\mu} \parallel f \parallel_{L_q(B(x,r)\bigcap Q(a,b))}, \sup_{x\in Q(a,b), r\geq|b-a|} r^{-\mu} \parallel f \parallel_{L_q(B(x,r)\bigcap Q(a,b))} \right\}.$$

Step 2. Next, assume that the operator I^{α}_{a+} is bounded from $M_p^{\lambda}(Q(a,b))$ to $M_q^{\mu}(Q(a,b))$, that is for some $C_4 > 0$

$$\left\| I^{\alpha}_{a+} f \right\|_{M_q^{\mu}(Q(a,b))} \leq C_4 \parallel f \parallel_{M_p^{\lambda}(Q(a,b))} \tag{2.6}$$

for all $f \in M_p^{\lambda}(Q(a,b))$.

Let here $f = 1$ and $b_1 - a_1 = b_2 - a_2 = ... = b_n - a_n$. Then

$$\parallel 1 \parallel_{M_p^{\lambda} Q(a,b)} = \sup_{x\in Q(a,b),\ r>0} r^{-\lambda} \parallel 1 \parallel_{L_p(Q(a,b)\bigcap B(x,r))}$$
$$= \sup_{x\in Q(a,b),\ r>0} r^{-\lambda} \left| B(x,r) \bigcap Q(a,b) \right|^{\frac{1}{p}}$$
$$\leq max \left\{ \sup_{x\in Q(a,b), 0<r<|b-a|} r^{-\lambda} \left| B(x,r) \right|^{\frac{1}{p}}, \sup_{x\in Q(a,b),\ r\geq|b-a|} r^{-\lambda} \left| Q(a,b) \right|^{\frac{1}{p}} \right\}$$
$$= max \left\{ v_n^{\frac{1}{p}} \sup_{0<r\leq|b-a|} r^{-\lambda+\frac{n}{p}}, |b-a|^{-\lambda+\frac{n}{p}} \right\} = C_5 \, |b-a|^{-\lambda+\frac{n}{p}} ,$$

where $C_5 = max \left\{ v_n^{\frac{1}{p}}, 1 \right\}$. Moreover,

$$\left\| I^{\alpha}_{a+}(1) \right\|_{M_q^{\mu} Q(a,b)} = \frac{1}{\prod_{i=1}^{n} \Gamma(\alpha_i)} \sup_{x\in Q(a,b),\ r>0} r^{-\mu} \left\| \prod_{i=1}^{n} \frac{(y_i - a_i)^{\alpha_i}}{\alpha_i} \right\|_{L_q(Q(a,b)\bigcap B(x,r))}$$
$$\geq \frac{1}{\prod_{i=1}^{n} \alpha_i \Gamma(\alpha_i)} r^{-\mu} \left\| \prod_{i=1}^{n} (y_i - a_i)^{\alpha_i} \right\|_{L_q(Q(a,b)\bigcap B(x,r))} \Bigg|_{x=a+\epsilon, r=|b-a|}$$
$$= \frac{1}{\prod_{i=1}^{n} \alpha_i \Gamma(\alpha_i)} |b-a|^{-\mu} \left\| \prod_{i=1}^{n} (y_i - a_i)^{\alpha_i} \right\|_{L_q(Q(a,b)\bigcap B(a+\varepsilon, |b-a|))} ,$$

where $0 < \varepsilon < min\{b_1 - a_1, ..., b_n - a_n\}$.

By the proof of Lemma 2.1, we have $Q(a,b) \subset B(a+\varepsilon, |b-a|)$.
Therefore

$$\left\| \prod_{i=1}^{n} (y_i - a_i)^{\alpha_i} \right\|_{L_q(Q(a,b) \bigcap B(a+\varepsilon,|b-a|))} = \left\| \prod_{i=1}^{n} (y_i - a_i)^{\alpha_i} \right\|_{L_q(Q(a,b))}$$

$$= \prod_{i=1}^{n} \frac{1}{(\alpha_i q + 1)^{\frac{1}{q}}} (b_i - a_i)^{\alpha_i + \frac{1}{q}} = \prod_{i=1}^{n} \frac{1}{(\alpha_i q + 1)^{\frac{1}{q}}} \left(\frac{|b-a|}{\sqrt{n}} \right)^{\alpha_1 + \ldots + \alpha_n + \frac{n}{q}}$$

$$= C_6 |b-a|^{\alpha_1 + \ldots + \alpha_n + \frac{n}{q}},$$

where $C_6 = \prod_{i=1}^{n} \frac{1}{(\alpha_i q+1)^{\frac{1}{q}}} \left(\frac{1}{\sqrt{n}}\right)^{\alpha_1 + \ldots + \alpha_n + \frac{n}{q}}$. Thus

$$\left\| I_{a+}^{\alpha} 1 \right\|_{M_q^{\mu}(Q(a,b))} \geq C_7 |b-a|^{\alpha_1 + \ldots + \alpha_n + \frac{n}{q} - \mu},$$

where

$$C_7 = C_6 \prod_{i=1}^{n} \frac{1}{\alpha_i \Gamma(\alpha_i)}.$$

By (2.5), it follows that

$$C_7 |b-a|^{\alpha_1 + \ldots + \alpha_n + \frac{n}{q} - \mu} \leq C_4 C_5 |b-a|^{-\lambda + \frac{n}{p}},$$

hence

$$C_4 \geq \frac{C_7}{C_5} |b-a|^{\nu}.$$

□

Remark 1. Similarly one can obtain the results for the right multidimensional Riemann-Liouville fractional integral operator of order $\alpha = (\alpha_1, ..., \alpha_n),\ i = 1, ...n, \frac{1}{p} < \alpha_i < 1$.

Lemma 2.2. *Let* $0 < p < \infty, 0 \leq \mu < \lambda \leq \frac{n}{p}$. *Then*

$$\| f \|_{M_p^{\mu}(Q(a,b))} \leq |b-a|^{\lambda-\mu} \| f \|_{M_p^{\lambda}(Q(a,b))} \tag{2.7}$$

for any finite parallelepiped Q(a,b) and for any $f \in M_p^{\lambda}(Q(a,b))$.

Proof. By applying inequality (2.1), we have

$$r^{-\mu} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))} \leq r^{-\mu} \| f \|_{L_p(Q(a,b))} \leq r^{-\mu} |b-a|^{\lambda} \| f \|_{M_p^{\lambda}(Q(a,b))} .$$

$$\sup_{r \geq |b-a|} r^{\lambda-\mu} r^{-\lambda} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))} \leq | b-a |^{\lambda-\mu} \sup_{r \geq |b-a|} r^{-\lambda} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))}$$

$$= | b-a |^{\lambda-\mu} \| f \|_{M_p^{\lambda}(Q(a,b))},$$

then

$$\| f \|_{M_p^{\mu}(Q(a,b))}$$

$$= \max \left\{ \sup_{r \geq |b-a|} r^{-\mu} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))}, \sup_{r < |b-a|} r^{-\mu} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))} \right\}$$

$$= \max \left\{ \sup_{r \geq |b-a|} r^{-\lambda-\mu} r^{\lambda} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))}, \sup_{r < |b-a|} r^{-\mu} \| f \|_{L_p(B(x,r) \bigcap Q(a,b))} \right\}$$

$$\leq max\left\{| b-a |^{\lambda-\mu} \sup_{r\geq|b-a|} r^{-\lambda} \parallel f \parallel_{L_p(B(x,r)\bigcap Q(a,b))}, |b-a|^{\lambda-\mu} \sup_{r<|b-a|} r^{-\lambda} \parallel f \parallel_{L_p(B(x,r)\bigcap Q(a,b))}\right\}$$

$$= max\left\{|b-a|^{\lambda-\mu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}, |b-a|^{\lambda-\mu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}\right\}.$$

$$= |b-a|^{\lambda-\mu} \parallel f \parallel_{M_p^{\mu}(Q(a,b))} .$$

□

Lemma 2.3. *Let $0 < p < q \leq \infty$, $0 \leq \lambda \leq \frac{n}{p}$. Then*

$$\parallel f \parallel_{M_p^{\lambda}(Q(a,b))} \leq |b-a|^{n(\frac{1}{p}-\frac{1}{q})} \parallel f \parallel_{M_q^{\lambda}(Q(a,b))} \tag{2.8}$$

for any finite parallelepiped $Q(a,b)$ and any $f \in M_p^{\lambda}(Q(a,b))$.

Proof. If $0 < p < q \leq \infty$, E is a Lebesgue measurable set, $|E| < \infty$, then by Hölder's inequality

$$\parallel f \parallel_{L_p(E)} \leq |E|^{\frac{1}{p}-\frac{1}{q}} \parallel f \parallel_{L_q(E)} . \tag{2.9}$$

By applying (2.9), we get

$$\sup_{x\in Q(a,b),\ r>0} r^{-\lambda} \parallel f \parallel_{L_p(B(x,r)\bigcap Q(a,b))}$$

$$\leq |b-a|^{n(\frac{1}{p}-\frac{1}{q})} \sup_{x\in Q(a,b),\ r>0} r^{-\lambda} \parallel f \parallel_{L_q(B(x,r)\bigcap Q(a,b))}$$

and inequality (2.8) follows. □

We shall need the following variant of Young's inequality for truncated convolutions (see [3] for details).

Lemma 2.4. *Let $A, B \subset \mathbb{R}^n$ be Lebesgue measurable sets,*

$$1 \leq p, \rho \leq s \leq \infty, \frac{1}{p} + \frac{1}{\rho} = \frac{1}{s} + 1. \tag{2.10}$$

If $f \in L_p(B)$,

$$\sup_{t\in B} \parallel \varphi \parallel_{L\rho(A-t)} < \infty,\ \sup_{y\in A} \parallel \varphi \parallel_{L\rho(y-B)} < \infty,$$

then the integral $\int_B \varphi(y-t)f(t)dt$ is finite for almost all $y \in A$ and

$$\left\|\int_B \varphi(y-t)f(t)dt\right\|_{L_s(A)} \leq \sup_{t\in B} \parallel \varphi \parallel_{L_\rho(A-t)}^{\frac{\rho}{s}} \sup_{y\in A} \parallel \varphi \parallel_{L_\rho(y-B)}^{1-\frac{\rho}{s}} \parallel f \parallel_{L_p(B)} . \tag{2.11}$$

Theorem 2.2. *Let $1 \leq p \leq s \leq \infty$, $0 < q \leq s$, $\frac{1}{p} - \frac{1}{s} < \alpha_i < 1, i = 1, ..., n$, $0 \leq \lambda \leq \frac{n}{p}$,*

$$0 \leq \mu \leq \frac{n}{q} - \frac{p(n-\alpha_1 ... -\alpha_n)}{p+s(p-1)}, \tag{2.12}$$

then there exists $C_8 > 0$ such that

$$\left\|I_{a_+}^{\alpha} f\right\|_{M_q^{\mu}(Q(a,b))} \leq C_8 \mid b-a \mid^{\nu} \parallel f \parallel_{M_p^{\lambda}(Q(a,b))}, \tag{2.13}$$

where $\nu > 0$ is defined by (2.3), for all finite parallelepipeds $Q(a,b)$ and for all $f \in M_p^{\lambda}(Q(a,b))$ and the exponent ν cannot be replaced by any other one.

Proof. Note that the right-hand side of inequality (2.12) is positive and that inequality (2.12) implies that $0 \leq \mu < \frac{n}{q}$. Let the number ρ be defined by equality (2.10), then inequality (2.12) can be rewritten as

$$0 \leq \mu \leq \frac{n}{q} - \frac{n}{s} + \frac{\rho}{s}\left(\alpha_1 + \alpha_2 + ... + \alpha_n + \frac{n}{s} - \frac{n}{p}\right). \tag{2.14}$$

It is required to prove inequality (2.13). The proof of the fact that ν cannot be replaced by any other one is the same as in Theorem 2.1. Let

$$\varphi_i(\tau_i) = \begin{cases} \tau_i^{\alpha_i - 1}, & 0 < \tau_i < b_i - a_i, \\ 0, & \text{otherwise}, \end{cases}$$

$i = 1, ..., n, \varphi(\tau) = \varphi_1(\tau_1)...\varphi_n(\tau_n), \tau \in \mathbb{R}^n$. Then for all $y \in Q(a,b)$

$$\begin{aligned} \left(I_{a_+}^{\alpha} f\right)(y) &= \frac{1}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_{a_n}^{y_n} ... \int_{a_1}^{y_1} \varphi_1(y_1 - t_1)...\varphi_n(y_n - t_n) f(t_1, ..., t_n) dt_1...dt_n \\ &= \frac{1}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_{Q(a,y)} \varphi(y-t) f(t) dt. \\ &= \frac{1}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_{Q(a,b)} \varphi(y-t) f(t) dt, \end{aligned}$$

since $\varphi(y-t) = 0$ for $t \in Q(a,b) \setminus Q(a,y)$.

Let

$$K = \left\| \int_{Q(a,b)} \varphi(y-t) f(t) dt \right\|_{L_s(B(x,r) \bigcap Q(a,b))}.$$

By (2.10),

$$K \leq \sup_{t \in Q(a,b)} \| \varphi \|^{\frac{\rho}{s}}_{L_\rho(B(x,r) \bigcap Q(a,b) - t)} \sup_{y \in B(x,r) \bigcap Q(a,b)} \| \varphi \|^{1 - \frac{\rho}{s}}_{L_\rho(y - Q(a,b))} \| f \|_{L_p(Q(a,b))}. \tag{2.15}$$

$$\| \varphi \|_{L_\rho(y - Q(a,b))} \leq \| \varphi \|_{L_\rho(\mathbb{R}^n)} = \| \varphi \|_{L_\rho(Q(0, b-a))},$$

$$\sup_{y \in B(x,r) \bigcap Q(a,b)} \| \varphi \|_{L_\rho(y - Q(a,b))} \leq \| \varphi \|_{L_\rho(Q(0,b-a))},$$

$$\| \varphi \|_{L_\rho(Q(0,b-a))} = \left(\int_0^{b_n - a_n} ... \int_0^{b_1 - a_1} \tau_1^{(\alpha_1 - 1)\rho} ... \tau_n^{(\alpha_n - 1)\rho} d\tau_1 ... d\tau_n \right)^{\frac{1}{\rho}}$$

$$= \prod_{i=1}^n \Big((\alpha_i - 1)\rho + 1 \Big)^{-\frac{1}{\rho}} (b_i - a_i)^{\alpha_i + \frac{1}{\rho} - 1} \leq C_9 |b-a|^{\alpha_1 + ... + \alpha_n + \frac{n}{s} - \frac{n}{p}}, \tag{2.16}$$

where $C_9 = \prod_{i=1}^n \Big((\alpha_i - 1)\rho + 1 \Big)^{-\frac{1}{\rho}}$.

1) If $r \geq | b - a |$,

$$\| \varphi \|_{L_\rho(B(x,r) \bigcap Q(a,b) - t)} \leq \| \varphi \|_{L_\rho(\mathbb{R}^n)} = \| \varphi \|_{L_\rho(Q(0,b-a))}.$$

Consequently,

$$K \leq \| \varphi \|^{\frac{\rho}{s}}_{L_\rho(Q(0,b-a))} \| \varphi \|^{1 - \frac{\rho}{s}}_{L_\rho(Q(0,b-a))} \| f \|_{L_p(Q(a,b))}$$

$$= \| \varphi \|_{L_\rho(Q(0,b-a))} \| f \|_{L_p(Q(a,b))}$$

$$\leq C_9|b-a|^{\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p}} \parallel f \parallel_{L_p(Q(a,b))} .$$

By inequality (2.1), we get

$$\sup_{x\in Q(a,b),r\geq|b-a|} r^{-\mu}K \leq C_9 \mid b-a \mid^{\lambda+\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p}-\mu} \parallel f \parallel_{M_p^\lambda(Q(a,b))} . \tag{2.17}$$

2) If $0 < r <\mid b-a \mid$,

$$\parallel \varphi \parallel_{L_\rho(B(x,r)\bigcap Q(a,b)-t)} \leq \parallel \varphi \parallel_{L_\rho(B(x,r)-t)} = \parallel \varphi \parallel_{L_\rho(B(x-t,r))}$$

$$\leq \parallel \varphi \parallel_{L_\rho(B(0,2r))} \leq \parallel \varphi \parallel_{L_\rho(Q(0,2r))},$$

thus

$$\|\varphi\|_{L_\rho(B(x,r)\bigcap Q(a,b)-t)} \leq \parallel \varphi \parallel_{L_\rho(Q(0,2r))},$$

$$\|\varphi\|^{\frac{\rho}{s}}_{L_\rho(Q(0,2r))} = \left[\int_0^{2r} ... \int_0^{2r} \varphi_n^\rho(\tau_n)...\varphi_1^\rho(\tau_1)d\tau_n...d\tau_1\right]^{\frac{1}{s}}$$

$$\leq \left[\int_0^{2r} \tau_n^{(\alpha_n-1)\rho}d\tau_n\right]^{\frac{1}{s}} ... \left[\int_0^{2r} \tau_1^{(\alpha_1-1)\rho}d\tau_1\right]^{\frac{1}{s}}$$

$$= C_9^{\frac{\rho}{s}} \prod_{i=1}^n 2^{\frac{1}{s}[(\alpha_i-1)\rho+1]} r^{\frac{1}{s}[(\alpha_i-1)\rho+1]} = C_{10} r^{\frac{1}{s}((\alpha_1+...+\alpha_n)\rho-n\rho+n)}, \tag{2.18}$$

where

$$C_{10} = C_9^{\frac{\rho}{s}} \prod_{i=1}^n 2^{\frac{1}{s}[(\alpha_i-1)\rho+1]}.$$

By applying (2.15) and (2.16), we obtain

$$r^{-\mu}K \leq C_9^{1-\frac{\rho}{s}} C_{10}|b-a|^{(1-\frac{\rho}{s})(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})} r^{\frac{\rho}{s}(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})-\mu} \parallel f \parallel_{L_p(Q(a,b))}, \tag{2.19}$$

since $\frac{\rho}{s}(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p}) \geq \mu$, then

$$r^{-\mu}K \leq C_{10} C_9^{1-\frac{\rho}{s}} |b-a|^{(1-\frac{\rho}{s})(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})+\frac{\rho}{s}(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})-\mu},$$

by (2.1), we get

$$r^{-\mu}K \leq C_9^{1-\frac{\rho}{s}} C_{10}|b-a|^{\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p}+\lambda-\mu} \parallel f \parallel_{M_p^\lambda(Q(a,b))} .$$

Consequently,

$$\sup_{x\in Q(a,b),\ 0<r<|b-a|} r^{-\mu}K \leq C_8 \mid b-a \mid^{\lambda+\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p}-\mu} \parallel f \parallel_{M_p^\lambda(Q(a,b))}, \tag{2.20}$$

where $C_8 = C_9^{1-\frac{\rho}{s}} C_{10}$.

Inequality (2.13) with $q = s$ follows by (2.15) and (2.17):

$$\left\|I_{a+}^\alpha f\right\|_{M_s^\mu(Q(a,b))} \leq C_8 \mid b-a \mid^\gamma \parallel f \parallel_{M_p^\lambda(Q(a,b))}, \tag{2.21}$$

where $\gamma = \lambda + \alpha_1 + ... + \alpha_n + \frac{n}{s} - \frac{n}{p} - \mu$.

Next, let $0 < q < s \leq \infty$ and

$$L = \left\| \int_{Q(a,b)} \varphi(y-t) f(t) dt \right\|_{L_q(B(x,r) \bigcap Q(a,b))}.$$

By (2.9), we get

$$L \leq \mid B(x,r) \cap Q(a,b) \mid^{\frac{1}{q}-\frac{1}{s}} K. \tag{2.22}$$

1) if $r \geq |b-a|$, then (2.21) implies

$$r^{-\mu} L \leq |Q(a,b)|^{\frac{1}{q}-\frac{1}{s}} r^{-\mu} K \leq |b-a|^{n(\frac{1}{q}-\frac{1}{s})} r^{-\mu} K, \tag{2.23}$$

by (2.15), we have

$$\sup_{x \in Q(a,b), r \geq |b-a|} r^{-\mu} L \leq C_9 |b-a|^{\lambda+\alpha_1+...+\alpha_n+\frac{n}{q}-\frac{n}{p}-\mu} \parallel f \parallel_{M_p^\lambda(Q(a,b))}. \tag{2.24}$$

2) If $r < |b-a|$, by (2.19), we obtain

$$r^{-\mu} L \leq v_n^{\frac{1}{q}-\frac{1}{s}} C_9^{1-\frac{\varrho}{s}} C_{10} |b-a|^{(1-\frac{\varrho}{s})(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})} r^{n(\frac{1}{q}-\frac{1}{s})} r^{\frac{\varrho}{s}(\alpha_1+...+\alpha_n+\frac{n}{s}-\frac{n}{p})-\mu} \parallel f \parallel_{L_p(Q(a,b))}.$$

By (2.14)

$$\frac{n}{q} - \frac{n}{s} + \frac{\rho}{s}(\alpha_1 + \alpha_2 + ... + \alpha_n + \frac{n}{s} - \frac{n}{p}) - \mu \geq 0,$$

therefore

$$r^{-\mu} L \leq v_n^{\frac{1}{q}-\frac{1}{s}} C_9^{1-\frac{\varrho}{s}} C_{10} |b-a|^{\alpha_1+...+\alpha_n+\frac{n}{q}-\frac{n}{p}-\mu} \parallel f \parallel_{L_p(Q(a,b))},$$

by applying inequality (2.1), we get

$$\sup_{x \in Q(a,b), 0<r<|b-a|} r^{-\mu} L \leq v_n^{\frac{1}{q}-\frac{1}{s}} C_9^{1-\frac{\varrho}{s}} C_{10} |b-a|^{\lambda+\alpha_1+...+\alpha_n+\frac{n}{q}-\frac{n}{p}-\mu} \parallel f \parallel_{M_p^\lambda(Q(a,b))}. \tag{2.25}$$

Then inequality (2.13) follows for any $0 < q < s$.

□

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