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ON THE SOLUTION TO A TWO-DIMENSIONAL HEAT CONDUCTION PROBLEM IN A DEGENERATE DOMAIN

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Abstract. In a degenerate domain, namely, the inverted cone, we consider a boundary value problem of heat conduction. For this problem the solvability theorems are established in weighted spaces of essentially bounded functions. The proofs of the theorems are based on the results of the solvability for a nonhomogeneous integral equation of the third kind. The problem under study is reduced to the study of this integral equation using the representation of the solution to the boundary value problem in the form of a sum of constructed thermal potentials.

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1 Introduction

When studying thermophysical processes in an electric arc of high-current shutdown devices, there is an effect of contracting the axial section of the arc into a contact spot in the cathode region. Moreover, the diameter of this spot is much less than the diameter of the section of the developed arc stream, and this allows us to consider this spot as a point.

From the mathematical point of view, the new features of the problem under consideration are, firstly, that the boundary of the domain changes with time, the change in the boundary depends on the conditions for opening the contacts. Secondly, the solution domain degenerates to a point at the initial instant of time, since at the initial instant of time the contacts are in a closed state.

The fundamental difference between boundary value problems for parabolic equations in evolving domains and classical problems (for cylindrical domains) is that methods of separation of variables and integral transformations are not applicable to such problems. The application of the method of thermal potentials allows us to reduce the boundary-value problem with a moving boundary to an Volterra type integral equation of the second kind [10]. If the solution domain degenerates to a point at the initial moment of time, the Volterra-type integral equation becomes special (singular), since the corresponding homogeneous equation, and hence the original homogeneous boundary-value problem, can have nonzero solutions [1]–[5].

A feature of the problem studied in this paper is, namely, the degeneracy of the solution domain to a point at the initial moment of time and the need to study the problem for sufficiently small values of time.

In this paper, we study the following two-dimensional boundary value problem with respect to spatial variables in the inverted cone $G = \{(x; y, t) : x^2 + y^2 < t^2,\ 0 < t < T\}$ for the equation

$$\frac{\partial u(x,\,y,\,t)}{\partial t} = a^2 \left(\frac{\partial^2 u(x,\,y,\,t)}{\partial x^2} + \frac{\partial^2 u(x,\,y,\,t)}{\partial y^2} \right) \tag{1.1}$$

with the boundary value on the lateral surface of the cone

$$u(x, y, t) = u_{\text{c}}(x, y, t),\ \sqrt{x^2 + y^2} = t,\ 0 < t < T, \tag{1.2}$$

where $u_{\text{c}}(x, y, t)$ is a given function.

Under certain physical and technical assumptions, (see [6]) the boundary value problem (1.1)–(1.2) simulates the temperature field in a plasma body of an electrical discharge between high-voltage disconnecting contacts. These contacts were initially in the closed state. Taking into account the short duration of the process, there are no instruments that can measure the specified temperature field. It is necessary, at least qualitatively, to describe the nature of these thermal processes using methods of mathematical modeling.

In order to study (1.1)–(1.2), the solution to the problem was represented as the sum of the constructed thermal potentials, and the problem was reduced to a Volterra type singular integral equation of the second kind, which can be considered as an integral equation of the third kind:

$$t\,\psi(t) - \frac{\lambda}{\sqrt{\pi}} \int\limits_0^t \frac{\psi(\tau)d\,\tau}{\sqrt{t-\tau}} = F(t),\ 0 < t < T < \infty, \tag{1.3}$$

where λ is a given positive constant and $\{F(t),\ t \in (0, T)\}$ is a given function.

Equations of form (1.3) have been the subject of study by many authors. Here, we indicate only the papers [7], [8] and note the numerous studies that are cited in those papers.

2 Preliminary results

In this section, we study solvability issues in the class of essentially bounded functions $\psi \in L_\infty(0;\ +\infty)$ for the integral equation of the third kind (1.3), where λ is a given positive constant and F is a given function such that $F(t)/t \in L_\infty(0;\ +\infty)$.

The following lemmas hold.

Lemma 2.1. *The homogeneous integral equation*

$$t\psi(t) - \frac{\lambda}{\sqrt{\pi}} \int\limits_0^t \frac{1}{\sqrt{t-\tau}} \psi(\tau)d\,\tau = 0,\ \psi \in L_\infty(0;\ +\infty),$$

along with the trivial solution has the following nontrivial solution

$$\psi_{hom}(t) = \frac{\lambda}{\sqrt{\pi}} t^{-3/2} \exp\left\{-\frac{\lambda^2}{t}\right\},\ t > 0.$$

Lemma 2.2. *A particular solution ψ_{part} of nonhomogeneous integral equation* (1.3) *has the form*

$$\psi_{part}(t) = \frac{F(t)}{t} + \int\limits_0^t [\tau R(t,\tau)] \frac{F(\tau)}{\tau}\, d\tau,$$

where

$$R(t,\tau) = \frac{\lambda}{\sqrt{\pi}\cdot\tau t^2\sqrt{t-\tau}}\exp\left\{-\frac{4\lambda^2}{t-\tau}\right\}$$
$$+\frac{\lambda^2}{t^{3/2}\tau^{3/2}}\exp\left\{\frac{\lambda^2}{t-\tau}\left(\sqrt{\frac{\tau}{t}}-\sqrt{\frac{t}{\tau}}\right)^2\right\}\operatorname{erfc}\left\{\frac{\lambda}{\sqrt{t-\tau}}\left(\sqrt{\frac{\tau}{t}}+\sqrt{\frac{t}{\tau}}\right)\right\},$$

and erfc *is the* erfc *integral:*

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{+\infty} e^{-\xi^2}\,d\xi.$$

Moreover, for some $C > 0$

$$\tau R(t,\tau) \le \frac{C\,\lambda}{\sqrt{\pi}(t-\tau)^{3/2}}\exp\left\{-\frac{2\lambda^2}{t-\tau}\right\},\quad 0<\tau<t<\infty.$$

The following statement follows from Lemmas 2.1–2.2.

Lemma 2.3. *For all* $F(t)/t \in L_\infty(0;\ +\infty)$ *integral equation* (1.3) *is solvable in the class of essentially bounded functions* $\psi \in L_\infty(0;\ T),\ T < +\infty$.

3 Main results

In this section we study BVP (1.1)–(1.2) in the case of the isotropy property for the angular coordinate using the polar coordinates, and we reduce the problem to an integral equation of form (1.3). Then we prove a solvability theorem (Theorem 3.1) for the obtained integral equation on the base of Preliminary results. Further results of this section are stated in Theorem 3.2 (classes of solutions to the BVP of heat conduction in the polar coordinates) and Theorem 3.3 (classes of solutions to BVP (1.1)–(1.2)). The proofs of these theorems are based on Theorem 3.1.

3.1 Reducing a boundary value problem to an integral equation

Converting to the polar coordinates in problem (1.1)–(1.2) and assuming that the isotropy property is fulfilled for the angular coordinate (the case of the axial symmetry), we encounter the following problem: to find in the domain $\Omega = \{(r,t) : 0<r<t,\ 0<t<T\}$ a solution to the BVP:

$$\frac{\partial u(r,\,t)}{\partial t} = \frac{a^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u(r,\,t)}{\partial r}\right), \tag{3.1}$$

$$\lim_{r\to 0}\frac{u(r,t)}{\ln(1/r)} = u_0(t),\ \ 0<t<T, \tag{3.2}$$

$$\lim_{r\to t} u(r,t) = u_1(t) \equiv u_c(x,y,t)\Big|_{\sqrt{x^2+y^2}=t},\ \ 0<t<T. \tag{3.3}$$

Usually, instead of condition (3.2), a limit relation on the boundedness of the solution is required, that is, $|u(r,t)| \ne \infty$ as $r\to 0$. We assume that the solution $u(r,t)$ may have a singularity as $r\to 0$, that is, we assume that $u(r,t)$ may have some growth as $r\to 0$. We associate this assumption with the property of a fundamental solution to the Laplace operator in the center of the circle. Thus, we admit some growth property of the required solution $u(r,t)$ to equation (3.1) (this will be specified below in Theorems 3.2 and 3.3.

As is known ([9], p. 76, problem 1.2.2-7), the function

$$G(r,\xi,t)=\frac{\xi}{2a^2t}\exp\left\{-\frac{r^2+\xi^2}{4a^2t}\right\}\cdot I_0\left(\frac{r\xi}{2a^2t}\right)$$

is a fundamental solution to equation (3.1), where ξ is a parameter. Here and below, $I_\mu(\eta)$ is the modified Bessel function.

Using Green's formula as in ([10], p. 476–480) we write the integral representation of the solution to equation (3.1):

$$u(r,t)=\int\limits_0^t\frac{\partial G(r,\xi,t-\tau)}{\partial\xi}\Big|_{\xi=0}\nu(\tau)\,d\tau+\int\limits_0^t\frac{\partial G(r,\xi,t-\tau)}{\partial\xi}\Big|_{\xi=\tau}\varphi(\tau)\,d\tau. \tag{3.4}$$

In equality (3.4), the functions ν and φ are unknown and to be determined. Note that the function u defined by (3.4) for any given functions ν and φ satisfies equation (3.1).

A solution of (3.4) should satisfy conditions (3.2) – (3.3). We obtain $\nu(t)=a^2u_0(t)$ and an integral equation of form (1.3) (an integral equation of the third kind):

$$t\varphi_1(t)-\frac{\lambda}{\sqrt{\pi}}\int\limits_0^t\frac{1}{\sqrt{t-\tau}}\varphi_1(\tau)\,d\tau=f_1(t),\ 0<t<T,\ \lambda=\frac{a}{2}, \tag{3.5}$$

where

$$\varphi_1(t)=t^{-1/2}\exp\left\{\frac{t}{4a^2}\right\}\varphi(t)\in L_\infty(0,T), \tag{3.6}$$

$$f_1(t)=-2a^2t^{1/2}\exp\left\{\frac{t}{4a^2}\right\}\left[f_0(t)-\frac{\exp\{-t/(4a^2)\}}{4a^3\sqrt{\pi}}\int\limits_0^t\frac{\tau}{t^{1/2}(t-\tau)^{1/2}}\varphi_1(\tau)d\tau\right],$$

$$f_0(t)=-\int\limits_0^t\frac{1}{2(t-\tau)}\exp\left\{-\frac{t^2}{4a^2(t-\tau)}\right\}u_0(\tau)d\tau+u_1(t)-\sum_{k=0}^{2}[K_k\varphi_1](t),$$

$$[K_0\varphi_1](t)=\int\limits_0^t\frac{1}{2a^2(t-\tau)}\exp\left\{-\frac{t^2+\tau^2}{4a^2(t-\tau)}\right\}\Delta I_0\left(\frac{t\tau}{2a^2(t-\tau)}\right)\varphi_1(\tau)d\tau,$$

$$[K_1\varphi_1](t)=\int\limits_0^t\frac{\tau}{4a^4(t-\tau)}\exp\left\{-\frac{t^2+\tau^2}{4a^2(t-\tau)}\right\}\Delta I_0\left(\frac{t\tau}{2a^2(t-\tau)}\right)\varphi_1(\tau)d\tau,$$

$$[K_2\varphi_1](t)=\int\limits_0^t\frac{t\tau}{4a^4(t-\tau)^2}\exp\left\{-\frac{t^2+\tau^2}{4a^2(t-\tau)}\right\}\Delta I_{01}\left(\frac{t\tau}{2a^2(t-\tau)}\right)\varphi_1(\tau)d\tau,$$

$$\Delta I_0(\alpha)=I_0(\alpha)-\tilde{I}_0(\alpha),\quad \tilde{I}_0(\alpha)=\frac{\exp\{\alpha\}}{\sqrt{2\pi\alpha}},\ \tilde{I}_{01}(\alpha)=\frac{\exp\{\alpha\}}{2\sqrt{2\pi}\alpha^{3/2}},\ \alpha\in(0,\infty).$$

3.2 Solvability theorems

In order to apply the preliminary results, namely, the result of Lemma 2.3, we show that the function $f_1(t)/t$ is bounded on the interval $(0, T)$.

Note also that according to (3.6), the function φ_1 is a bounded function on R_+.

Hence, the following theorem on the solvability of integral equation (3.5) is valid:

Theorem 3.1. *Let $t^{-1/2} f_1(t) \in L_\infty(0, T)$. Then integral equation* (3.5) *has the general solution*

$$\varphi_1(t) = C\varphi_{1hom}(t) + \varphi_{1part}(t) \in L_\infty((0, T); t^{-1/2}),$$

i.e., $t^{-1/2}\varphi_1(t) \in L_\infty(0, T)$, where $C = const$, $\varphi_{1hom}(t)$ and $\varphi_{1part}(t)$ are solutions to homogeneous (when $f(t) \equiv 0$) and nonhomogeneous integral equations (3.5)*, respectively.*

We formulate the main results of this section.

Theorem 3.2. *Let $t^{-1}u_0(t),\ t^{-1/2}u_1(t) \in L_\infty(0, T)$. Then BVP (3.1)–(3.3) has the general solution*

$$u(r, t) = Cu_{hom}(r, t) + u_{part}(r, t) \in L_\infty(\Omega; r^{1/2}),$$

i.e., $r^{1/2}u(r, t) \in L_\infty(\Omega)$, where $C = const$, $u_{hom}(r, t)$ and $u_{part}(r, t)$ are solutions to homogeneous (when $u_0(t) \equiv 0,\ u_1(t) \equiv 0$) and nonhomogeneous boundary value problems (3.1)–(3.3)*, respectively.*

For the axisymmetric case, the following result follows from Theorem 3.2.

Theorem 3.3. *Let $t^{-1/2}u_1(t) \equiv t^{-1/2}u_c(x, y, t)\big|_{\sqrt{x^2+y^2}=t} \in L_\infty(0, T)$. Then BVP (1.1)–(1.2) has the general solution*

$$u(x, y, t) = Cu_{hom}(x, y, t) + u_{part}(x, y, t) \in L_\infty(G; (x^2 + y^2)^{1/4}),$$

i.e., $(x^2 + y^2)^{1/4}u(x, y, t) \in L_\infty(G)$, where $C = const$, $u_{hom}(x, y, t)$ and $u_{part}(x, y, t)$ are solutions to homogeneous (when $u_c(x, y, t) \equiv 0$) and nonhomogeneous boundary value problems (1.1)–(1.2)*, respectively.*

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