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MIKHAIL L'VOVICH GOLDMAN

(to the 75th birthday)

Mikhail L'vovich Goldman was born on April 13, 1945 in Moscow. In 1963 he graduated from school in Moscow and entered the Physical Faculty of the M.V. Lomonosov Moscow State University (MSU) from which he graduated in 1969 and became a PhD student (1969–1972) at the Mathematical Department of this Faculty. In 1972 he has defended the PhD thesis, and in 1988 his DSc thesis "The study of spaces of differentiable functions of many variables with generalized smoothness" at the S.L. Sobolev Institute of Mathematics in Novosibirsk. Scientific degree "Professor in Mathematics" was awarded to him in 1991.

From 1974 to 2000 M.L. Goldman was successively an assistant Professor, Full Professor, Head of the Mathematical Department at the Moscow Institute of Radio Engineering, Electronics and Automation (technical university). Since 2000 he is a Professor of the S.M. Nikol'skii Mathematical Institute at the Peoples Friendship University of Russia (RUDN University).

Research interests of M.L. Goldman are: the theory of function spaces, optimal embeddings, integral inequalities, spectral theory of differential operators. Main achievements: optimal embeddings of spaces with generalized smoothness, sharp conditions of the convergence of spectral expansions, descriptions of integral and differential properties of the generalized Bessel and Riesz-type potentials, sharp estimates for operators on cones and optimal envelopes for the cones of functions with properties of monotonicity. Professor M.L. Goldman has over 140 scientific publications in leading mathematical journals.

Under his scientific supervision, 8 candidate theses in Russia and 1 thesis in Kazakhstan were successfully defended. Some of his former students are now professors in Ethiopia, Columbia, Mongolia.

Participation in scientific and organizational activities of M.L. Goldman is well known. He is a member of the DSc Councils at RUDN and MSU, of the PhD Council in the Lulea Technical University (Sweden), a member of the Editorial Board of the Eurasian Mathematical Journal, an invited lector and visiting professor at universities of Russia, Germany, Sweden, UK etc., an invited speaker at many international conferences.

The mathematical community, friends and colleagues and the Editorial Board of the Eurasian Mathematical Journal cordially congratulate Mikhail L'vovich Goldman on the occasions of his 75th birthday and wish him good health, happiness, and new achievements in mathematics and mathematical education.

Short communications

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SOLUTION OF THE NEUMANN PROBLEM FOR ONE FOUR-DIMENSIONAL ELLIPTIC EQUATION

A.S. Berdyshev, A. Hasanov, A.R. Ryskan

Communicated by M. Otelbaev

Key words: Neumann problem, energy-integral method, degenerate four-dimensional elliptic equation, Gauss-Ostrogradsky formula, fundamental solutions, Lauricella hypergeometric functions.

AMS Mathematics Subject Classification: 35J25, 35J70.

Abstract. In this article we investigate the Neumann problem for a degenerate elliptic equation in four variables. A fundamental solution is used to construct a solution to the problem. The fundamental solutions are written by using the Lauricella's hypergeometric functions. The energy-integral method is used to prove the uniqueness of the solution to the problem under consideration. In the course of proving the existence of the problem solution, differentiation formulas, decomposition formulas, some adjacent relations formulas and the autotransformation formula of hypergeometric functions are used. The Gauss-Ostrogradsky formula is used to express problem's solution in an explicit form.

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1 Introduction

Special functions are used for solving many problems of mathematical physics. These include the Gauss hypergeometric series, Bessel and Hermite functions, multidimensional Lauricella hypergeometric functions, etc. The Hermite functions are actively applied in algorithms and information systems that are used in medical diagnostics [12]. The Bessel functions are used in solving a number of problems of hydrodynamics, radiophysics, and thermal conductivity [10]. Some functions that are used in astronomy can be arranged in hypergeometric series [13]. Multidimensional hypergeometric functions are used in the superstrings theory [5]. In works [1, 4, 6, 11, 14], applications of the Gauss functions and hypergeometric functions of many variables to the solution of modern relevant problems are presented. The study of boundary value problems for degenerate equations is one of the important directions of the modern theory of partial differential equations. It is known that in the formulation and construction of local and nonlocal boundary value problems solutions, the main role is played by fundamental solutions. For instance, in [9], three-dimensional fundamental solutions were obtained for the modified Helmholtz equation, which were used to construct the theory of single layer, double layer, and volume potentials [7]; in [3], Appel's hypergeometric functions are used to construct the theory of double layer potential.

We consider the four-dimensional degenerate elliptic Hellerstedt equation

$$y^m z^k t^l u_{xx} + x^n z^k t^l u_{yy} + x^n y^m t^l u_{zz} + x^n y^m z^k u_{tt} = 0 \tag{1.1}$$

in the region $R_+^4 = \{(x, y, z, t) : x > 0, y > 0, z > 0, t > 0\}$, where m, n, k, l are positive numbers. This equation has four degeneracy hypersurfaces. For equation (1.1) sixteen fundamental solutions were obtained [8]. Fundamental solutions are expressed in terms of the Lauricella hypergeometric functions [2].

2 The Neumann problem

We introduce the following notation:

$$D = \{(x, y, z, t) :\ x > 0, y > 0,\ z > 0, t > 0\},$$

$$S_1 = \{(0, y, z, t) :\ x = 0, y > 0,\ z > 0, t > 0\},$$
$$S_2 = \{(x, 0, z, t) :\ x > 0, y = 0,\ z > 0, t > 0\},$$
$$S_3 = \{(x, y, 0, t) :\ x > 0, y > 0,\ z = 0, t > 0\},$$
$$S_4 = \{(x, y, z, 0) :\ x > 0, y > 0,\ z > 0, t = 0\},$$

$$R^2 = \frac{4}{(n+2)^2}x^{n+2} + \frac{4}{(m+2)^2}y^{m+2} + \frac{4}{(k+2)^2}z^{k+2} + \frac{4}{(l+2)^2}t^{l+2}.$$

The Neumann problem. Find a solution $u(x, y, z, t)$ of equation (1.1) belonging to the class $C^1(\bar{D}) \cap C^2(D)$ and satisfying the conditions:

$$\left.\frac{\partial}{\partial x}u(x, y, z, t)\right|_{x=0} = \nu_1(y, z, t),\quad (y, z, t) \in S_1, \tag{2.1}$$

$$\left.\frac{\partial}{\partial y}u(x, y, z, t)\right|_{y=0} = \nu_2(x, z, t),\quad (x, z, t) \in S_2, \tag{2.2}$$

$$\left.\frac{\partial}{\partial z}u(x, y, z, t)\right|_{z=0} = \nu_3(x, y, t),\quad (x, y, t) \in S_3, \tag{2.3}$$

$$\left.\frac{\partial}{\partial t}u(x, y, z, t)\right|_{t=0} = \nu_4(x, y, z),\quad (x, y, z) \in S_4, \tag{2.4}$$

$$\lim_{R\to\infty} u(x, y, z, t) = 0, \tag{2.5}$$

where $\nu_1(y, z, t), \nu_2(x, z, t), \nu_3(x, y, t), \nu_4(x, y, z)$ are given continuous functions.

We assume that the functions ν_i $(i = \overline{1, 4})$ satisfy the following:

1)

$$\iiint\limits_{S_1} y^m z^k t^l \nu_1(y, z, t)\, dydzdt + \iiint\limits_{S_2} x^n z^k t^l \nu_2(x, z, t)\, dxdzdt$$
$$+ \iiint\limits_{S_3} x^k y^m t^l \nu_3(x, y, t)\, dxdydt + \iiint\limits_{S_4} x^k y^m t^l \nu_4(x, y, z)\, dxdydz = 0;$$

2) ν_i $(i = \overline{1, 4})$ can tend to infinity with an order less than $1 - 2\alpha$, $1 - 2\beta$, $1 - 2\gamma$, $1 - 2\delta$, respectively, as $R \to 0$;

3) for sufficiently large values of R, the following inequalities are valid:

$$|\nu_1(y,z,t)| \le \frac{c_1}{\left[1+\frac{4}{(m+2)^2}y^{m+2}+\frac{4}{(k+2)^2}z^{k+2}+\frac{4}{(l+2)^2}t^{l+2}\right]^{\frac{1-2\alpha+\varepsilon_1}{2}}},$$

$$|\nu_2(x,z,t)| \le \frac{c_2}{\left[1+\frac{4}{(n+2)^2}x^{n+2}+\frac{4}{(k+2)^2}z^{k+2}+\frac{4}{(l+2)^2}t^{l+2}\right]^{\frac{1-2\beta+\varepsilon_2}{2}}},$$

$$|\nu_3(x,y,t)| \le \frac{c_3}{\left[1+\frac{4}{(n+2)^2}x^{n+2}+\frac{4}{(m+2)^2}y^{m+2}+\frac{4}{(l+2)^2}t^{l+2}\right]^{\frac{1-2\gamma+\varepsilon_3}{2}}},$$

$$|\nu_4(x,y,z)| \le \frac{c_4}{\left[1+\frac{4}{(n+2)^2}x^{n+2}+\frac{4}{(m+2)^2}y^{m+2}+\frac{4}{(k+2)^2}z^{k+2}\right]^{\frac{1-2\delta+\varepsilon_4}{2}}}.$$

Here $c_i > 0$ $(i=\overline{1,4})$, $\alpha = \frac{n}{2(n+2)}$, $\beta = \frac{m}{2(m+2)}$, $\gamma = \frac{k}{2(k+2)}$, $\delta = \frac{l}{2(l+2)}$, and $\varepsilon_i > 0$ $(i=\overline{1,4})$ are sufficiently small.

Theorem 2.1. *Let conditions* $1)-3)$ *be satisfied, then Neumann problem* $(1.1),(2.1)-(2.5)$ *has no more than one solution.*

Theorem 2.1 is proved by using the energy integral.

Theorem 2.2. *Let conditions* $1)-3)$ *be satisfied, then there exists a solution to Neumann problem* $(1.1),(2.1)-(2.5)$ *and it is expressed by the following formula:*

$$\begin{aligned} u(x_0,y_0,z_0,t_0) = &-\int_0^\infty\int_0^\infty\int_0^\infty y^m z^k t^l \nu_1(y,z,t)\, g_1(0,y,z,t;x_0,y_0,z_0,t_0)\,dydzdt \\ &-\int_0^\infty\int_0^\infty\int_0^\infty x^n z^k t^l \nu_2(x,z,t)\, g_1(x,0,z,t;x_0,y_0,z_0,t_0)\,dxdzdt \\ &-\int_0^\infty\int_0^\infty\int_0^\infty x^n y^m t^l \nu_3(x,y,t)\, g_1(x,y,0,t;x_0,y_0,z_0,t_0)\,dxdydt \\ &-\int_0^\infty\int_0^\infty\int_0^\infty x^n y^m z^k \nu_4(x,y,z)\, g_1(x,y,z,0;x_0,y_0,z_0,t_0)\,dxdydz. \end{aligned} \tag{2.6}$$

Here

$$\begin{aligned} &g_1(x,y,z,t;x_0,y_0,z_0,t_0) \\ &= k_1\left(r^2\right)^{-\alpha-\beta-\gamma-\delta-1} F_A^{(4)}(\alpha+\beta+\gamma+\delta+1;\alpha,\beta,\gamma,\delta;2\alpha,2\beta,2\gamma,2\delta;\xi,\eta,\zeta,\varsigma) \end{aligned}$$

is one of the fundamental solutions to equation (1.1), $F_A^{(4)}$ *is the Lauricella hypergeometric function, and* $\xi = \frac{r^2-r_1^2}{r^2}$, $\eta = \frac{r^2-r_2^2}{r^2}$, $\zeta = \frac{r^2-r_3^2}{r^2}$, $\varsigma = \frac{r^2-r_4^2}{r^2}$,

$$\begin{aligned} k_1 = &\frac{1}{4\pi^2}\left(\frac{4}{n+2}\right)^{2\alpha}\left(\frac{4}{m+2}\right)^{2\beta}\left(\frac{4}{k+2}\right)^{2\gamma}\left(\frac{4}{l+2}\right)^{2\delta} \\ &\times\frac{\Gamma(\alpha+\beta+\gamma+\delta+1)\,\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(\gamma)\,\Gamma(\delta)}{\Gamma(2\alpha)\,\Gamma(2\beta)\,\Gamma(2\gamma)\,\Gamma(2\delta)}, \end{aligned}$$

$$r^2 = \left(\frac{2}{n+2}x^{\frac{n+2}{2}} - \frac{2}{n+2}x_0^{\frac{n+2}{2}}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} - \frac{2}{m+2}y_0^{\frac{m+2}{2}}\right)^2$$
$$+\left(\frac{2}{k+2}z^{\frac{k+2}{2}} - \frac{2}{k+2}z_0^{\frac{k+2}{2}}\right)^2 + \left(\frac{2}{l+2}t^{\frac{l+2}{2}} - \frac{2}{l+2}t_0^{\frac{l+2}{2}}\right)^2,$$

$$r_1^2 = \left(\frac{2}{n+2}x^{\frac{n+2}{2}} + \frac{2}{n+2}x_0^{\frac{n+2}{2}}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} - \frac{2}{m+2}y_0^{\frac{m+2}{2}}\right)^2$$
$$+\left(\frac{2}{k+2}z^{\frac{k+2}{2}} - \frac{2}{k+2}z_0^{\frac{k+2}{2}}\right)^2 + \left(\frac{2}{l+2}t^{\frac{l+2}{2}} - \frac{2}{l+2}t_0^{\frac{l+2}{2}}\right)^2,$$

$$r_2^2 = \left(\frac{2}{n+2}x^{\frac{n+2}{2}} - \frac{2}{n+2}x_0^{\frac{n+2}{2}}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} + \frac{2}{m+2}y_0^{\frac{m+2}{2}}\right)^2$$
$$+\left(\frac{2}{k+2}z^{\frac{k+2}{2}} - \frac{2}{k+2}z_0^{\frac{k+2}{2}}\right)^2 + \left(\frac{2}{l+2}t^{\frac{l+2}{2}} - \frac{2}{l+2}t_0^{\frac{l+2}{2}}\right)^2,$$

$$r_3^2 = \left(\frac{2}{n+2}x^{\frac{n+2}{2}} - \frac{2}{n+2}x_0^{\frac{n+2}{2}}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} - \frac{2}{m+2}y_0^{\frac{m+2}{2}}\right)^2$$
$$+\left(\frac{2}{k+2}z^{\frac{k+2}{2}} + \frac{2}{k+2}z_0^{\frac{k+2}{2}}\right)^2 + \left(\frac{2}{l+2}t^{\frac{l+2}{2}} - \frac{2}{l+2}t_0^{\frac{l+2}{2}}\right)^2,$$

$$r_4^2 = \left(\frac{2}{n+2}x^{\frac{n+2}{2}} - \frac{2}{n+2}x_0^{\frac{n+2}{2}}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} - \frac{2}{m+2}y_0^{\frac{m+2}{2}}\right)^2$$
$$+\left(\frac{2}{k+2}z^{\frac{k+2}{2}} - \frac{2}{k+2}z_0^{\frac{k+2}{2}}\right)^2 + \left(\frac{2}{l+2}t^{\frac{l+2}{2}} + \frac{2}{l+2}t_0^{\frac{l+2}{2}}\right)^2.$$

Theorem 2.2 is proved by using the fundamental solution g_1 of equation (1.1), the Gauss-Ostrogradsky formula, and various properties of multidimensional hypergeometric functions, such as autotransformation, decomposition, differentiation formulas and adjacent relations. Thus, it was established that the solution of the Neumann problem for equation (1.1) exists, is unique and can be expressed in explicit form (2.6) if conditions 1) – 3) are satisfied.

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