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EURASIAN MATHEMATICAL JOURNAL

Aims and Scope

The Eurasian Mathematical Journal (EMJ) publishes carefully selected original research papers in all areas of mathematics written by mathematicians, principally from Europe and Asia. However papers by mathematicians from other continents are also welcome.

From time to time the EMJ publishes survey papers.

The EMJ publishes 4 issues in a year.

The language of the paper must be English only.

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The EMJ is included in the list of journals recommended by the Committee for Control of Education and Science (Ministry of Education and Science of the Republic of Kazakhstan) and in the list of journals recommended by the Higher Attestation Commission (Ministry of Education and Science of the Russian Federation).

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Figures. Figures should be prepared in a digital form which is suitable for direct reproduction.

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The Editorial Board of the EMJ will monitor and safeguard publishing ethics.

The procedure of reviewing a manuscript, established by the Editorial Board of the Eurasian Mathematical Journal

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1.2. The Managing Editor of the journal determines whether a paper fits to the scope of the EMJ and satisfies the rules of writing papers for the EMJ, and directs it for a preliminary review to one of the Editors-in-chief who checks the scientific content of the manuscript and assigns a specialist for reviewing the manuscript.

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2.2. A review should include a qualified analysis of the material of a paper, objective assessment and reasoned recommendations.

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- compliance of the paper with the scope of the EMJ;
- compliance of the title of the paper to its content;
- compliance of the paper to the rules of writing papers for the EMJ (abstract, key words and phrases, bibliography etc.);
- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);
- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

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At the end of year 2019 there is 10th anniversary of the activities of the Eurasian Mathematical Journal. Volumes EMJ 10-4 and EMJ 11-1 are dedicated to this event.

VLADIMIR DMITRIEVICH STEPANOV

(to the 70th birthday)

Vladimir Dmitrievich Stepanov was born on December 13, 1949 in a small town Belovo, Kemerovo region. In 1966 he finished the Lavrentiev school of physics and mathematics at Novosibirsk academic town-ship and the same year he entered the Faculty of Mathematics of the Novosibirsk State University (NSU) from which he has graduated in 1971 and started to teach mathematics at the Khabarovsk Technical University till 1981 with interruption for postgraduate studies (1973-1976) in the NSU.

In 1977 he has defended the PhD dissertation and in 1985 his doctoral thesis "Integral convolution operators in Lebesgue spaces" in the S.L. Sobolev Institute of Mathematics. Scientific degree "Professor of Mathematics" was awarded to him in 1989. In 2000 V.D. Stepanov was elected a corresponding member of the Russian Academy of Sciences (RAS).

Since 1985 till 2005 V.D. Stepanov was the Head of Laboratory of Functional Analysis at the Computing Center of the Far Easten Branch of the Russian Academy of Science.

In 2005 V.D. Stepanov moved from Khabarovsk to Moscow with appointment at the Peoples Friendship University of Russia as the Head of the Department of Mathematical Analysis (retired in 2018). Also, he was hired at the V.A. Steklov Mathematical Institute of RAS at the Function Theory Department.

Research interests of V.D. Stepanov are: the theory of integral and differential operators, harmonic analysis in Euclidean spaces, weighted inequalities, duality in function spaces, approximation theory, asymptotic estimates of singular, approximation and entropy numbers of integral transformations, and estimates of the Schatten-Neumann type. Main achievements: the theory of integral convolution operators is constructed, the criteria for the boundedness and compactness of integral operators in function spaces are obtained, weighted inequalities and the behaviour of approximation numbers of the Volterra, Riemann-Liouville, Hardy integral operators are studied, etc.

Under his scientific supervision 15 candidate theses in Russia and 5 PhD theses in Sweden were successfully defended. Professor V.D. Stepanov has over 100 scientific publications including 3 monographs. Participation in scientific and organizational activities of V.D. Stepanov is well known. He is a member of the American Mathematical Society (since 1987) and a member of the London Mathematical Society (since 1996), Deputy Editor of the Analysis Mathematica, member of the Editorial Board of the Eurasian Mathematical Journal, invited speaker at many international conferences and visiting professor of universities in USA, Canada, UK, Spain, Sweden, South Korea, Kazakhstan, etc.

The mathematical community, many his friends and colleagues and the Editorial Board of the Eurasian Mathematical Journal cordially congratulate Vladimir Dmitrievich on the occasion of his 70th birthday and wish him good health, happiness and new achievements in mathematics and mathematical education.

INTERNATIONAL CONFERENCE "ACTUAL PROBLEMS OF ANALYSIS, DIFFERENTIAL EQUATIONS AND ALGEBRA" (EMJ-2019), DEDICATED TO THE 10TH ANNIVERSARY OF THE EURASIAN MATHEMATICAL JOURNAL

From October 16 to October 19, 2019 at the L.N. Gumilyov Eurasian National University (ENU) the International Conference "Actual Problems of Analysis, Differential Equations and Algebra" (EMJ-2019) was held. The conference was dedicated to the 10th anniversary of the Eurasian Mathematical Journal (EMJ).

The purposes of the conference were to discuss the current state of development of mathematical scientific directions, expand the number of potential authors of the Eurasian Mathematical Journal and further strengthen the scientific cooperation between the Faculty of Mechanics and Mathematics of the ENU and scientists from other cities of Kazakhstan and abroad.

The partner universities for the organization of the conference were the M.V. Lomonosov Moscow State University, the Peoples' Friendship University of Russia (the RUDN University, Moscow) and the University of Padua (Italy).

The conference was attended by more than 80 mathematicians from the cities of Almaty, Aktobe, Karaganda, Nur-Sultan, Shymkent, Taraz, Turkestan, as well as from several foreign countries: from Azerbaijan, Germany, Greece, Italy, Japan, Kyrgyzstan, Russia, Tajikistan and Uzbekistan.

The chairman of the International Programme Committee of the conference was Ye.B. Sydykov, rector of the ENU, co-chairmen were Chief editors of the EMJ: V.I. Burenkov, professor of the RUDN University, M. Otelbaev, academician of the National Academy of Sciences of the Republic of Kazakhstan (NAS RK), V.A. Sadovnichy, academician of the Russian Academy of Sciences (RAS), rector of the M.V. Lomonosov Moscow State University (MSU).

There were three sections at the conference: "Function Theory and Functional Analysis", "Differential Equations and Equations of Mathematical Physics" and "Algebra and Model Theory". 16 plenary presentations of 30 minutes each and more than 60 sectional presentations of 20 minutes each, devoted to contemporary areas of mathematics, were given.

It was decided to recommend selected reports of the participants for publication in the Eurasian Mathematical Journal and the Bulletin of the Karaganda State University (series "Mathematics").

Before the conference, a collection of abstracts of the participants' talks was published.

PROGRAMME OF THE INTERNATIONAL CONFERENCE EMJ-2019

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Conference Schedule:

16.10.2019
09.00 – 10.00 Registration
10.00 – 10.30 Opening of the conference
10.30 – 12.50 Plenary talks
12.50 – 14.00 Lunch
14.00 – 18.00 Session talks

17.10.2019
09.30 – 12.20 Plenary talks
12.20 – 14.00 Lunch
14.00 – 18.00 Session talks
18.00 – Dinner for participants of the conference

18.10.2019
09.30 – 13.00 Plenary talks
12.20 – 14.00 Lunch
14.00 – 17.00 Excursion around the city

19.10.2019
09.30 – 12.30 Plenary talks
12.30 – 13.00 Closing of the conference

At the opening ceremony welcome speeches were given by Ye.B. Sydykov, rector of the ENU, chairman of the Program Committee of the conference; V.I. Burenkov, professor of the RUDN Uni-

versity, editor-in-chief of the EMJ; L. Mukasheva, official representative of the international company Clarivate Analytics in the Central Asian region; A. Ospanova, official representative of Scopus.

Plenary talks were given by

T.Sh. Kalmenov (Kazakhstan), M. Otelbaev and B.D. Koshanov (Kazakhstan), P.D. Lamberti and V. Vespri (Italy) – on 16.10.2019;

V.I. Burenkov (Russia), T. Ozawa (Japan), H. Begehr (Germany), M.A. Sadybekov and A.A. Dukenbaeva (Kazakhstan), D. Suragan (Kazakhstan) – on 17.10.2019;

M.L. Goldman (Russia), A. Bountis (Greece), A.K. Kerimbekov (Kyrgyzstan), S.N. Kharin (Kazakhstan), M.I. Dyachenko (Russia) – on 18.10.2019;

E.D. Nursultanov (Kazakhstan), M.A. Ragusa (Italy), P.D. Lamberti and V. Vespri (Italy), M.G. Gadoev (Russia) and F.S. Iskhokov (Tajikistan) – on 19.10.2019.

At the closing ceremony all participants unanimously congratulated the staff of the L.N. Gumilyov Eurasian National University and the Editorial Board of the Eurasian Mathematical Journal with the 10th anniversary of the journal and wished further creative successes.

They expressed hope that the journal will continue to play an important role in the development of mathematical science and education in Kazakhstan in the future.

V.I. Burenkov, K.N. Ospanov, A.M. Temirkhanova

Short communications

EURASIAN MATHEMATICAL JOURNAL
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EXISTENCE AND MAXIMAL REGULARITY OF SOLUTIONS IN $L_2(\mathbb{R}^2)$ FOR A HYPERBOLIC TYPE DIFFERENTIAL EQUATION WITH QUICKLY GROWING COEFFICIENTS

M.B. Muratbekov, Ye.N. Bayandiyev

Communicated by K.N. Ospanov

Key words: hyperbolic type equation, maximal regularity, an unbounded domain, nonsmooth coefficients.

AMS Mathematics Subject Classification: 35M10.

Abstract. In this paper the problem of the existence of solutions is studied for a hyperbolic type differential equation defined in an unbounded domain. The problem of the smoothness of solutions is also considered here. Such problems are of particular interest when the coefficients are unbounded. The novelty of the work is that the weighted coercive estimate is obtained for the solutions of a hyperbolic type differential equation with quickly growing coefficients.

DOI: https://doi.org/10.32523/2077-9879-2020-11-1-95-100

We consider the following differential equation with unbounded coefficients

$$(L+\lambda I)u=\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}+b(y)u_x+q(y)u+\lambda u=f(x,y)\in L_2(\mathbb{R}^2) \tag{1}$$

where $(x,y)\in\mathbb{R}^2$, $\lambda\geq 0$, $\mathbb{R}=(-\infty,\infty)$. Throughout the paper we assume that

i) $b(y)$, $q(y)$ are continuous functions and $|b(y)|\geq\delta_0>0$, $q(y)\geq\delta>0$.

We note that a comprehensive bibliography on the existence, uniqueness and qualitative behaviour of solutions of hyperbolic type differential equations is contained, for example, in the papers of J. Hadamard [10], C. Friedrichs [8], S.L. Sobolev [26], L. Gording [9], O.A. Ladyzhenskaya [16], A.V. Bitsadze [1], J. Leray [18], A. Nahushev [23], T.Sh. Kalmenov [12], T.I. Kiguradze [13, 14], A.V. Filinovskii [7] etc. In these papers, Darboux, Goursat and Cauchy problems, periodic and boundary value problems with constant or variable bounded coefficients were studied. As J. Leray [18] noted in his work, the study of hyperbolic equation in the whole space is an important and interesting problem. Here, the behaviour of the equation coefficients at infinity plays a significant role. The problem of the solution existence of a hyperbolic type equation with continuous and bounded coefficients posed by M. Nagumo was studied in [22].

In this paper we study in the space $L_2(\mathbb{R}^2)$ the problem of the existence of solutions for a class of hyperbolic type singular differential equations with quickly increasing coefficients. We also study the estimates of the norms of solutions and their first-order derivatives with weights. The problem of estimation of weighted Sobolev norms of solutions was previously proposed by I.M. Gelfand for elliptic equations [17].

Before studying these issues, we give the following definitions.

Definition 1. A function $u \in L_2(\mathbb{R}^2)$ is called a solution of equation (1), if there exists a sequence $\{u_n(x,y)\} \subset C_0^\infty(\mathbb{R}^2)$ ($C_0^\infty(\mathbb{R}^2)$ is the set of all infinitely continuously differentiable functions with compact support) such that $\|u_n - u\|_2 \to 0$ and $\|(L+\lambda I)u_n - f\|_2 \to 0$ as $n \to \infty$. Here $\|\cdot\|_2$ is the norm in $L_2(\mathbb{R}^2)$.

Definition 2 [11, 15, 27]. We say that a solution u of the hyperbolic equation (1) is maximally regular if the following estimate

$$\|u_{xx} - u_{yy}\|_2 + \|u_y\|_2 + \|b(y)u_x\|_2 + \|q(y)u\|_2 \le c\|f\|_2 \tag{2}$$

holds, where c>0 is independent of u.

The problem of the maximal regularity was investigated for elliptic equations in [2–6,19,24,25]. The main results of this paper are following.

Theorem 1. *Let condition i) be satisfied and $\lambda \ge 0$. Then for any $f \in L_2(\mathbb{R}^2)$ there exists a unique solution of equation (1).*

Suppose that the coefficients $b(y)$ and $q(y)$ satisfy $i)$ and the following conditions

ii) $\mu = \sup\limits_{|y-t|\le 1} \frac{b(y)}{b(t)} < \infty$, $\mu = \sup\limits_{|y-t|\le 1} \frac{q(y)}{q(t)} < \infty$;

iii) $q(y) \le c_0 \cdot b^2(y)$, $\forall y \in \mathbb{R}$ ($c_0 > 0$ is a constant).

Theorem 2. *Let conditions $i) - iii)$ be satisfied. Then there exists a unique solution u of equation (1), which is maximally regular.*

Example. It is easy to verify that the conditions of Theorems 1 and 2 are satisfied for the following equation

$$Lu = u_{xx} - u_{yy} + e^{100|y|}u_x + e^{10|y|}u = f_1(x,y) \in L_2(\mathbb{R}^2).$$

Hence, there exists a unique solution u of this equation such that the following estimate

$$\|u_{xx} - u_{yy}\|_2 + \|u_y\|_2 + \|e^{100|y|}u_x\|_2 + \|e^{10|y|}u\|_2 \le c\|f_1\|_2$$

holds, where $c > 0$ is independent of u.

Using the Fourier method the study of equation (1) in the space $L_2(\mathbb{R}^2)$ can be reduced to the study of the following differential operator with a negative parameter $-t^2$

$$(l_t + \lambda I)u = -u'' + \left[-t^2 + itb(y) + q(y) + \lambda\right] u,\ u \in \mathbb{D}(l_t), -\infty < t < \infty.$$

The indicated operator is the well-known Sturm-Liouville operator when $t = 0$. The parameter $-t^2$ in the coefficient $-t^2 + itb(y) + q(y) + \lambda$ tends to $-\infty$ when $|t| \to \infty$. Therefore, the differential operator $l_t + \lambda I$ is not semi-bounded. In this case a completely different situation arises in comparison with the Sturm-Liouville operator.

Let $\triangle_j = (j-1, j+1)$, $j \in \mathbb{Z}$. Then $\bigcup\limits_{\{j\}} \triangle_j = \mathbb{R}$. We take a set of non-negative functions $\{\varphi_j\}_{j=-\infty}^{j=\infty}$ of $C_0^\infty(\mathbb{R})$ such that *supp* $\varphi_j \subset \triangle_j$ $(j \in \mathbb{Z})$, $\sum\limits_j \varphi_j^2 \equiv 1$.

Remark. It is easy to verify that the supports of functions φ_j $(j \in \mathbb{Z})$ have no more than a triple intersection, i.e. every point $y \in \mathbb{R}$ can belong to no more than three segments from the system of segments $\{supp\ \varphi_j\}_{j=-\infty}^{j=\infty}$.

We extend $b(y)$ and $q(y)$ from $\triangle_j$ to all $\mathbb{R}$ such that their extensions $b_j(y)$ and $q_j(y)$, respectively, are bounded and periodic functions of the same period. We denote by $l_{t,j,\alpha} + \lambda I$ the closure of the following operator

$$(l_{t,j,\alpha} + \lambda I)u = -u'' + \left[-t^2 + it(b_j(y) + \alpha) + q_j(y) + \lambda\right] u$$

defined on $C_0^\infty(\mathbb{R})$, where the sign of a real number α coincides with the sign of the function $b(y)$, i.e. $\alpha \cdot b(y) > 0$ for $y \in \mathbb{R}$. The number α was introduced in order to obtain estimates of the norm of the operator $\frac{d}{dy}(l_{t,j,\alpha} + \lambda I)^{-1}$.

Lemma 1. *Let condition i) be satisfied and $\lambda \geq 0$. Then*

1) the operator $l_{t,j,\alpha} + \lambda I$ has a continuous inverse $(l_{t,j,\alpha} + \lambda I)^{-1}$ defined in the whole space $L_2(\mathbb{R})$ for $\lambda \geq 0$.

2) the following estimates

a) $\|(l_{t,j,\alpha} + \lambda I)^{-1}\|_{L_2(\mathbb{R})\to L_2(\mathbb{R})} \leq \frac{c}{(\delta+\lambda)^{\frac{1}{2}}}, c(\delta) > 0;$

b) $\|\frac{d}{dy}(l_{t,j,\alpha} + \lambda I)^{-1}\|_{L_2(\mathbb{R})\to L_2(\mathbb{R})} \leq \frac{c}{(\delta+\lambda)^{\frac{1}{4}}}, c > 0;$

c) $\|(l_{t,j,\alpha} + \lambda I)^{-1}\|_{L_2(\mathbb{R})\to L_2(\mathbb{R})} \leq \frac{c}{|t|\cdot\eta_j}, \quad t \neq 0;$

d) $\|(l_{t,j,\alpha} + \lambda I)^{-1}\|_{L_2(\mathbb{R})\to L_2(\mathbb{R})} \leq \frac{2\cdot c}{\mu_j+\lambda}, \quad c > 0,$

hold, where $\eta_j = \min\limits_{y\in\overline{\triangle}_j} |b_j(y)|$, $\mu_j = \min\limits_{y\in\overline{\triangle}_j} q_j(y)$.

Lemma 1 is proved by transforming the functionals $\langle (l_{t,j,\alpha} + \lambda I)u, u\rangle$ and $\langle (l_{t,j,\alpha} + \lambda I)u, -itu\rangle$ and using the computations and argument used to prove Lemma 2.1 in [20] and Lemma 2.3 in [21].

Denote by $l_{t,\alpha} + \lambda I$ the closure of the following differential operator

$$(l_{t,\alpha} + \lambda I)u = -u'' + \left[-t^2 + it(b(y) + \alpha) + q(y) + \lambda\right] u$$

in the space $L_2(\mathbb{R})$ defined on $C_0^\infty(\mathbb{R})$. Suppose $K_{\lambda,\alpha} f = \sum\limits_{\{j\}} \varphi_j (l_{t,j,\alpha} + \lambda I)^{-1} \varphi_j f$. The following statement is proved by reproducing the computations and argument used in [20, 21].

Lemma 2. *Let condition i) be satisfied. Then there is a number $\lambda_0 > 0$ such that the operator $l_{t,\alpha} + \lambda I$ is boundedly invertible for $\lambda \geq \lambda_0$, and the following equality*

$$(l_{t,\alpha} + \lambda I)^{-1} g = K_{\lambda,\alpha}(I - B_{\lambda,\alpha})^{-1} g$$

holds for the inverse operator $(l_{t,\alpha} + \lambda I)^{-1}$, where $g \in L_2(\mathbb{R})$, $\lambda \geq \lambda_0$ and $B_{\lambda,\alpha} g = \sum\limits_{\{j\}} \varphi_j''(l_{t,\alpha} + \lambda I)^{-1}\varphi_j g + 2\sum\limits_{\{j\}} \varphi_j' \frac{d}{dy}(l_{t,j,\alpha} + \lambda I)^{-1}\varphi_j g.$

Lemma 3. *Let condition i) be satisfied and $\lambda > 0$. Then the operator $l_t + \lambda I$ is boundedly invertible and the following equality*

$$(l_t + \lambda I)^{-1} g = (l_{t,\alpha} + \lambda I)^{-1}(I - A_{\lambda,\alpha})^{-1} g, \ g \in L_2(\mathbb{R})$$

holds, where $A_{\lambda,\alpha} = it\alpha(l_{t,\alpha} + \lambda I)^{-1}$, and $\|A_{\lambda,\alpha}\|_{L_2(\mathbb{R})\to L_2(\mathbb{R})} < 1$.

This lemma is proved by the same method as Lemma 3.4 [21].

Proof of Theorem 1. Taking into account condition *i*) it is easy to show that the following estimate

$$\|(L + \lambda I)u\|_2 \geq c\|u\|_2 \tag{3}$$

holds for all $u \in D(L)$, where $c > 0$ is independent of u.

Let $F_{x\to t}$ and $F_{t\to x}^{-1}$ be the direct and inverse Fourier transforms, respectively. By virtue of their properties, in view of Lemma 3, we have

$$u(x, y) = (L + \lambda I)^{-1} f = F_{t\to x}^{-1}(l_t + \lambda I)^{-1}\tilde{f},$$

where $\tilde{f}$ is the Fourier transform with respect to the variable x of the function $f(x, y)$. This equality, by virtue to the continuity of the operator $(l_t + \lambda I)^{-1}$ and by properties of the Fourier transform, holds for all $f \in L_2(\mathbb{R}^2)$. The existence of a solution of equation (1) is proved. The uniqueness of a solution of equation (1) follows from estimate (3). Theorem 1 is proved.

Theorem 2 is proved on the basis of Lemmas 1, 2 and 3 taking into account conditions *ii*) and *iii*).

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