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- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);
- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

2.4. The final part of the review should contain an overall opinion of a reviewer on the paper and a clear recommendation on whether the paper can be published in the Eurasian Mathematical Journal, should be sent back to the author for revision or cannot be published.

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At the end of year 2019 there is 10th anniversary of the activities of the Eurasian Mathematical Journal. Volumes EMJ 10-4 and EMJ 11-1 are dedicated to this event.

ANDREI ANDREEVICH SHKALIKOV

(to the 70th birthday)

Andrei Andreevich Shkalikov, corresponding member of the Russian Academy of Sciences, an outstanding mathematician with a wide range of interests, a remarkable person, professor of the Faculty of Mechanics and Mathematics of the M.V. Lomonosov Moscow State University was born on November 19, 1949.

Andrei Andreevich is a leading specialist in the theory of operators and their applications, especially to problems of mechanics and mathematical physics. He is well known for his work in the theory of functions and in the theory of spaces with indefinite metrics. He is also a specialist in the theory of entire and meromorphic functions and its applications to operator theory.

Andrei Andreevich is known for solving a number of difficult problems that for many years remained unsolved. His work on the basic properties of systems of root functions of differential operators is well known worldwide. He gave a justification for Mandelstam's hypothesis about the existence of solutions satisfying the radiation principle. He solved, in general form, the Rayleigh problem on the reflection of waves from a periodic surface, obtained a solution to the Sobolev problem on the stability of the motion of a top with a cavity filled with liquid. His contribution to the construction of an abstract theory of the Orr-Sommerfeld problem is invaluable. He obtained a description of the limiting spectral portraits for a large class of functions describing the profiles of fluid flows. He is one of the founders of the modern theory of differential operators, coefficients of which are distributions, and inverse problems for such operators.

Andrei Andreevich has been a plenary speaker at many international conferences. He conducts fruitful scientific work and collaborates with many international mathematical research centers.

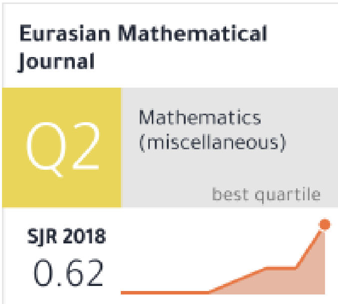
Andrei Andreevich is an author of more than 130 scientific publications. Among his pupils there are more than 20 Candidates of Sciences and 6 Doctors of Sciences. The results obtained by A.A. Shkalikov, his pupils, collaborators and followers gained worldwide recognition.

Professor Shkalikov is also an outstanding organizer. Under his supervision, many international conferences were held. In particular, conferences dedicated to the memory of I.G. Petrovsky, I.M. Gelfand, S.M. Nikol'skii, B.M. Levitan, anniversary conferences of V.A. Sadovnichy, and others.

Andrei Andreyevich is a deputy editor-in-chief of the journals Mathematical Notes, Moscow University Mathematics Bulletin, Moscow University Mechanics Bulletin, and a member of the editorial boards of the Russian Mathematical Surveys, Proceedings of the Moscow Mathematical Society and other journals, including the Eurasian Mathematical Journal.

The Editorial Board of the Eurasian Mathematical Journal cordially congratulates Andrei Andreevich on the occasion of his 70th birthday and wishes him good health, and new achievements in mathematics and mathematical education.

GOOD NEWS: EMJ IS NOW AN SJR Q2 JOURNAL



Recently the lists were published of all mathematical journals included in 2018 SCImago Journal Rank (SJR) quartiles Q1 (385 journals), Q2 (430 journals), Q3 (445 journals), and Q4 (741 journals), and Scopus CiteScore quartiles Q1 (443 journals), Q2 (375 journals), Q3 (348 journals), and Q4 (283 journals).

With great pleasure we inform our readers and authors that the Eurasian Mathematical Journal was included in the most popular scientific ranking database SJR in quartile Q2, currently the only mathematical journal in the Republic of Kazakhstan and Central Asia. The SJR data for the Eurasian Mathematical Journal (2018) is as follows:

550. Eurasian Mathematical Journal (Kazakhstan), Q2, SJR=0.624.
(550 is the number in the list of all Q1 - Q4 journals.)

The SJR indicator is calculated by using the data of the Scopus Database of the Elsevier, the modern publishing business founded in 1880. It uses a sophisticated formula, taking into account various characteristics of journals and journals publications. This formula and related comments can be viewed on the web-page

http : //www.scimagojr.com/journalrank.php.

Some other SJR Q2 mathematical journals:
490. Studia Mathematica (Poland), SJR=0.706,
492. Comptes Rendus Mathematique (France), SJR=0.704,
522. Journal of Mathematical Physics (USA), SJR=0.667,
540. Doklady Mathematics (Russia), SJR=0.636,
570. Journal of Mathematical Sciences (Japan), SJR=0.602,
662. Journal of Applied Probability (UK), SJR=0.523,
733. Mathematical Notes (Russia), SJR=0.465,
791. Canadian Mathematical Bulletin (Canada), SJR=0.433.

Our journal ranks:

7726th place in the list of 31971 scientific journals, representing all subjects and all regions, included in this database (in the first 25% of journals of this category),

225th place in the list of 2519 scientific journals, representing all subjects, of the Asiatic region, included in this database (in the first 10% of journals of this category),

550th place in the list of 2011 mathematical journals, representing all regions, included in this database (in the first 30% of journals of this category),

19th place in the list of 165 mathematical journals of the Asiatic region, included in this database (in the first 15% of journals of this category).

On a separate page the SJR statistics for the Eurasian Mathematical Journal is attached.

Recall that the Eurasian Mathematical Journal started its work in 2010 (see [1]-[3]) and was first included in SJR indicator in 2014 (Q4, SJR=0.101, see [4], [5], [6]). So, the ambitious plan set in [6] was implemented and even essentially exceeded.

As for the Scopus CiteScore indicator, it uses another sophisticated formula, differently taking into account various characteristics journals publications. This formula and related comments can be viewed on the web-page

https : //ru.service.elsevier.com/app/answers/detail/a$_{i}$d/19266/supporthub/scopus.

In this indicator the Eurasian Mathematical Journal was included in quartile Q3. The CiteScore data for the Eurasian Mathematical Journal (2018) is as follows:

333. Eurasian Mathematical Journal (Kazakhstan), Q3, CiteScore = 0,41
(333 is the number in the list of only Q3 journals.)

Some other Scopus CiteScore Q3 mathematical journals:

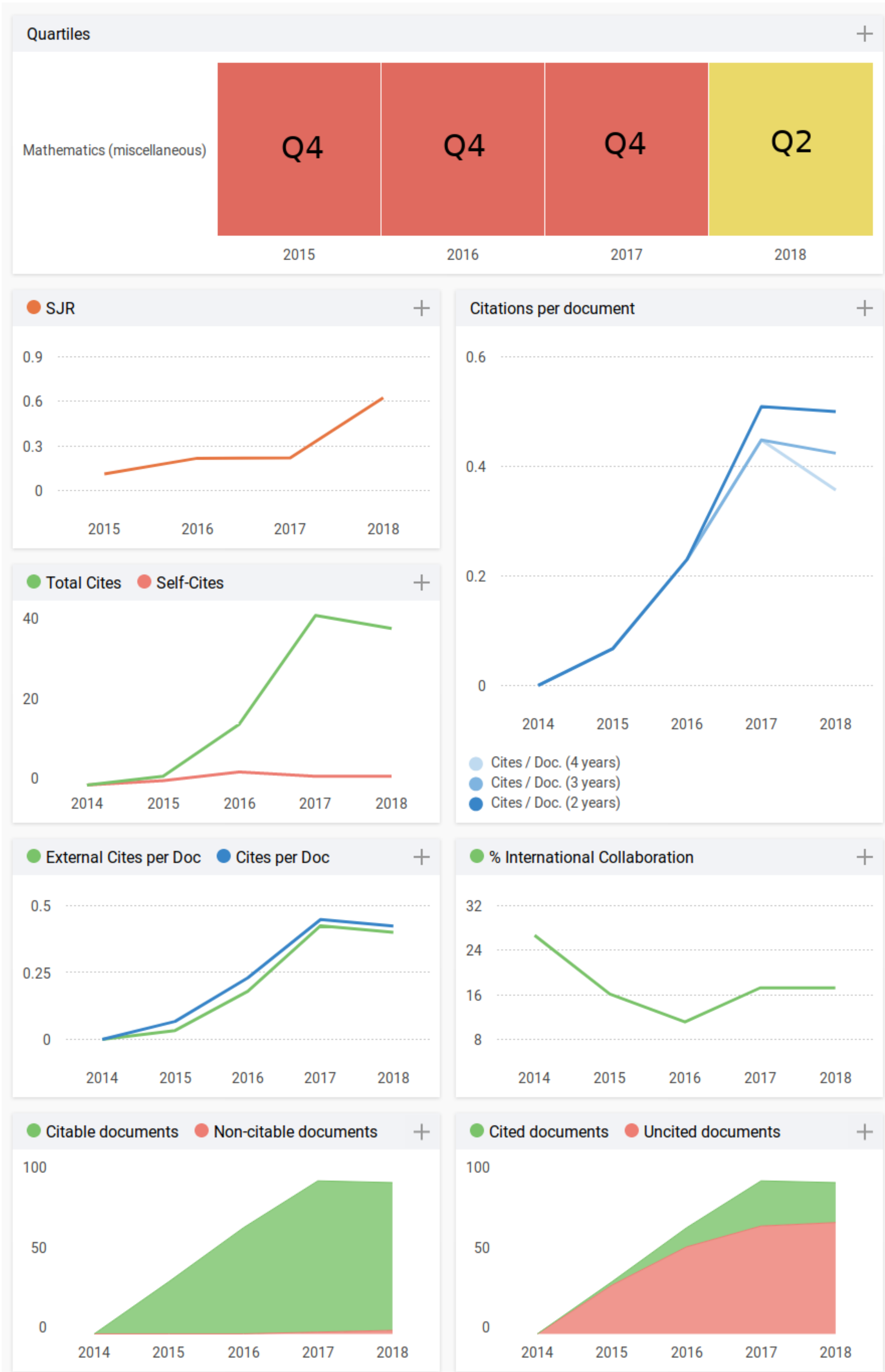
320. Czechoslovak Mathematical Journal (Czech Republic), CiteScore = 0.44,
321. Italian Journal of Pure and Applied Mathematics (Italy), CiteScore = 0.44,
323. Studia Scientiarum Mathematicarum Hungarica (Hungary), CiteScore = 0.44,
332. Bulletin Mathematique de la Societe des Sciences Mathematiques de Roumanie (Romania), CiteScore = 0.41,
334. Indian Journal of Pure and Applied Mathematics (India), CiteScore = 0.41,
33. Transactions of the Moscow Mathematical Society (Russia), CiteScore = 0.41,
337. Illinois Journal of Mathematics (USA), CiteScore = 0.40,
339. Publications de l'Institut Mathematique (France), CiteScore = 0.40.

Our main current aim is to preserve the status of an SJR Q2 journal and of a Scopus CiteScore Q3 journal.

We hope that all respected members of the international Editorial Board, reviewers, current authors of our journal, representing more than 35 countries, and future authors will provide high quality publications in the EMJ which will allow to achieve this aim.

V.I. Burenkov, K.N. Ospanov, T.V. Tararykova, A.M. Temirkhanova.

Extract from http://www.scimagojr.com/journalrank.php
EURASIAN MATHEMATICAL JOURNAL



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Short communications

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ON SMOOTH SOLUTIONS OF A CLASS OF ALMOST HYPOELLIPTIC EQUATIONS OF CONSTANT STRENGTH

H.G. Ghazaryan, V.N. Margaryan

Communicated by M.L. Goldman

Key words: hypoelliptic by Burenkov operator, almost hypoelliptic operator, differential operator of constant strength.

AMS Mathematics Subject Classification: 12E10, 26C05.

Abstract. In this paper we state a new theorem about smoothness of solutions of almost hypoelliptic and hypoelliptic by Burenkov equation $P(x^{'}, D)u = 0$, where the coefficients of the linear differential operator $P(x, D) = P(x_1, ..., x_n, D_1, ..., D_n)$ of uniformly constant strength depend only on the variables $x^{'} = (x_1, ..., x_k), k \leq n$: if the operator $P(x^{'}, D)$ is hypoelliptic by Burenkov and almost hypoelliptic for any $x^{'} \in \mathbb{E}^k$, then all the solutions of the differential equation $P(x^{'}, D)u = 0$ belonging to a certain weighted Sobolev class are infinitely differentiable functions.

DOI: https://doi.org/10.32523/2077-9879-2019-10-4-92-95

1 Introduction

After Levi's significant example (see [7]) of the equation $P(x, D)u = (-iD_1+D_2-2(x_1+x_2)D_3)u = f$, which for some $f \in C^\infty(\mathbb{E}^3)$ does not have smooth solutions in any region of $\mathbb{E}^3$, the question naturally arose about the existence and selection of smooth solutions of different classes of differential equations with variable coefficients (see, for example, [1] - [3], [9] and others).

Precise results on the smoothness of solutions of hypoelliptic or partially hypoelliptic equations with variable coefficients of constant strength are obtained by many authors (see, for example, [4] or [9])

As for more general equations (which are not hypoelliptic or partally hypoelliptic), for some classes of such equations with constant cofficients, the above questions have been studied (see, for example, [4] or [9]).

In [8] we considered this problem for a class of formally almost hypoelliptic equations, the principal part of which has constant coefficients.

Here we consider the case when all the coefficients of a considered equation can be variable, but with the reservation that they can depend only on a certain group of variables. Our goal in this paper is to select infinitely differentiable solutions of equations $P(x, D)u = 0$ for such formally almost hypoelliptic operators $\{ P(x, D) \}$, which have uniformly constant power and are Burenkov's hypoelliptic.

2 Main results

For a fixed $k : 1 \le k \le n$ we set $x^{'} := (x_1, ..., x_k)$ $(\xi^{'} := (\xi_1, ..., \xi_k),\ \alpha^{'} := (\alpha_1, ..., \alpha_k))$, $x^{''} = (x_{k+1}, ..., x_n)$, $(\xi^{''} := (\xi_{k+1}, ..., \xi_n), ...)$.

Definition 1. (see [1] or [2]]) Let $1 \le k \le n$. An operator $R(D)$ (a polynomial $R(\xi)$) is said to be **hypoelliptic by Burenkov** with respect to variables $x^{'} := (x_1, ..., x_k)$ (with respect to variables $(\xi^{'} := (\xi_1, ..., \xi_k))$, if for any $0 \ne \alpha^{'} \in \mathbb{N}_0^k$

$$|D^{\alpha^{'}} P(\xi)|/[1 + |P(\xi)|\,] \to 0 \ \text{ as } \ |\xi| \to \infty.$$

When $k = n$ operator $R(D)$ (a polynomial $R(\xi)$) is said to be **hypoelliptic by Hörmander** (see [4], Definition 11.1.2).

Definition 2. (see [5] and [4]) Let $P(D)$ and $Q(D)$ be linear differential operators with constant coefficients and $P(\xi)$ and $Q(\xi)$ be their symbols (characteristic polynomials). We say that

1) P **is more powerful than** Q and write $P > Q$ or $Q < P$, if for some constant $c > 0$

$$|Q(\xi)| \le c[1 + |P(\xi)|], \ \ \forall \xi \in \mathbb{R}^n,$$

2) P **is stronger than** Q and write $P \succ Q$, or $Q \prec P$ if $\tilde{P}^2 > \tilde{Q}^2$, where $\tilde{R}$ is L. Hörmander's function of operator $R(D)$.

Definition 3 (see [6] or [8]) *An operator $R(D)$ (a polynomial $R(\xi)$) is said to be* ***almost hypoelliptic*** *if* $\tilde{R}^2 < |R|^2$.

Theorem 2.1. *Let $Q < P < Q$. The polynomial P is Burenkov hypoelliptic with respect to the variables $\xi' := (\xi_1, ..., \xi_k)$ if and only if the polynomial Q is Burenkov hypoelliptic with respect to the same variables.*

Let $P(x, D) := \sum_{\alpha \in (x,P)} \gamma_\alpha(x)\, D^\alpha$ be a linear differential operator with coefficients defined in $\mathbb{E}^n$ and $P(x, \xi) := \sum_{\alpha \in (x,P)} \gamma_\alpha(x)\, \xi^\alpha$ be its symbol (characteristic polynomial), where the sum is extended over a finite set $(x, P) := \{\alpha \in \mathbb{N}_0^n, \gamma_\alpha(x) \ne 0\}$ of multi - indices.

In present paper we consider operators (polynomials) whose coefficients depend only on the variables $x' := (x_1, ..., x_k) \in \mathbb{E}^k$, $(k \le n)$:

$$P(x', \xi) := \sum_{\alpha \in (x',\, P)} \gamma_\alpha(x')\, \xi^\alpha, \ \ P_0(\xi) := P(0', \xi). \tag{2.1}$$

Definition 4. A differential operator $P(x', D)$ (a polynomial $P(x', \xi)$ of the form (2.1)) is said to have

a) uniformly constant power in $\mathbb{E}^k$, if for a constant $\kappa_0 = \kappa_0(P) > 0$

$$\kappa_0^{-1}\, |P_0(\xi)| \le |P(x', \xi)| + 1 \le \kappa_0\, [1 + |P_0(\xi)|\,] \ \ \forall x' \in \mathbb{E}^k, \xi \in \mathbb{R}^n,$$

b) uniformly constant strength in $\mathbb{E}^k$, if for a constant $\kappa = \kappa(P) > 0$

$$\kappa^{-1}\, \tilde{P}_0(\xi) \le [\sum_{\alpha \in \mathbb{N}_0^n} |D_\xi^\alpha P(x', \xi)|^2\,]^{1/2} \le \kappa\, \tilde{P}_0(\xi) \ \ \forall x' \in \mathbb{E}^k, \ \xi \in \mathbb{R}^n.$$

Let $\mathcal{A}$ be a set of polynomials of the form (1) with uniformly constant strength in $\mathbb{E}^k$ for which

1) the set (x', P) does not depend on the $x' \in \mathbb{E}^k$: $(x', P) \equiv (0', P)$,

2) the coefficients $\{\gamma_\alpha(x')\}$ are bounded in $\mathbb{E}^k$ together with their derivatives of arbitrary order,

3) the polynomial P_0 is almost hypoelliptic and Burenkov hypoelliptic with respect to the variables $\xi' := (\xi_1, ..., \xi_k)$.

From the definition of class $\mathcal{A}$ and from Theorem 2.1 it immediately follows

Corollary 2.1 *For any polynomial* $P \in \mathcal{A}$ *and for any* $x' \in \mathbb{E}^k$ *the polynomial* $P(x', \xi)$ *is almost hypoelliptic and hypoelliptic by Burenkov.*

For any linear differential operator $R(D)$ with constant coefficients and for any $\delta > 0$, $m \in \mathbb{N}_0$ we denote (see [3])

$$L_{2,\delta} = L_{2,\delta}(\mathbb{E}^n) := \{u; ||u||_{L_{2,\delta}} := (\int_{\mathbb{E}^n} |u(x)\, e^{-\delta\,|x|}|^2 dx)^{1/2} < \infty\},$$

$$W^m_{2,\delta}(\mathbb{E}^n) := \{D^\alpha u \in L_{2,\delta} \ \ \forall \alpha \in \mathbb{N}^n_0 : |\alpha| \leq m\}, W^\infty_{2,\delta} := \bigcap_{m=0}^{\infty} W^m_{2,\delta}(\mathbb{E}^n)$$

$$W^m_{2,\delta}(R) = W^m_{2,\delta}(R, \mathbb{E}^n) := \{u \in L_{2,\delta};\ R(D)u \in W^m_{2,\delta}\}.$$

$$\Phi_0(P, \delta) := \{u \in L_{2,\delta}, P(x', D)u = 0\}.$$

The main result of this paper is

Theorem 2.2. Let the operator $P(x', D) \in \mathcal{A}$ satisfies condition: there exists a number $c > o$ such that $|\xi| \leq c\,[|P_0(\xi)| + 1]$ for all $\xi \in \mathbb{R}^n$. Then there exists a number $\delta_0 > 0$, such that for any $\delta \in (0, \delta_0)$ we have

$$\Phi_0(P, \delta) \subset W^\infty_{2,\delta}(P_0) \subset W^\infty_{2,\delta} \subset C^\infty.$$

Acknowledgments

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