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Short communications

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ON SPECTRAL STABILITY PROBLEM FOR A PAIR OF SELF-ADJOINT ELLIPTIC DIFFERENTIAL OPERATORS ON BOUNDED OPEN SETS

V.I. Burenkov, B.Th. Tuyen

Communicated by P.D. Lamberti

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Abstract. We prove estimates for the variation of the eigenvalues for a pair of self-adjoint elliptic differential operators in the case of diffeomorphic open sets.

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1 Introduction

Let Ω be a bounded open set in $\mathbb{R}^N$,

$$Hu = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha \left(A_{\alpha\beta}(x)D^\beta u\right), \qquad x \in \Omega, \tag{1.1}$$

and

$$Mu = (-1)^k \sum_{|\gamma|=|\eta|=k} D^\gamma \left(B_{\gamma\eta}(x)D^\eta u\right), \qquad x \in \Omega. \tag{1.2}$$

Here $m, k \in \mathbb{N}_0, k < m, A_{\alpha\beta}, B_{\gamma\eta}$ are bounded measurable real-valued functions defined on Ω satisfying the symmetry conditions $A_{\alpha\beta} = A_{\beta\alpha}, B_{\gamma\eta} = B_{\eta\gamma}$ and the uniform ellipticity conditions on Ω: for some $\theta_1, \theta_2 > 0$

$$\sum_{|\alpha|=|\beta|=m} A_{\alpha\beta}(x)\xi_\alpha\xi_\beta \geq \theta_1|\xi|^2, \tag{1.3}$$

$$\sum_{|\gamma|=|\eta|=k} B_{\gamma\eta}(x)\zeta_\gamma\zeta_\eta \geq \theta_2|\zeta|^2, \tag{1.4}$$

for all $x \in \Omega, \xi = (\xi_\alpha)_{|\alpha|=m} \in \mathbb{R}^{\overline{m}}, \zeta = (\zeta_\gamma)_{|\gamma|=k} \in \mathbb{R}^{\overline{k}}$ where $\overline{m}, \overline{k}$ are the numbers of the multi-indices $\alpha = (\alpha_1, ..., \alpha_N) \in \mathbb{N}_0^N, \gamma = (\gamma_1, ..., \gamma_N) \in \mathbb{N}_0^N$ with lengths $|\alpha| = \alpha_1 + ... + \alpha_N = m, |\gamma| = \gamma_1 + ... + \gamma_N = k$.

Let $W^{m,2}(\Omega)$ be the Sobolev space of all functions $u \in L^{1,loc}(\Omega)$ having all weak derivatives $D^\alpha u$ on Ω for $|\alpha| = m$ and such that

$$\|u\|_{W^{m,2}(\Omega)} = \|f\|_{L^2(\Omega)} + \sum_{|\alpha|=m} \|D^\alpha u\|_{L^2(\Omega)} < \infty, \tag{1.5}$$

and $W_0^{m,2}(\Omega)$ be the closure with respect to norm (5) of the set of all infinitely continuously differentiable functions with compact support in Ω.

Let

$$Q_{H,\Omega}(u,v)=\int_\Omega \sum_{|\alpha|=|\beta|=m} A_{\alpha\beta}D^\alpha u D^\beta \overline{v}dx, \qquad Q_{H,\Omega}(u)=Q_{H,\Omega}(u,u),$$

and

$$Q_{M,\Omega}(u,v)=\int_\Omega \sum_{|\gamma|=|\eta|=k} B_{\gamma\eta}D^\gamma u D^\eta \overline{v}dx, \qquad Q_{M,\Omega}(u)=Q_{M,\Omega}(u,u),$$

for all $u,v\in W_0^{m,2}(\Omega)$.

We consider the weak formulation of the Dirichlet eigenvalue problem $Hu=\lambda Mu$, that is:

$$Q_{H,\Omega}(u,v)=\lambda Q_{M,\Omega}(u,v), \tag{1.6}$$

for all test functions $v\in W_0^{m,2}(\Omega)$, with the unknowns $u\in W_0^{m,2}(\Omega)$ and $\lambda\in\mathbb{R}$.

For references on problems of the type $Hu=\lambda Mu$ we quote the book [4], where, in particular, in Chapter 14 spectral stability for problems of this type are studied.

2 Main results

Lemma 2.1. *Let Ω be a bounded open set in $\mathbb{R}^N, l,m\in\mathbb{N}, m<l$. Then the emdedding*

$$W_0^{l,2}(\Omega)\subset W^{m,2}(\Omega)$$

is compact.

The proof is based on the description of precompact sets in $W^{m,2}(\Omega)$ given in [2], page 168, and Sobolev embedding theorems.

Theorem 2.1. *Let Ω be a bounded open set in $\mathbb{R}^N$. Let $m,k\in\mathbb{N}_0, k<m, \theta_1,\theta_2>0$ and for all $\alpha,\beta,\gamma,\eta\in\mathbb{N}_0^N$ such that $|\alpha|=|\beta|=m, |\gamma|=|\eta|=k$, let $A_{\alpha\beta}, B_{\gamma\eta}$ be bounded measurable real-valued functions defined on Ω, satisfying $A_{\alpha\beta}=A_{\beta\alpha}, B_{\gamma\eta}=B_{\eta\gamma}$ and conditions (3), (4).*

Then the spectrum $\sigma(H,M)$ of the eigenvalue problem (6) consists of a countable number of eigenvalues of finite multiplicity

$$0\le\lambda_1\le\lambda_2\le...\le\lambda_n\le..., \qquad \lim_{n\to\infty}\lambda_n=\infty.$$

(Here each eigenvalue is repeated as many times as its multiplicity).

Moreover, there exists a basis in $W_0^{m,2}(\Omega)$ consisting of eigenfunctions φ_k corresponding to eigenvalues $\lambda_k, k\in\mathbb{N}$, and

$$\lambda_n=\inf_{\substack{L\subset W_0^{m,2}(\Omega)\\ dimL=n}}\ \sup_{\substack{u\in L\\ u\neq0}}\frac{Q_{H,\Omega}(u)}{Q_{M,\Omega}(u)}, \qquad n=1,2,... \tag{2.1}$$

For $M=I$ this the celebrated Min-Max principle (see [5] for the proof and further discussions). The case of general M is reduced to the case $M=I$.

Theorem 2.2. *Let U be a bounded open set in $\mathbb{R}^N$. Let $m,\ k\in\mathbb{N}_0,\ k<m,\ B_1,B_2,B_3,B_4,\theta_1,\theta_2,c>0$ and for all $\alpha,\beta,\gamma,\eta\in\mathbb{N}_0^N, |\alpha|=|\beta|=m, |\gamma|=|\eta|=k,\ A_{\alpha\beta},B_{\gamma\eta}$ be measurable real-valued functions defined on U, satisfying $A_{\alpha\beta}=A_{\beta\alpha}, B_{\gamma\eta}=B_{\eta\gamma}$, (3), (4) and the following conditions in U*

$$\max_{|\alpha|=|\beta|=m}|A_{\alpha\beta}(x)|\le B_1, \qquad \max_{|\gamma|=|\eta|=k}|B_{\gamma\eta}(x)|\le B_2,$$

for all $x \in U$.

Moreover, assume that $\Omega \subset U$ *a bounded open set and* ϕ *is a diffeomorphism of* Ω *onto* $\phi(\Omega)$ *of class* C^m, $\phi(\Omega) \subset U$ *satisfying*

$$\max_{1\le|\alpha|\le m} |D^\alpha \phi(x)| \le B_3, \qquad |det \nabla \phi(x)| \ge B_4,$$

for all $x \in \Omega$ *and*

$$Q_{M,\phi(\Omega)}(u \circ \phi^{-1}) \ge c Q_{M,\Omega}(u),$$

for all $u \in W_0^{m,2}(\Omega)$.

Let $\lambda_n(\Omega), n \in \mathbb{N}$ *be the eigenvalues of problem (6) and* $\lambda_n[\phi(\Omega)]$ *be the eigenvalues of problem (6) with* Ω *replaced by* $\phi(\Omega)$.

Then there exit $C_1, C_2 > 0$ *depending only on* $N, m, k, B_1, B_2, B_3, B_4, \theta_1, \theta_2, c$ *such that*

$$|\lambda_n[\phi(\Omega)] - \lambda_n[\Omega]| \le C_1 \lambda_n[\Omega] \mathcal{L}(\phi), \tag{2.2}$$

for all $n \in \mathbb{N}$, *if* $\mathcal{M}(\phi) < C_2$, *where*

$$\mathcal{M}(\phi) = \max\{\mathcal{L}(\phi), \mathcal{L}_H(\phi), \mathcal{L}_M(\phi)\},$$
$$\mathcal{L}_m(\phi) = \max_{1\le|\alpha|\le m} \|D^\alpha(\phi - Id)\|_{L^\infty(\Omega)},$$
$$\mathcal{L}_H(\phi) = \max_{1\le|\alpha|\le m} \|A_{\alpha\beta} \circ \phi - A_{\alpha\beta}\|_{L^\infty(\Omega)},$$
$$\mathcal{L}_M(\phi) = \max_{1\le|\gamma|\le k} \|B_{\gamma\eta} \circ \phi - B_{\gamma\eta}\|_{L^\infty(\Omega)}.$$

For $M = I$ inequality (8) under slightly different assumptions with $1 + \lambda_n(\Omega)$ replacing $\lambda_n(\Omega)$ was proved in [3].

3 Applications

Next we consider the case in which $H = \Delta^2, M = -\Delta$, hence

$$Q_{\Delta^2,\Omega}(u,v) = \int_\Omega \left(\sum_{i,j=1}^n \frac{\partial^2 u}{\partial x_i^2} \frac{\partial^2 \overline{v}}{\partial x_j^2} \right) dx,$$

$$Q_{-\Delta,\Omega}(u,v) = \int_\Omega \left(\sum_{j=1}^n \frac{\partial u}{\partial x_j} \frac{\partial \overline{v}}{\partial x_j} \right) dx,$$

for all $u, v \in W_0^{2,2}(\Omega)$, and the weak formulation of the Dirichlet eigenvalue problem $\Delta^2 u = -\lambda \Delta u$, that is

$$Q_{\Delta^2,\Omega}(u,v) = \lambda Q_{-\Delta,\Omega}(u,v) \tag{3.1}$$

for all test functions $v \in W_0^{2,2}(\Omega)$, with the unknowns $u \in W_0^{2,2}(\Omega), \lambda \in \mathbb{R}$.

If Ω is a bounded domain with C^1-boundary, this problem is equivalent to the problem

$$\begin{cases} \Delta^2 u = -\lambda \Delta u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \\ \frac{\partial u}{\partial \nu} = 0, & \text{on } \partial\Omega. \end{cases} \tag{3.2}$$

This is the famous buckling problem. See paper [1] for references. In particular, in [1] the authors study the differentiability of the eigenvalues of this problem with respect to a perturbation of Ω of the form $\phi(\Omega)$ and provide Hadamard-type formulas.

Theorem 3.1. *Let Ω be a bounded open set in $\mathbb{R}^N$, $B_1, B_2 > 0$ and ϕ be a diffeomorphism of Ω onto $\phi(\Omega)$ of class C^1 such that*

$$\max_{i,j=1,..,N} \max_{x\in\Omega} \left| \frac{\partial \phi_i(x)}{\partial x_j} \right| \leq B_1, \qquad |det \nabla \phi(x)| \geq B_2,$$

for all $x \in \Omega$.

Then there exists $c > 0$ depending only on N, B_1 such that

$$Q_{-\Delta,\phi(\Omega)}(u \circ \phi^{-1}) \geq c Q_{-\Delta,\Omega}(u),$$

for all $u \in W_0^{1,2}(\Omega)$.

The proof is based on Min-Max formula (7) and Lemma 4.1 in [3].

Theorems 2 and 3 imply the following statement.

Theorem 3.2. *Let $B_1, B_2 > 0, \Omega$ be a bounded open set in $\mathbb{R}^N$ and ϕ be a diffeomorphism of Ω onto $\phi(\Omega)$ of class C^2 satisfying*

$$\max_{1\leq|\alpha|\leq 2} |D^\alpha \phi(x)| \leq B_1, \qquad |det \nabla \phi(x)| \geq B_2,$$

for all $x \in \Omega$.

Let $\lambda_n(\Omega), n \in \mathbb{N}$ be the eigenvalues of problem (9) and $\lambda_n[\phi(\Omega)]$ be the eigenvalues of problem (9) with Ω replaced by $\phi(\Omega)$.

Then there exit $C_1, C_2 > 0$ depending only on N, B_1, B_2 such that

$$|\lambda_n[\phi(\Omega)] - \lambda_n[\Omega]| \leq C_1 \lambda_n[\Omega] \mathcal{L}(\phi), \tag{3.3}$$

for all $n \in \mathbb{N}$, if $\mathcal{L}(\phi) < C_2$, where

$$\mathcal{L}_2(\phi) = \max_{1\leq|\alpha|\leq 2} \|D^\alpha(\phi - Id)\|_{L^\infty(\Omega)}.$$

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Events

5TH INTERNATIONAL CONFERENCE "ACTUAL PROBLEMS OF MATHEMATICS AND COMPUTER SCIENCE: THEORY, METHODOLOGY, PRACTICE" (APRIL 18–20, 2019, YELETS, RUSSIA)

The XX century is marked by the enrichment of world science with outstanding achievements in the field of mathematics, solving many important problems that remain relevant in the modern world. Such problems include, in particular, the problems considered in fundamental works of academician S. Chaplygin. Based on his works new research paths were set, and serious applied problems were addressed in the fields of aerodynamics, gas dynamics, hydrodynamics, and mechanics. They were further intensively developed due to the achievements of contemporary information technology.

To commemorate his activities was organized a large-scale scientific event in the historic homeland of academician S. Chaplygin — the 5th international conference "Actual problems of mathematics and computer science: theory, methodology, practice", dedicated to the 150th anniversary of the birth of academician S. Chaplygin.

The Ivan Bunin Yelets State University (Russia), the Samarkand State University (Uzbekistan), the Higher School of Insurance and Finance (Bulgaria),the Khachatur Abovyan Armenian State Pedagogical University (Armenia), and the Scientific and Methodological Council on Mathematics of the Ministry of Science and Higher Education of Russia held through April 18–20, 2019 the 5th International Conference "Actual problems of mathematics and computer science: theory, methodology, practice" dedicated to the 150th anniversary of academician S. Chaplygin.

The conference marked the three major milestones related to the development of mathematical science in the Lipetsk region and in the oldest university center in the region — the Ivan Bunin Yelets State University.

1. April 2019 is the 150th anniversary of the birth of S. Chaplygin (1869–1942) — a well-known Russian scientist, academician of the Academy of Sciences of the USSR. S. Chaplygin is an outstanding representative of the Lipetsk region, whose surname is immortalized in the name of the city Chaplygin (previously Ranenburg) in the Lipetsk region.

2. 2019 year is the 80th anniversary of the foundation of the Faculty of Physics and Mathematics. It is the oldest faculty of the Ivan Bunin Yelets State University, where students of the scientific school of academician N. Zhukovsky were taught, whose famous representative was academician S. Chaplygin.

3. In October 2019 there will be 10 years since the organization of the Lipetsk Branch of the Scientific and Methodological Council for Mathematics of the Ministry of Science and Higher Education of Russia on the basis of the Ivan Bunin Yelets State University.

The main goals of the conference were the creation of conditions for international scientific communication of representatives of fundamental and applied areas in the field of mathematics, understanding the importance of scientific works of S. Chaplygin, the actualization of his scientific achievements, taking into account the rapid development of information technologies and their adaptation to modern mathematical education.

The plenary session of the conference was opened by the Rector of the Ivan Bunin Yelets State University Professor E. Gerasimova and continued by the President of the International Academy of the History of Science Professor S. Demidov (Moscow, Russia), who presented the talk "Pure and Applied Mathematics at the M.V. Lomonosov Moscow State University in the first half of the twentieth century: N. Luzin and S. Chaplygin".

Professor A. Soleev (Samarkand, Uzbekistan) devoted his talk to basic ideas and general provisions of the Power Geometry. Professor A. Soldatov (Moscow, Russia) focused on the consideration of the Dirichlet problem for equations of mixed type. In her talk Professor G. Zhukova (Moscow, Russia) discussed the dependence of solutions to singularly perturbed linear differential systems on a small parameter. The talk of Professors O. Masina (Yelets, Russia) and O. Druzhinina (Moscow, Russia) was devoted to the analysis of the known and developed by the authors approaches to the study of the stability of intelligent control systems. The talks of Professors V. Tikhomirov (Moscow, Russia), T. Sergeeva (Moscow, Russia) and E. Smirnov (Yaroslavl, Russia) addressed the issues of improving mathematical education, introducing novelty into the teaching process while maintaining the best traditions of high-quality teaching mathematics, laid by S. Chaplygin in his productive teaching activities.

The relevance of the event was noted in the talks of Professors A. Abylkasymova (Alma-Ata, Kazakhstan), A. Borovskikh (Moscow, Russia), S. Grozdev (Sofia, Bulgaria), M. Mkrtchyan (Yerevan, Armenia) and other scientists. At the end of the plenary session, talks were presented by the authors of this communication on the history of the Scientific and Methodological Council on Mathematics of the Ministry of Science and Higher Education of Russia and its contribution to the development of mathematics and its applications in Russian education, as well as on the activities of the Lipetsk Branch of the Scientific and Methodological Council.

The following sections were working at the conference: “Modern Directions in Mathematics”, “Applied problems of mathematics”, “Computer modeling, information technologies and systems”, “New theories, models and technologies of teaching mathematics and computer science at schools and universities”, “Actualization of the problems of the history of mathematics and mathematical education in modern conditions”.

At the conference there were more than 250 participants, including leading foreign specialists from Armenia, Bulgaria, Uzbekistan, Kazakhstan, well-known scientists from more than twenty regions of Russia, as well as young researchers. Overall, it was a successful conference, which helped to increase the scientific and innovative activity of the region, stimulated the participants to develop mathematics, information technologies and mathematical education.

S. Dvoryatkina, S. Rozanova, S. Shcherbatykh