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ON GENERALIZED B^*-CONTINUITY, B^*-COVERINGS AND B^*-SEPARATIONS

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Key words: contra B^*-continuity, slight B^*-continuity, B^*-Hausdorff, B^*-normal, B^*-compact.

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Abstract. There are various generalizations of continuous functions in topological spaces and B^*-continuity is one of them which deals with the Baire property and denseness of the space. We have defined and discussed several properties and interrelations of some further generalizations of B^*-continuity, namely, contra B^*-continuity, slight B^*-continuity and weak B^*-continuity. We have also defined certain notions of generalized coverings and separations in terms of B^*-sets and studied the effect of generalized B^*-continuous functions on spaces having these covering and separation properties.

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1 Introduction

Starting from early years of modern mathematics, many different types of generalized continuities have been introduced for functions, e.g., quasi-continuity [9], weak continuity [11], semi-continuity [12], B-continuity, Br-continuity [13, 14], B^*-continuity [4, 5], β-continuity [15] and many more. These continuities have been defined in terms of generalized open sets such as semi-open sets, B sets, B^*-sets, β-open sets etc.

These continuities have been further generalized over the years. In 1973, Popa and Stan [19] introduced weak quasi-continuity which is implied by semi-continuity [12]. In 1980, Jain [8] introduced the notion of slightly continuous functions. Nour [18] defined and investigated slightly semi-continuous functions as a weak form of slight continuity. Noiri and Chae [17] studied various properties of slightly semi-continuous functions. Moreover, Noiri [16] introduced the notion of slight β-continuity which is implied by both slight semi-continuity and faint pre-continuity.

In 1996, Dontchev [1] introduced the concept of contra-continuity and obtained some results concerning compactness, S-closedness and strong S-closedness. In 1999, Dontchev and Noiri [2] introduced and investigated contra-semi-continuous functions. Jafari and Noiri [7], introduced and studied the notion of contra β-continuous functions. In 2006, Ganguly and Mitra generalized B*-continuity as weak B*-continuity [6].

In the literature modified forms of continuity, separation axioms etc. have also been studied by utilizing generalized closed and open sets. Recently, as generalizations of closed sets the notion of β^*-closed sets are introduced and studied [21].

The aim of the present paper is to supplement the above mentioned works. Precisely, we have defined certain generalized coverings and separations using B^*-sets. We have also defined various new classes of generalized continuities such as slight B^*-continuity, and contra B^*-continuity and studied the effect of these generalized B^*-continuities on the spaces having these generalized coverings and separations.

The paper is organized as follows. We have given some definitions and notations in Section 2. In Section 3, we have defined and studied generalized coverings and separations in terms of B^*-sets. In Section 4, we have discussed some properties of generalized B^*-continuities. Finally in Section 5, we have dealt with the relationship among various generalized continuities with the help of a relationship diagram and some examples and counterexamples.

2 Definitions and notations

Throughout the paper, X and Y will denote topological spaces, unless specified otherwise. By $\mathrm{int}(A)$ and $\mathrm{cl}(A)$ we shall denote the interior and closure of the set A.

The following notions are known.

Definition 1. [8] A function $f : X \to Y$ is said to be slightly continuous at $x \in X$ if for every clopen set (i.e. simultaneously open and closed set) V containing $f(x)$, there is an open set $U \subset X$ containing x such that $f(U) \subset V$.

Definition 2. [1] A function $f : X \to Y$ is said to be contra continuous if the pre-image of every open subset of Y is closed in X.

An equivalent definition of contra continuity is given in the following theorem [1]:

Theorem A. *For a function $f : X \to Y$, the following conditions are equivalent:*

(1) *f is contra-continuous,*

(2) *for each $x \in X$ and each closed set V in Y with $f(x) \in V$, there exists an open set U in X such that $x \in U$ and $f(U) \subset V$,*

(3) *the pre-image of each closed set in Y is open in X.*

Definition 3. [4] A subset $A \subset X$ is said to be a B^*-set if it is not nowhere dense having the property of Baire (i.e., $A = (G \setminus I) \cup J$, where G is an open set and I, J are the sets of first Category).

Definition 4. [6] Let $P \subset X$. A point $x \in X$ is said to be a B^*-cluster point of P if for every B^*-set B containing x, $P \cap B \neq \emptyset$. The set of all B^*-cluster point of P is called B^*-cluster derived set of P and denoted by B^*-cls-d-P. A set P is said to be B^*-closed if B^*-cls-d-$P \subset P$. The B^*-closure of $P = P \cup (B^*\text{-cls-d-}P)$ is denoted by $B^*_{\mathrm{cl}}(P)$.

Definition 5. [4] A function $f : X \to Y$ is said to be B^*-continuous at $x \in X$ if for every open set U containing x and for every open set V containing $f(x)$, there is a B^*-set $B \subset U$ containing x such that $f(B) \subset V$.

The notion of weak B^*-continuity was introduced by Ganguly and Mitra [6] for multifunctions. Obviously, for single valued function, the definition can be written as follows.

Definition 6. A function $f : X \to Y$ is said to be weakly B^*-continuous at $x \in X$ if for every open set U containing x and for every open set V containing $f(x)$, there is a B^*-set $B \subset U$ containing x such that $f(B) \subset \mathrm{cl}(V)$.

Next, we define slight B^*-continuity and contra B^*-continuity as follows:

Definition 7. A function $f : X \to Y$ is said to be slightly B^*-continuous at $x \in X$ if for every open set U containing x and for every clopen set V containing $f(x)$, there exists a B^*-set $B \subset U$ containing x such that $f(B) \subset V$.

Definition 8. A function $f : X \to Y$ is said to be contra B^*-continuous at a point $x \in X$ if for every open set U containing x and for every closed set V containing $f(x)$, there is a B^*-set $B \subset U$ containing x such that $f(B) \subset V$.

3 Coverings and separations

We begin this section by defining certain notions of generalized coverings and separations in terms of B^*-sets.

Definition 9. (a) A cover of $A \subset X$ by B^*-sets is said to be a B^*-cover of A.

(b) $A \subseteq X$ is said to be B^*-compact if every B^*-cover of A has a finite subcover.

(c) $A \subseteq X$ is said to be B^*-Lindelöf if every infinite B^*-cover of A has a countable subcover.

Definition 10. A space X is said to be B^*-Hausdorff if every two distinct points of X can be separated by disjoint B^*-sets.

Definition 11. A space X is said to be B^*-regular if for any point $x \in X$ and for any closed set V not containing x, there exist disjoint B^*-sets B_1 and B_2 containing x and V, respectively.

Definition 12. A space X is said to be B^*-normal if for any pair of disjoint closed sets U and V in X, there exist disjoint B^*-sets B_1 and B_2 containing U and V, respectively.

Since every non-empty open set is a B^*-set, the following statement is obvious.

Theorem 3.1. *If $A \subseteq X$ is B^*-compact, then A is compact.*

Theorem 3.2. *Every closed subset of a B^*-compact (B^*-Lindelöf) set is B^*-compact (B^*-Lindelöf).*

Proof. Let X be a B^*-compact set and $A \subset X$ be closed. Then $X \setminus A$ is open in X and hence B^*-set in X. Let $\{B_\alpha\}_{\alpha \in \Lambda}$ be a B^*-cover of A. Then

$$\Big(\bigcup_{\alpha \in \Lambda} B_\alpha\Big) \bigcup \Big(X \setminus A\Big)$$

is a B^*-cover of X. Since X is B^*-compact, there exists a finite cover $\{B_1, B_2, \ldots, B_n\}$ of X. Since $X \setminus A$ is open, it is clear that one of $B_i, i = 1, 2, \ldots, n$, say B_p, is such that

$$B_p = X \setminus A.$$

Then $\{B_1, B_2, \ldots, B_{p-1}, B_{p+1} \ldots, B_n\}$ is a finite B^*-cover covering A and the assertion follows. The case of B^*-Lindelöf sets can be disposed of similarly. □

Theorem 3.3. *Every B^*-compact subset of a B^*-Hausdorff space is B^*-closed.*

Proof. Let X be a B^*-Hausdorff space and $A \subseteq X$ be B^*-compact. Let $x \in X \setminus A$ and $y \in A$ be arbitrary. As X is B^*-Hausdorff, there exist disjoint B^*-sets B_x and B_y containing x and y, respectively. Now

$$A = \bigcup_{y \in A} \{y\} \subseteq \bigcup_{y \in A} B_y,$$

so that $\{B_y\}_{y \in A}$ is a B^*-cover of A and, A being B^*-compact, there exists a finite subcover, say $\{B_{y_1}, B_{y_2}, \ldots, B_{y_n}\}$ such that

$$A \subseteq \bigcup_{i=1}^{n} B_{y_i}.$$

Since $B_x \cap B_{y_i} = \emptyset, i = 1, 2, \ldots, n$, it follows that $x \notin B^*_{\text{cl}}(A)$. □

The next result is based on the following notions.

Definition 13. [1] A set $A \subset X$ is said to be strongly S-closed if every closed cover of A has a finite subcover.

Definition 14. [20] A space X is said to be mildly compact (mildly Lindelöf) if every infinite clopen cover of X has a finite (countable) subcover.

Theorem 3.4. *(a) B^*-continuous image of a B^*-compact (B^*-Lindelöf) set is compact (Lindelöf).*

(b) Contra B^*-continuous image of a B^*-compact set is strongly S-closed.

(c) Slightly B^*-continuous image of a B^*-compact (B^*-Lindelöf) set is mildly compact (mildly Lindelöf).

Proof. (*a*) Let $x \in X$, U_x an open set containing x and $V_{f(x)}$ be an open set containing $f(x)$. Then $\{V_{f(x)} : x \in X\}$ forms an open cover for $f(X)$. By the definition of B^*-continuity of f at x, there exists a B^*-set B_x such that

$$x \in B_x \subseteq U_x \qquad \text{and} \qquad f(B_x) \subseteq V_{f(x)}.$$

Now, $X = \bigcup_{x \in X} B_x$ so that $\{B_x\}$ is a B^*-cover of X and since X is B^*-compact, it admits a finite subcover $\{B_{x_1}, B_{x_2}, \ldots, B_{x_n}\}$, i.e., $X \subseteq \bigcup_{i=1}^{n} B_{x_i}$. Consequently, for every $x \in X$, we have

$$f(x) \in f\Big(\bigcup_{1=1}^{n} B_{x_i}\Big) \subseteq \bigcup_{i=1}^{n} f(B_{x_i}) \subseteq \bigcup_{i=1}^{n} V_{f(x_i)}$$

which implies that $f(X) \subseteq \bigcup_{i=1}^{n} V_{f(x_i)}$ and we are done. The case of B^*-Lindelöf can be disposed of similarly.

Similarly we can prove (*b*) and (*c*). □

In the following three theorems sufficient conditions are given for the space X to be respectively B^*-Hausdorff, B^*-normal and B^*-regular. But first let us recall few definitions.

Definition 15. [20] A space X is said to be ultra Hausdorff, if for every two distinct points x and y in X there exist disjoint clopen sets U and V in X such that $x \in U$ and $y \in V$.

Definition 16. [3] A space X is said to be ultra regular, if for any point $x \in X$ and for any closed set V not containing x, there exist disjoint clopen sets B_1 and B_2 containing, respectively, x and V.

Definition 17. [20] A space X is said to be ultra normal, if for any pair of disjoint closed sets, U and V in X, there exist disjoint clopen sets B_1 and B_2 containing, respectively, U and V.

Theorem 3.5. *Consider an injective function $f : X \to Y$. Then X is B^*-Hausdorff if one of the following holds:*

(a) *f is B^*-continuous and Y is a Hausdorff space,*

(b) *f is contra B^*-continuous and Y is a Urysohn space,*

(c) *f is slightly B^*-continuous and Y is an ultra Hausdorff space.*

Proof. (a) Let x and y be two distinct points in X. Then $f(x) \neq f(y)$ since f is injective. Now, Y being Hausdorff, there exist disjoint open sets V and W containing, respectively, $f(x)$ and $f(y)$. Since f is B^*-continuous, for open sets U_1 and U_2 containing, respectively, x and y, there exist B^*-sets B_1 and B_2 containing, respectively, x and y such that

$$f(B_1) \subseteq V \qquad \text{and} \qquad f(B_2) \subseteq W.$$

Then

$$f(B_1) \cap f(B_2) = \emptyset.$$

Clearly, $B_1 \cap B_2 = \emptyset$, and hence X is B^*-Hausdorff.

(b) Let x and y be two distinct points in X. Since f is injective, $f(x) \neq f(y)$. Now, Y being Urysohn, there exist two open sets V and W containing, respectively, $f(x)$ and $f(y)$ such that

$$\mathrm{cl}(V) \cap \mathrm{cl}(W) = \emptyset.$$

Since f is contra B^*-continuous, for open sets U_1 and U_2 containing, respectively, x and y, there exist B^*-sets B_1 containing x and B_2 containing y such that

$$f(B_1) \subseteq \mathrm{cl}(V) \qquad \text{and} \qquad f(B_2) \subseteq \mathrm{cl}(W).$$

Clearly

$$f(B_1) \cap f(B_2) = \emptyset$$

which gives that $B_1 \cap B_2 = \emptyset$, so that X is B^*-Hausdorff.

(c) We can prove this easily with the help of (b). □

In general, the class of B^*-sets is not closed under arbitrary union. We say that a space X has Property P if an arbitrary union of B^*-sets in X is a B^*-set.

In the next two theorems we assume that X is a space having Property P.

Theorem 3.6. *Consider the injective mapping $f : X \to Y$. Then X will be B^*-normal if one of the following conditions holds:*

(a) *f is contra B^*-continuous and Y be a space with the property that for any two sets V and W in Y,*

$$V \cap W = \emptyset \implies \mathrm{cl}(V) \cap \mathrm{cl}(W) = \emptyset,$$

(b) *f is slightly B^*-continuous closed function and Y is ultra normal.*

Proof. (a) Let U and V be two disjoint closed sets in X. Since f is injective, we have

$$f(U) \cap f(V) = \emptyset \qquad \Longrightarrow \qquad \mathrm{cl}\big(f(U)\big) \cap \mathrm{cl}\big(f(V)\big) = \emptyset.$$

Let $x \in U$. Then $\mathrm{cl}(f(U))$ is a closed set containing $f(x)$ and $X \setminus V$ is an open set containing x. Since f is contra B^*-continuous, there exists a B^*-set B_x such that

$$x \in B_x \subset X \setminus V \qquad \text{and} \qquad f(B_x) \subset \mathrm{cl}\big(f(U)\big).$$

Now

$$U = \bigcup_{x \in U} \{x\} \subseteq \bigcup_{x \in U} \{B_x\} =: B_1.$$

Since X has Property P, it follows that B_1 is a B^*-set in $X \setminus V$ and

$$f(B_1) \subseteq \mathrm{cl}\big(f(U)\big).$$

Similarly, we can obtain a B^*-set B_2 such that

$$V \subset B_2 \subset X \setminus U \qquad \text{and} \qquad f(B_2) \subseteq \mathrm{cl}\big(f(V)\big).$$

Now, since f is injective, it follows that $B_1 \cap B_2 = \emptyset$ and the assertion is proved.

(b) Let U_1 and U_2 be two disjoint closed sets in X. f being a closed injective mapping, $f(U_1)$ and $f(U_2)$ are disjoint closed sets in Y. Since Y is ultra regular, there exist disjoint clopen sets V_1 and V_2 such that

$$f(U_1) \subset V_1 \qquad \text{and} \qquad f(U_2) \subset V_2.$$

Now, let $x \in U_1$. Then $X \setminus U_2$ is an open set containing x and $f(U_1)$ is closed set containing $f(x)$. Since f is slightly B^*-continuous, there exists a B^*-set B_x such that

$$x \in B_x \subset X \setminus U_2 \qquad \text{and} \qquad f(B_x) \subset V_1.$$

We have

$$U_1 = \bigcup_{x \in U_1} \{x\} \subseteq \bigcup_{x \in U_1} B_x = B_1 \subset X \setminus U_2.$$

Hence $U_2 \cap B_1 = \emptyset$. Similarly,

$$U_2 \subset B_2 \subset X \setminus U_1, \qquad \text{i.e.,} \qquad U_1 \cap B_2 = \emptyset.$$

Since, $f(B_1) \subset V_1$, $f(B_2) \subset V_2$ and $V_1 \cap V_2 = \emptyset$ we find that $B_1 \cap B_2 = \emptyset$ and we are done. □

Theorem 3.7. *Consider the injective mapping $f : X \to Y$. Then X will be B^*-regular if one of the following conditions holds:*

(a) *if X is a T_1 space, f is contra B^*-continuous and Y is a space with the property that for any two sets V and W in Y,*

$$V \cap W = \emptyset \implies \mathrm{cl}(V) \cap \mathrm{cl}(W) = \emptyset,$$

(b) *f is a slightly B^*-continuous closed function and Y is ultra regular.*

Proof. (a) Let $x \in X$ and V be a closed set in X not containing x. Then there exists an open set U containing x such that $U \cap V = \emptyset$. Since f is injective, we have

$$f(U) \cap f(V) = \emptyset \qquad \Longrightarrow \qquad \mathrm{cl}\big(f(U)\big) \cap \mathrm{cl}\big(f(V)\big) = \emptyset.$$

f being contra B^*-continuous at x, there exists a B^*-set B_x such that

$$x \in B_x \subset U \qquad \text{and} \qquad f(B_x) \subset \mathrm{cl}\big(f(U)\big).$$

Let $y \in V$. Then $X \setminus \{x\}$ is an open set containing y. As in the proof of Theorem 3.6, we can obtain a B^*-set B_1 such that

$$V \subset B_1 \subset X \setminus \{x\} \qquad \text{and} \qquad f(B_1) \subseteq \mathrm{cl}\big(f(V)\big).$$

Now, since f is injective, it follows that $B_x \cap B_1 = \emptyset$ and the assertion is proved.

(b) We can proof this easily using (a) and Theorems 3.6. □

Remark 1. The assertions of Theorems 3.6 and 3.7 remain valid if

(i) Property P is replaced by the fact that X is either B^*-Lindelöf or B^*-compact,

(ii) the restriction on the space Y is removed and instead f is assumed to be either closed or open contra B^*-continuous.

4 Some properties of generalized B^*-continuous functions

Definition 18. The graph $G(f)$ of a function $f : X \to Y$ is said to be contra B^*-closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists a B^*-set B containing x and a closed set V containing y such that $(B \times V) \cap G(f) = \emptyset$.

Theorem 4.1. *The graph $G(f)$ of a function $f : X \to Y$ will be contra B^*-closed in $X \times Y$ if any one of the following conditions holds:*

(a) *$f : X \to Y$ is B^*-continuous and Y be T_1,*

(b) *$f : X \to Y$ is weakly B^*-continuous and Y is Urysohn,*

(c) *$f : X \to Y$ is a contra B^*-continuous and Y is Urysohn.*

Proof. (a) Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since Y is T_1, there exists open set V such that $f(x) \in V$ and $y \notin V$. Since f is B^*-continuous, for any open set U containing x, there exists a B^*-set $B \subseteq U$ containing x such that $f(B) \subseteq V$, ie.,

$$f(B) \cap (Y \setminus V) = \emptyset.$$

Since $Y \setminus V$ is a closed set containing y, the assertion follows.

(b) Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since Y is Urysohn, there exist open sets V and W containing, respectively, $f(x)$ and y such that

$$\mathrm{cl}(V) \cap \mathrm{cl}(W) = \emptyset.$$

Since f is weakly B^*-continuous, for any open set U containing x, there exists a B^*-set $B \subseteq U$ containing x such that $f(B) \subseteq \mathrm{cl}(V)$. It follows that

$$f(B) \cap \mathrm{cl}(W) = \emptyset.$$

Now, $\mathrm{cl}(W) = W_1$ is a closed set containing y and

$$(B \times W_1) \cap G(f) = \emptyset.$$

Hence, $G(f)$ is contra B^*-closed in $X \times Y$.

(c) We can prove this in similar lines to (b). □

Remark 2. The composition of two contra B^*-continuous functions need not be contra B^*-continuous. To see this, consider the space $X = \{a, b\}$, with the topologies

$$\tau = \{\emptyset, \{a\}, X\} \qquad \text{and} \qquad \sigma = \{\emptyset, \{b\}, X\}.$$

It can be noticed that the identity functions $f : (X, \tau) \to (X, \sigma)$ and $g : (X, \sigma) \to (X, \tau)$ are contra B^*-continuous on X. However, $gof : (X, \tau) \to (X, \tau)$ is not.

It is natural to ask, when the composition of two functions is contra B^*-continuous, if one of them is so. In this regard, we first define the following.

Definition 19. A function $f : X \to Y$ is said to be B^*-irresolute at a point $x \in X$ if for every open set U in X containing x and for every B^*-set V containing $f(x)$, there exists a B^*-set $B \subset U$ containing x such that $f(B) \subset V$.

The above definition is motivated by the notion of irresolute functions and their various variants [7], [16]. The following theorem gives various sufficient conditions under which the composition becomes contra B^*-continuous. The proof is simple and hence omitted.

Theorem 4.2. *Consider the functions $f : X \to Y$ and $g : Y \to Z$.*

(a) *If f is contra B^*-continuous and g is continuous, then gof is contra B^*-continuous.*

(b) *If f is B^*-irresolute and g is contra B^*-continuous, then gof is contra B^*-continuous.*

Definition 20. Let $A \subset X$. Then B^*-frontier of A is defined by $B^*_{\mathrm{fr}}(A) = B^*_{\mathrm{cl}}(A) \cap B^*_{\mathrm{cl}}(X - A)$.

The next theorem relates the set of points where a function is not contra B^*-continuous with the B^*-frontier.

Theorem 4.3. *Consider the function $f : X \to Y$. The set of all points $x \in X$ for which the function is not contra B^*-continuous contains the union of all B^*-frontiers of the pre-images of closed sets of Y containing $f(x)$.*

Proof. Let

$$E_1 = \{x \in X : \ f \text{ is not contra } B^*\text{-continuous}\}$$

and

$$E_2 = \bigcup_{i \in I} \Big\{ B^*_{\mathrm{fr}}\Big(f^{-1}(F_i)\Big) : F_i \text{ is closed subset containing } f(x) \Big\}.$$

Let $x \in E_2$. Then

$$x \in B^*_{\mathrm{fr}}\Big(f^{-1}(F_i)\Big)$$

for some closed set F_i containing $f(x)$. We have

$$x \in B^*_{\mathrm{cl}}\Big(f^{-1}(F_i)\Big) \cap B^*_{\mathrm{cl}}\Big(X - f^{-1}(F_i)\Big)$$

$$= B^*_{\mathrm{cl}}\Big(f^{-1}(F_i)\Big) \cap B^*_{\mathrm{cl}}(f^{-1}\big(Y - F_i)\Big)$$

which implies that for any B^*-set B containing x,

$$B \cap B^*_{\mathrm{cl}}\Big(f^{-1}(Y - F_i)\Big) \neq \emptyset$$

which gives that $f(B) \nsubseteq F_{n_i}$ i.e., f is not contra B^*-continuous at x i.e., $x \in E_1$. □

A set $A \subset X$ is said to be semi-open if $O \subseteq A \subseteq \mathrm{cl}(O)$ for some open set $O \subset X$, see [12]. We prove the following:

Theorem 4.4. *If $f : X \to Y$ is slightly B^*-continuous at $x \in X$, then*

(i) for each clopen set V of Y with $f(x) \in V$, there exists a semi-open set O containing x such that $O \subseteq \mathrm{cl}\big(f^{-1}(V)\big)$,

(ii) for each clopen set V of $f(x)$, $x \in \mathrm{cl}\Big(\mathrm{int}\big(\mathrm{cl}(f^{-1}(V))\big)\Big)$,

Proof. (*i*) Let $x \in X$, U be an open set containing x and V be a clopen set in Y containing $f(x)$. Since f is slightly B^*-continuous there exists a B^*-set $B \subseteq U$ such that $f(B) \subset V$. We have $U \cap f^{-1}(V)$ is not nowhere dense. Hence

$$\emptyset \neq \mathrm{int}\left(\mathrm{cl}\Big(U \cap f^{-1}(V)\Big)\right) \subseteq \mathrm{int}\left(\mathrm{cl}\Big(f^{-1}(V)\Big)\right) \subseteq \mathrm{cl}\Big(f^{-1}(V)\Big).$$

Put

$$G = \mathrm{int}\Big(\mathrm{cl}(f^{-1}(V) \cap U\Big) \cap U.$$

Then $G \subset U$ is a nonempty open set and $G \subset \mathrm{cl}(f^{-1}(V))$. Let $\mathcal{U}_x$ be a family of open sets containing x. Then for each $U \in \mathcal{U}_x$, there exists a nonempty open set G_u such that

$$G_u \subseteq U \qquad \text{and} \qquad G_u \subset \mathrm{cl}\big(f^{-1}(V)\big).$$

Let $W = \bigcup G_u$. Then W is an open set in X such that

$$x \in \mathrm{cl}(W) \qquad \text{and} \qquad W \subset \mathrm{cl}(f^{-1}(V)).$$

Take $O = W \cup \{x\}$. Then $W \subseteq O \subseteq \mathrm{cl}(W)$ so that O is semi-open and $O \subseteq \mathrm{cl}\big(f^{-1}(V)\big)$.

(ii) Let V be a clopen set containing $f(x)$. Then there exists a semi-open set O containing x such that $O \subseteq \mathrm{cl}\big(f^{-1}(V)\big)$. Therefore,

$$x \in \mathrm{cl}(\mathrm{int}(O)) \subseteq \mathrm{cl}\Big(\mathrm{int}(\mathrm{cl}(f^{-1}(V)))\Big).$$

□

Theorem 4.5. *Let $f : X \to Y$ be weakly B^*-continuous and Y be T_2. Then for each $(x, y) \notin G(f)$, the graph of f, there exists a B^*-set $B \subseteq X$ and an open set V in Y with $x \in B$ and $y \in V$ such that $f(B) \cap \mathrm{int}(\mathrm{cl}(V)) = \emptyset$.*

Proof. Let $(x, y) \notin g(f)$. Then $y \neq f(x)$. Since Y is T_2 there exist two open sets V and W containing, respectively, y and $f(x)$ such that

$$\mathrm{int}(\mathrm{cl}(V)) \cap \mathrm{cl}(W) = \emptyset.$$

Since f is weakly B^*-continuous then for any open set U containing x there exists a B^*-set $B \subseteq U$ containing x such that $f(B) \subseteq \mathrm{cl}(W)$ which implies that $f(B) \cap \mathrm{int}(\mathrm{cl}(V)) = \emptyset$. □

Motivated by the notion of continuous retraction [10], we define below the notion of weakly B^*-continuous retraction.

Definition 21. A function $f : X \to A \subseteq X$ is said to be weakly B^*-continuous retraction if f is weakly B^*-continuous and $f|_A$ is the identity function on A.

Theorem 4.6. *Let $f : X \to A \subseteq X$ be a weakly B^*-continuous retraction. If X is T_2, then A is B^*-closed in X.*

Proof. Assume, to the contrary, that A is not B^*-closed. Then there exists some $x \in X$ such that $x \in B^*_{\mathrm{cl}}(A) \setminus A$. Since f is a weakly B^*-continuous retraction, we find that $x \neq f(x)$. Since X is T_2 there exist disjoint open sets U and V containing, respectively, x and $f(x)$ such that

$$U \cap \mathrm{cl}(V) = \emptyset.$$

Also, since f is weakly B^*-continuous, there exists a B^*-set $B \subseteq U$ containing x such that

$$f(B) \subset \mathrm{cl}(V) \qquad \text{and} \qquad B \cap A \neq \emptyset \text{ (as } x \in B^*_{\mathrm{cl}}(A)).$$

Let $y \in B \cap A$. Since $y \in A$, We have

$$f(y) = y \in B \cap A \subseteq U$$

and hence $f(y) \notin \mathrm{cl}(V)$ i.e., $f(B) \not\subseteq \mathrm{cl}\ (V)$. This contradicts the fact that f is weakly B^*-continuous and the assertion follows. □

The proof of the following theorem is simple and thus we omit it.

Theorem 4.7. *Let $\{Y_\lambda : \lambda \in \Lambda\}$ be a family of topological spaces with the index set Λ. By $\prod_{\lambda \in \Lambda} Y_\lambda$, or simply $\prod Y_\lambda$, we denote the product space. If a function $f : X \to \prod Y_\lambda$ is contra (slightly or weakly) B^*-continuous, then $P_\lambda o f : X \to Y_\lambda$ is contra (slightly or weakly) B^*-continuous for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

5 Interrelation

In this paper we have discussed some weaker forms of B^*-continuous functions, namely, contra B^*-continuous, slightly B^*-continuous and weakly B^*-continuous functions.

The following diagram shows the interrelationship among these functions with B^*-continuous and B^*-irresolute functions:

$$\begin{array}{ccccc} & & & & \text{Contra } B^*\text{–Continuity} \\ & & & & \Downarrow \\ B^*\text{–Irresolute} & \Longrightarrow & B^*\text{–Continuity} & \Longrightarrow & \text{Weak } B^*\text{–Continuity} \\ & & & & \Downarrow \\ & & & & \text{Slight } B^*\text{–Continuity.} \end{array}$$

With suitable examples now we shall show that in general, no other implication holds among them.

Remark 3. Contra B^*-continuity and B^*-continuity have no relation.
To see this, consider the space $X = \{a, b\}$, with the topologies

$$\tau = \{\emptyset, \{a\}, X\} \qquad \text{and} \qquad \sigma = \{\emptyset, \{b\}, X\}.$$

It can be noticed that the identity function $f : (X, \tau) \to (X, \sigma)$ is contra B^*-continuous. However, f is not B^*-continuous at $b \in X$. On the other hand, the identity function on the real line with the usual topology is B^*-continuous, but not contra B^*-continuous.

Example 1. Consider the space $X = \{a, b, c, d\}$, with the topologies

$$\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, X\}$$

and

$$\sigma = \{\emptyset, \{a, c\}, X\}.$$

It is easy to see that the identity function $f : (X, \tau) \to (Y, \sigma)$ is weakly B^*-continuous on X. However, f is not B^*-continuous at $a \in X$.

Example 2. The identity function on the real line with the usual topology is weakly B^*-continuous, but not contra B^*-continuous.

Example 3. Let $X = \{a, b, c, d\}$ with the topologies

$$\tau = \{\emptyset, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, X\}$$

and

$$\sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}.$$

It is easy to see that the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly B^*-continuous on X. However f is not weakly B^*-continuous at $a \in X$.

Example 4. Let $X = \{a, b, c, d\}$ with the topologies

$$\tau = \{\emptyset, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, X\}$$

and

$$\sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}.$$

It is easy to see that the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly B^*-continuous on X. However f is not weakly B^*-continuous at $a \in X$.

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Events

5TH INTERNATIONAL CONFERENCE “ACTUAL PROBLEMS OF MATHEMATICS AND COMPUTER SCIENCE: THEORY, METHODOLOGY, PRACTICE” (APRIL 18–20, 2019, YELETS, RUSSIA)

The XX century is marked by the enrichment of world science with outstanding achievements in the field of mathematics, solving many important problems that remain relevant in the modern world. Such problems include, in particular, the problems considered in fundamental works of academician S. Chaplygin. Based on his works new research paths were set, and serious applied problems were addressed in the fields of aerodynamics, gas dynamics, hydrodynamics, and mechanics. They were further intensively developed due to the achievements of contemporary information technology.

To commemorate his activities was organized a large-scale scientific event in the historic homeland of academician S. Chaplygin — the 5th international conference “Actual problems of mathematics and computer science: theory, methodology, practice”, dedicated to the 150th anniversary of the birth of academician S. Chaplygin.

The Ivan Bunin Yelets State University (Russia), the Samarkand State University (Uzbekistan), the Higher School of Insurance and Finance (Bulgaria),the Khachatur Abovyan Armenian State Pedagogical University (Armenia), and the Scientific and Methodological Council on Mathematics of the Ministry of Science and Higher Education of Russia held through April 18–20, 2019 the 5th International Conference “Actual problems of mathematics and computer science: theory, methodology, practice” dedicated to the 150th anniversary of academician S. Chaplygin.

The conference marked the three major milestones related to the development of mathematical science in the Lipetsk region and in the oldest university center in the region — the Ivan Bunin Yelets State University.

1. April 2019 is the 150th anniversary of the birth of S. Chaplygin (1869–1942) — a well-known Russian scientist, academician of the Academy of Sciences of the USSR. S. Chaplygin is an outstanding representative of the Lipetsk region, whose surname is immortalized in the name of the city Chaplygin (previously Ranenburg) in the Lipetsk region.

2. 2019 year is the 80th anniversary of the foundation of the Faculty of Physics and Mathematics. It is the oldest faculty of the Ivan Bunin Yelets State University, where students of the scientific school of academician N. Zhukovsky were taught, whose famous representative was academician S. Chaplygin.

3. In October 2019 there will be 10 years since the organization of the Lipetsk Branch of the Scientific and Methodological Council for Mathematics of the Ministry of Science and Higher Education of Russia on the basis of the Ivan Bunin Yelets State University.

The main goals of the conference were the creation of conditions for international scientific communication of representatives of fundamental and applied areas in the field of mathematics, understanding the importance of scientific works of S. Chaplygin, the actualization of his scientific achievements, taking into account the rapid development of information technologies and their adaptation to modern mathematical education.

The plenary session of the conference was opened by the Rector of the Ivan Bunin Yelets State University Professor E. Gerasimova and continued by the President of the International Academy of the History of Science Professor S. Demidov (Moscow, Russia), who presented the talk “Pure and Applied Mathematics at the M.V. Lomonosov Moscow State University in the first half of the twentieth century: N. Luzin and S. Chaplygin”.

Professor A. Soleev (Samarkand, Uzbekistan) devoted his talk to basic ideas and general provisions of the Power Geometry. Professor A. Soldatov (Moscow, Russia) focused on the consideration of the Dirichlet problem for equations of mixed type. In her talk Professor G. Zhukova (Moscow, Russia) discussed the dependence of solutions to singularly perturbed linear differential systems on a small parameter. The talk of Professors O. Masina (Yelets, Russia) and O. Druzhinina (Moscow, Russia) was devoted to the analysis of the known and developed by the authors approaches to the study of the stability of intelligent control systems. The talks of Professors V. Tikhomirov (Moscow, Russia), T. Sergeeva (Moscow, Russia) and E. Smirnov (Yaroslavl, Russia) addressed the issues of improving mathematical education, introducing novelty into the teaching process while maintaining the best traditions of high-quality teaching mathematics, laid by S. Chaplygin in his productive teaching activities.

The relevance of the event was noted in the talks of Professors A. Abylkasymova (Alma-Ata, Kazakhstan), A. Borovskikh (Moscow, Russia), S. Grozdev (Sofia, Bulgaria), M. Mkrtchyan (Yerevan, Armenia) and other scientists. At the end of the plenary session, talks were presented by the authors of this communication on the history of the Scientific and Methodological Council on Mathematics of the Ministry of Science and Higher Education of Russia and its contribution to the development of mathematics and its applications in Russian education, as well as on the activities of the Lipetsk Branch of the Scientific and Methodological Council.

The following sections were working at the conference: “Modern Directions in Mathematics”, “Applied problems of mathematics”, “Computer modeling, information technologies and systems”, “New theories, models and technologies of teaching mathematics and computer science at schools and universities”, “Actualization of the problems of the history of mathematics and mathematical education in modern conditions”.

At the conference there were more than 250 participants, including leading foreign specialists from Armenia, Bulgaria, Uzbekistan, Kazakhstan, well-known scientists from more than twenty regions of Russia, as well as young researchers. Overall, it was a successful conference, which helped to increase the scientific and innovative activity of the region, stimulated the participants to develop mathematics, information technologies and mathematical education.

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