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ON INVERSE PROBLEM OF CLOSURE OF DIFFERENTIAL SYSTEMS WITH DEGENERATE DIFFUSION

M.I. Tleubergenov, G.T. Ibraeva

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Abstract. The quasi-inversion method is used to obtain necessary and sufficient conditions for the solvability of the inverse closure problem in the class of stochastic differential Itô systems of the first order with random perturbations from the class of processes with independent increments and diffusion degenerate with respect to a part of the variables.

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1 Introduction

The theory of inverse problems for differential systems and its general methods goes back to works [2, 3] and has been further developed in [1, 8, 9, 18, 19] for deterministic systems described by ordinary differential equations (ODE). A set of ODE with a given integral curve was constructed in [2]. That paper plays a fundamental role in the formation and development of the theory of inverse problems in the dynamics of systems described by ODE. The statement and classification of the inverse problems for differential equations and their solutions in the class of ODE are discussed in [1, 3, 8, 9, 18, 19].

Solving inverse problems of differential systems (problems of construction of a set of differential equations according to a given integral manifold) is based on two methods: the Erugin method and the quasi-inversion method. Firstly, the Erugin method (the method of introduction of an auxiliary Erugin function) provides necessary and sufficient conditions for a given set to be an integral manifold [2, 3]. And, secondly, the quasi-inversion method, developed in [9], allows to write out the common solution of the functional-algebraic equation, to which the problem of construction of a set of differential equations for a given integral manifold is reduced.

However, the increasing requirements to the accuracy of description and serviceability of material systems lead to the situation in which numerous observed phenomena cannot be explained on the basis of the analysis of deterministic processes. Thus, in particular, probability laws should be used for the simulation of the behaviour of actual systems.

Thus, the problem of generalization of the methods used for the solution of inverse problems for differential systems to the class of stochastic differential equations seems to be quite urgent [7, 11].

Stochastic differential equations of the Itô-type are used to describe various models of mechanical systems taking into account the action of external random forces and are important for numerous applications, e.g., the motion of artificial satellites under the action of gravity and aerodynamic forces [10], the fluctuation drift of a heavy gyroscope in the gimbal suspension [12], and many others. In [13-15], the inverse problems of dynamics are studied under the additional assumptions of the presence of random perturbations from the class of Wiener processes. In particular, the following problems are solved by the method of quasi-inversion:

(a) the main inverse problem of dynamics, i.e., the construction of the set of Itô-type second-order stochastic differential equations with a given integral manifold;

(b) the problem of reconstruction of the equations of motion, i.e., the construction of the set of control parameters contained in a given system of Itô-type second-order stochastic differential equations according to a given integral manifold; and

(c) the problem of closure of the equations of motion, i.e., the construction of the set of closing Itô-type second-order stochastic differential equations for a given system of equations and a given integral manifold.

Other important (but different from the problem considered in this paper) inverse problems in the presence of random perturbations from the class of Wiener processes are considered in [5, 16]. In particular, the main inverse problem (according to Galiullin's classification [3]) with degenerate diffusion is solved by the method of separation in [5], and the inverse stochastic problem of reconstruction with an integral manifold, which depends only on a part of the variables, is considered in [16].

In [15, 17], one of the inverse problems - the problem of construction of a set of closing stochastic differential equations of Itô on the given integral manifold is considered in the assumption that random perturbations belong to a class of independent Wiener processes (as a special case of processes with independent increments).

In this paper, in contrast to [15, 17] we suppose that random perturbations belong to a more general class, namely the class of processes with independent increments.

The posed inverse problem of closure of stochastic differential first-order Itô equations by given properties of the motion is solved by a quasi-inversion method. The necessary and sufficient conditions of this problem's solvability are obtained in the terms of closing equations' coefficients.

2 Statement of the general problem of constructing of closing stochastic differential equations

We assume that the set

$$\Lambda(t) : \lambda(y, z, v, w, t) = 0, \quad \text{where } \lambda \in R^m,\ \lambda = \lambda(y, z, v, w, t) \in C^{1\,2\,1\,2\,1}_{y\,z\,v\,w\,t} \tag{2.1}$$

and the system of stochastic differential Itô equations of the first order

$$\begin{cases} \dot{y} = f_1(y, z, v, w, t), \\ \dot{z} = f_2(y, z, v, w, t) + \sigma_1(y, z, v, w, t)\dot{\xi}_0 + \displaystyle\int c_1(x, t)\dot{P}^0(t, dx) \end{cases} \tag{2.2}$$

are given. It is required to finish building the system of the closing equations of the form

$$\begin{cases} \dot{v} = f_3(y, z, v, w, t), \\ \dot{w} = f_4(y, z, v, w, t) + \sigma_2(y, z, v, w, t)\dot{\xi}_0 + \displaystyle\int c_2(x, t)\dot{P}^0(t, dx) \end{cases} \tag{2.3}$$

such that set (2.1) is an integral manifold of the system of equations (2.2), (2.3).

Here $y \in R^{l_1}$, $z \in R^{l_2}$, $v \in R^{p_1}$, $w \in R^{p_2}$, $l_1 + l_2 = l$, $l > 0$; $p_1 + p_2 = p$, $p > 0$; $l + p = n$, $x = (y^T, z^T, v^T, w^T)^T \in R^n$; $\xi_0 \in R^k$ is a vector Wiener process; P^0 is a Poisson process; $P^0(t, dx)$ is the number of jumps of the process P^0 in the interval $[0, t]$, falling on the set dx. Matrices σ_1, σ_2 have dimensions respectively $(l_2 \times k)$, $(p_2 \times k)$; $c_1(x, t)$ and $c_2(x, t)$ are vector functions that map the space R^n into spaces R^{l_2} and R^{p_2} respectively.

We say that a function $g(x,t)$ belongs to the class K, $g \in K$, if g is continuous in t, $t \in [0,\infty]$ satisfies the Lipschitz condition with respect to x in the entire space R^n; i.e.

$$\|g(x,t) - g(\widetilde{x},t)\| \leq M\|x - \widetilde{x}\|$$

and satisfies the condition of linear growth in x

$$\|g(x,t)\| \leq M(1 + \|x\|)$$

with a certain constant M.

It is presumed that there is a degeneracy of the diffusion with respect to the variable y in equation (2.2) and with respect to the variable v in equation (2.3).

It is assumed that the vector functions f_1, f_2, the matrix σ_1, and also unknown vector functions f_3, f_4 and matrix σ_2 belong to the class K. This guarantees the existence and uniqueness (up to stochastic equivalence) of solution $\big(y(t)^T, z(t)^T, v(t)^T, w(t)^T\big)^T$ of system of equations (2.2), (2.3) with the initial condition $\big(y(t_0)^T, z(t_0)^T, v(t_0)^T, w(t_0)^T\big)^T = (y_0^T, z_0^T, v_0^T, w_0^T)^T$. This solution is a strictly Markov process, which is continuous with probability 1 [7, p. 107].

Let us suppose that

(i) *the vector function $\lambda(y,z,v,w,t)$ is continuously differentiable with respect to all its arguments;*

(ii) *the given vector functions f_1, f_2, c_1 and the matrix σ_1, and also the unknown sets of vector functions $\{f_3\}$, $\{f_4\}$ and the unknown set of matrices $\{\sigma_2\}$ belong to the class* K.

Condition (ii) ensures in R^n, following [4], the existence and uniqueness up to the stochastic equivalence of the solution $\big(y(t)^T, z(t)^T, v(t)^T, w(t)^T\big)^T$ of system of equations (2.2), (2.3) with the initial condition $\big(y(t_0)^T, z(t_0)^T, v(t_0)^T, w(t_0)^T\big)^T = (y_0^T, z_0^T, v_0^T, w_0^T)^T$. This solution is a strictly Markov process, which is continuous with probability 1 [7, p. 107]. Moreover condition (i) ensures the possibility of deriving the equation of the perturbed motion with respect to the integral manifold Λ.

Thus, the posed problem:

1) is sufficiently fully investigated in [3, 9] in the absence of random perturbations ($\sigma_1 \equiv 0$, $\sigma_2 \equiv 0$);

2) generalizes the problem of construction of the set of closing Itô-type second-order stochastic differential equations

$$\ddot{u} = f_2(x,\dot{x},u,\dot{u},t) + \sigma_2(x,\dot{x},u,\dot{u},t)\dot{\xi} \tag{2.3$'$}$$

for a given system of equations

$$\ddot{x} = f_1(x,\dot{x},u,\dot{u},t) + \sigma_1(x,\dot{x},u,\dot{u},t)\dot{\xi} \tag{2.2$'$}$$

and a given integral manifold

$$\Lambda(t) : \lambda(x,\dot{x},u,\dot{u},t) = 0, \quad \text{where } \lambda \in R^m,\ \lambda = \lambda(x,\dot{x},u,\dot{u},t) \in C^{1\,2\,1\,2\,1}_{x\,\dot{x}\,u\,\dot{u}\,t}, \tag{2.1$'$}$$

so, that set (2.1$'$) is an integral manifold of the system of equations (2.2$'$) and (2.3$'$), which was considered in [15, 17];

3) extends results of work [6], where the posed problem is solved in the supposition, that random perturbations $\xi(t)$ are from a class of Wiener processes (a particular case of processes with independent increments).

For the solution of the posed problem we will use the quasi-inversion method [9], which is based on

Lemma 1.1. [9, p. 12–13]. *The set of all solutions of the linear systems*

$$Hv = g,\ \ H = (h_{\mu k}),\ \ v = (v_k),\ \ g = (g_\mu),\ \ \mu = \overline{1,m};\quad k = \overline{1,n},\ \ m \leq n \tag{2.4}$$

where the matrix H has a rank m, is defined by the expression

$$v = sv^\tau + v^\nu, \tag{2.5}$$

where s is any scalar,

$$v^\tau = [HC] = [h_1 \dots h_m c_{m+1} \dots c_{n-1}] = \begin{vmatrix} e_1 & \dots & e_n \\ h_{11} & \dots & h_{1n} \\ \dots & \dots & \dots \\ h_{m1} & \dots & h_{mn} \\ c_{m+1,1} & \dots & c_{m+1,n} \\ \dots & \dots & \dots \\ c_{n-1,1} & \dots & c_{n-1,n} \end{vmatrix}$$

is the vector product of vectors $h_\mu = (h_{\mu k})$ and any vectors $c_\rho = (c_{\rho k})$, $\rho = \overline{m+1, n-1}$; e_k is individual basis vectors of space R^n, $v^\tau = (v_k^\tau)$, where

$$v_k^\tau = \begin{vmatrix} 0 & \dots & 1 & \dots & 0 \\ h_{11} & \dots & h_{1k} & \dots & h_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ h_{m1} & \dots & h_{mk} & \dots & h_{mn} \\ c_{m+1,1} & \dots & c_{m+1,n} & \dots & c_{m+1,n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{n-1,1} & \dots & c_{n-1,k} & \dots & c_{n-1,n} \end{vmatrix},\ \ v^\nu = H^+ g,$$

$H^+ = H^T(HH^T)^{-1}$, H^T is the transposed matrix to H.

For the solving of posed problem of closing of equations' system (2.3) we will differentiate a composite function $\lambda = \lambda(y, z, v, w, t)$ by rule Itô [4] in the case of processes with independent increments

$$\dot{\lambda} = M + \left(\frac{\partial\lambda}{\partial v}\right) f_3 + \left(\frac{\partial\lambda}{\partial w}\right) f_4 + \left(\frac{\partial\lambda}{\partial z}\right) \sigma_1\dot{\xi} + \left(\frac{\partial\lambda}{\partial w}\right) \sigma_2\dot{\xi}_0 + S_3, \tag{2.6}$$

where $M = \dfrac{\partial\lambda}{\partial t} + \left(\dfrac{\partial\lambda}{\partial y}\right) f_1 + \left(\dfrac{\partial\lambda}{\partial z}\right) f_2 + S_1 + S_2,\ \ S_1 = \dfrac{1}{2}\left[\dfrac{\partial^2\lambda}{\partial z\partial z} : D_1 + \dfrac{\partial^2\lambda}{\partial w\partial w} : D_2\right]$,

$$S_2 = \int \{\lambda(y, z + c_1(x,t), v, w + c_2(x,t), t) - \lambda(y, z, v, w, t) - \frac{\partial\lambda}{\partial z} c_1(x,t) - \frac{\partial\lambda}{\partial w} c_2(x,t)\} dx,$$

$$S_3 = \int [\lambda(y, z + c_1(x,t), v, w + c_2(x,t), t) - \lambda(y, z, v, w, t)] \dot{P}^0(t, dx),$$

and under $\dfrac{\partial^2\lambda}{\partial z\partial z} : D_1,\quad D_1 = \sigma_1\sigma_1^T,\ \dfrac{\partial^2\lambda}{\partial w\partial w} : D_2,\quad D_2 = \sigma_2\sigma_2^T$ we understand, following [17], the vector, the elements of which are traces of the matrices' products of flexons of corresponding elements $\lambda_\mu(y,z,v,w,t)$ of the vector $\lambda(y,z,v,w,t)$ on components z, w of matrices D_1, D_2 :

$$\frac{\partial^2\lambda}{\partial z\partial z}:D_1=\begin{bmatrix} tr\big(\frac{\partial^2\lambda_1}{\partial z\partial z}D_1\big) \\ \vdots \\ tr\big(\frac{\partial^2\lambda_m}{\partial z\partial z}D_1\big)\end{bmatrix}, \qquad \frac{\partial^2\lambda}{\partial w\partial w}:D_2=\begin{bmatrix} tr\big(\frac{\partial^2\lambda_1}{\partial w\partial w}D_2\big) \\ \vdots \\ tr\big(\frac{\partial^2\lambda_m}{\partial w\partial w}D_2\big)\end{bmatrix}.$$

Further, providing that set (2.1) be an integral manifold of the system of equations (2.2), (2.3), we introduce, following the Erugin method [2], any m-dimensional vector functions A_1, A_2 and $(m\times k)$ matrix B, possessing property

$$A_1(0,y,z,v,w,t)\equiv A_2(0,y,z,v,w,t)\equiv 0, \quad B(0,y,z,v,w,t)\equiv 0,$$

such that equation (2.7)

$$\dot\lambda = A_1(\lambda,y,z,v,w,t)+B(\lambda,y,z,v,w,t)\dot\xi_0+\int A_2(\lambda,x,t)\dot P^0(t,dx) \tag{2.7}$$

holds.

Comparing equations (2.6) and (2.7), we arrive at the following relations:

$$\begin{cases} \frac{\partial\lambda}{\partial v}f_3+\frac{\partial\lambda}{\partial w}f_4=A_1-M, \\ \frac{\partial\lambda}{\partial z}\sigma_1+\frac{\partial\lambda}{\partial w}\sigma_2=B, \\ \lambda(y,z+c_1(x,t),v,w+c_2(x,t),t)-\lambda(y,z,v,w,t)=A_2, \end{cases} \tag{2.8}$$

from which it is necessary to define vector functions f_3, f_4 and a matrix σ_2. We will suppose that $f_3=\phi(y,z,v,w,t)$, where $\phi\in K$.

Then expression (2.8) will become:

$$\begin{cases} \frac{\partial\lambda}{\partial w}f_4=A_1-M-\frac{\partial\lambda}{\partial v}f_3, \\ \frac{\partial\lambda}{\partial w}\sigma_2=B-\frac{\partial\lambda}{\partial z}\sigma_1, \\ \lambda(y,z+c_1(x,t),v,w+c_2(x,t),t)-\lambda(y,z,v,w,t)=A_2. \end{cases} \tag{2.9}$$

Now we assume that along with conditions (i) and (ii), the following condition (iii) also holds

(iii) *the vector function λ is linear in z and w and has the form*

$$\lambda=\alpha(y,v,t)+\beta_1(t)z+\beta_2(t)w, \tag{2.10}$$

where $\alpha(y,v,t)$ is a vector function of arbitrarily specified from the class K, $\alpha\in$ K, *and $\beta_1(t)$, $\beta_2(t)$ are matrices arbitrarily given and continuous in t of order $(m\times l_2)$ and $(m\times p_2)$ respectively.*

If condition (iii) is satisfied, namely, if λ is linear in z and w, we have $\dfrac{\partial^2\lambda}{\partial z\partial z}\equiv 0$, $\dfrac{\partial^2\lambda}{\partial w\partial w}\equiv 0$ and, consequently, $S_1\equiv 0$, $S_2\equiv 0$, and S_3 in the linear case will take the form

$$S_3=\int[\beta_1(t)c_1(x,t)+\beta_2(t)c_2(x,t)]\dot{P}^0(t,dx).$$

Let $f_3=f(y,z,v,w,t)$, where the vector function f is an arbitrary function from the class K. Then taking into account expression (2.10), relations (2.9) take the form

$$\begin{cases}\beta_2 f_4=A_1-\widetilde{M}-\dfrac{\partial\alpha}{\partial v}f,\\ \beta_2\sigma_2=B-\beta_1\sigma_1,\\ \beta_2 c_2=A_2-\beta_1 c_1,\end{cases}\tag{2.11}$$

where $\widetilde{M}=\dfrac{\partial\alpha}{\partial t}+\dfrac{\partial\beta_1}{\partial t}z+\dfrac{\partial\beta_2}{\partial t}w+\dfrac{\partial\alpha}{\partial y}f_1+\dfrac{\partial\alpha}{\partial z}f_2.$

From relations (2.11) by formula (2.6) of Lemma 1.1, we will define unknown vector functions f_4, c_2 and a matrix σ_2 in the form of

$$\begin{aligned}&f_4=s_1\left[\beta_2 C\right]+(\beta_2)^+\widetilde{A_1},\\ &\sigma_{2i}=s_{2i}\left[\beta_2 C\right]+(\beta_2)^+\widetilde{B_i},\quad i=\overline{1,k},\\ &c_2=s_3\left[\beta_2 C\right]+(\beta_2)^+\widetilde{A_2},\end{aligned}\tag{2.12}$$

where $\widetilde{A_1}=A_1-\widetilde{M}-\beta_1 f$, $\widetilde{B_i}=(B-\beta_1\sigma_1)_i$, $\widetilde{A_2}=A_2-\beta_1 c_1$, σ_{2i} is i-th column of the matrix $\sigma_2=(\sigma_{2\nu j})$, $(\nu=\overline{1,p_2},\ j=\overline{1,k})$, $\widetilde{B_i}$ is i-th column of the matrix $\widetilde{B}=(\widetilde{B_{\mu l}})$, $(\mu=\overline{1,m}$, $l=\overline{1,k})$,

$$[\beta_2 C]=\begin{vmatrix} e_1 & \cdots & e_{p_2}\\ \beta_{2,11} & \cdots & \beta_{2,1p_2}\\ \cdots & \cdots & \cdots\\ \beta_{2,m1} & \cdots & \beta_{2,mp_2}\\ c_{m+1,1} & \cdots & c_{m+1,p_2}\\ \cdots & \cdots & \cdots\\ c_{p_2-1,1} & \cdots & c_{p_2-1,p_2}\end{vmatrix}.$$

Consequently, the following theorem takes place.

Theorem 2.1. *Let conditions (i), (ii) and (iii) be satisfied. Then the system of differential equations (2.2), (2.3) of Itô type has the given integral manifold (2.1) if and only if the sets of vector functions $\{f_4\}$, $\{c_2\}$ and columns σ_{2i} of the set of matrices $\{\sigma_2\}$ of the closing stochastic differential equations (2.3) can be represented in form (2.12).*

3 Scalar case of the closure's inverse problem

Let us give the set

$$\Lambda(t):\eta(y,z,v,w,t)=0,\quad\text{where}\quad\eta\in R^1,\tag{3.1}$$

and the system of scalar stochastic differential equations

$$\begin{cases} \dot{y} = g_1(y,z,v,w,t), \\ \dot{z} = g_2(y,z,v,w,t) + \gamma_1(y,z,v,w,t)\dot{\zeta}_0 + \int \kappa_1(x,t)\dot{P}^0(t,dx), \end{cases} \tag{3.2}$$

where $y, z, v, w \in R^1$, it is required to finish building the closing system of scalar stochastic equations

$$\begin{cases} \dot{v} = g_3(y,z,v,w,t), \\ \dot{w} = g_4(y,z,v,w,t) + \gamma_2(y,z,v,w,t)\dot{\zeta}_0 + \int \kappa_2(x,t)\dot{P}^0(t,dx), \end{cases} \tag{3.3}$$

so that set (3.1) is an integral manifold of the system of equations (3.2), (3.3), where $\zeta_0 = \zeta_0(t,\omega)$ is a scalar Wiener process, P^0 is a scalar Poisson process, $P^0(t,dx)$ is the number of jumps of the process in the interval $[0,t]$.

The problem consists in the definition of functions g_3, g_4 and γ_2 by the given functions g_1, g_2, γ_1 and by the given integral manifold $\eta(y,z,v,w,t) = 0$.

Let us differentiate a composite function $\eta = \eta(y,z,v,w,t)$ by the rule of Itô [4] in the case of a process with independent increments

$$\dot{\eta} = \frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial y}g_1 + \frac{\partial\eta}{\partial z}g_2 + \frac{\partial\eta}{\partial v}g_3 + \frac{\partial\eta}{\partial w}g_4 + \widetilde{S}_1 + \widetilde{S}_2 + \widetilde{S}_3 + \left(\frac{\partial\eta}{\partial z}\gamma_1 + \frac{\partial\eta}{\partial w}\gamma_2\right)\dot{\zeta}_0, \tag{3.4}$$

where

$$\widetilde{S}_1 = \frac{1}{2}\left(\frac{\partial^2\eta}{\partial z}\gamma_1^2 + \frac{\partial^2\eta}{\partial w}\gamma_2^2\right),$$

$$\widetilde{S}_2 = \int \{\eta(y, z+\kappa_1(x,t), v, w+\kappa_2(x,t), t) - \eta(y,z,v,w,t) - \frac{\partial\eta}{\partial z}\kappa_1(x,t) - \frac{\partial\eta}{\partial w}\kappa_2(x,t)\}dx,$$

$$\widetilde{S}_3 = \int [\eta(y, z+\kappa_1(x,t), v, w+\kappa_2(x,t), t) - \eta(y,z,v,w,t)]\dot{P}^0(t,dx),$$

Further, following Erugin method [2], we will introduce scalar functions $a_1 = a_1(\eta,y,z,v,w,t)$, $a_2 = a_2(\eta,y,z,v,w,t)$ and $b = b(\eta,y,z,v,w,t)$, which possess the property $a_1(0,y,z,v,w,t) \equiv a_2(0,y,z,v,w,t) \equiv b(0,y,z,v,w,t) \equiv 0$. Then equality (3.5)

$$\dot{\eta} = a_1 + b\dot{\zeta}_0 + \int a_2(\eta,x,t)\dot{P}^0(t,dx) \tag{3.5}$$

also holds.

The following relations follow from (3.4) and (3.5)

$$\begin{cases} \dfrac{\partial\eta}{\partial w}g_4 = a - m - \dfrac{\partial\eta}{\partial z}g_3, \\ \dfrac{\partial\eta}{\partial w}\gamma_2 = b - \dfrac{\partial\eta}{\partial z}\gamma_1, \\ \eta(y, z+\chi_1(t), v, w+\chi_2(t), t) - \eta(y,z,v,w,t) = a_2, \end{cases} \tag{3.6}$$

where $m = \dfrac{\partial\eta}{\partial t} + \dfrac{\partial\eta}{\partial y}g_1 + \dfrac{\partial\eta}{\partial z}g_2 + \widetilde{S}_1 + \widetilde{S}_2$.

Conditions (i), (ii), (iii) in this section will take the corresponding forms

(i') *the scalar function $\eta(y,z,v,w,t)$ is continuously differentiable with respect to all its arguments;*

(ii') *the given scalar functions* g_1, g_2, c_1, γ_1 *and also the unknown sets of scalar functions* $\{g_3\}$, $\{g_4\}$, $\{\gamma_2\}$ *belong to the class* K;

(iii') *the scalar function* η *is linear in* z *and* w *and has the form*

$$\eta = \mu(y, v, t) + \chi_1(t)z + \chi_2(t)w, \tag{3.7}$$

where $\mu \in$ K; $\chi_1(t)$, $\chi_2(t)$ *are scalar functions continuous in* t.

Let us suppose that conditions (i'), (ii'), (iii') are satisfied and that the function g_3 is equal to some arbitrary function g from a class $K: g_3 = g(y, z, v, w, t)$, $g \in K$. Then from (3.6), (3.7) we have $\widetilde{S}_1 \equiv \widetilde{S}_2 \equiv 0$, $\widetilde{S}_3 = \int[\chi_1\kappa_1 + \chi_2\kappa_2]\dot{P}^0(t, dx)$ and the following relations follow:

$$\begin{cases} \chi_2 g_4 = a_1 - \widetilde{m} - \dfrac{\partial\mu}{\partial v}g, \\ \chi_2\gamma_2 = b - \chi_1\gamma_1, \\ \chi_2\kappa_2 = a_2 - \chi_1\kappa_1, \end{cases} \tag{3.8}$$

where $\widetilde{m} = \dfrac{\partial\mu}{\partial t} + \dfrac{\partial\chi_1}{\partial t}z + \dfrac{\partial\chi_2}{\partial t}w + \dfrac{\partial\mu}{\partial y}g_1 + \chi_1(t)g_2$.

From (3.8) under the supposition that $\chi_2 \neq 0$, we have

$$\begin{cases} g_4 = \chi_2^{-1}\left(a_1 - \widetilde{m} - \dfrac{\partial\mu}{\partial v}g\right), \\ \gamma_2 = \chi_2^{-1}\left(b - \chi_1\gamma_1\right), \\ \kappa_2 = \chi_2^{-1}\left(a_2 - \chi_1\kappa_1\right). \end{cases} \tag{3.9}$$

Relations (3.9) represent a solution of stochastic problem of closure, i.e. the problem of construction of the set of closing equations (3.3) by the given integral manifold (3.1) and by the given stochastic equation (3.2).

Conclusion

We have constructed the set of the closing stochastic differential Itô equations of the first order with diffusion degenerate with respect to a part of the variables, such that the joint system of given and constructed differential equations possesses the given integral manifold. It should be noted that random perturbations are assumed to be in the class of processes with independent increments.

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