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KHARIN STANISLAV NIKOLAYEVICH

(to the 80th birthday)

Stanislav Nikolayevich Kharin was born on December 4, 1938 in the village of Kaskelen, Alma-Ata region. In 1956 he graduated from high school in Voronezh with a gold medal. In the same year he entered the Faculty of Physics and Mathematics of the Kazakh State University and graduated in 1961, receiving a diploma with honors. After postgraduate studies he entered the Sector (since 1965 Institute) of Mathematics and Mechanics of the National Kazakhstan Academy of Sciences, where he worked until 1998 and progressed from a junior researcher to a deputy director of the Institute (1980). In 1968 he has defended the candidate thesis "Heat phenomena in electrical contacts and associated singular integral equations", and in 1990 his doctoral thesis "Mathematical models of thermo-physical processes in electrical contacts" in Novosibirsk. In 1994 S.N. Kharin was elected a corresponding member of the National Kazakhstan Academy of Sciences, the Head of the Department of Physics and Mathematics, and a member of the Presidium of the Kazakhstan Academy of Sciences.

In 1996 the Government of Kazakhstan appointed S.N. Kharin to be a co-chairman of the Committee for scientific and technological cooperation between the Republic of Kazakhstan and the Islamic Republic of Pakistan. He was invited as a visiting professor in Ghulam Ishaq Khan Institute of Engineering Sciences and Technology, where he worked until 2001. For the results obtained in the field of mathematical modeling of thermal and electrical phenomena, he was elected a foreign member of the National Academy of Sciences of Pakistan. In 2001 S.N. Kharin was invited to the position of a professor at the University of the West of England (Bristol, England), where he worked until 2003. In 2005, he returned to Kazakhstan, to the Kazakh-British Technical University, as a professor of mathematics, where he is currently working.

Stanislav Nikolayevich paid much attention to the training of young researchers. Under his scientific supervision 10 candidate theses and 4 PhD theses were successfully defended.

Professor S.N. Kharin has over 300 publications including 4 monographs and 10 patents. He is recognized and appreciated by researchers as a prominent specialist in the field of mathematical modeling of phenomena in electrical contacts. Using models based on the new original methods for solving free boundary problems he described mathematically the phenomena of arcing, contact welding, contact floating, dynamics of contact blow-open phenomena, electrochemical mechanism of electron emission, arc-to-glow transition, thermal theory of the bridge erosion. For these achievements he got the International Holm Award, which was presented to him in 2015 in San Diego (USA).

Now he very successfully continues his research and the evidence of this in the new monograph "Mathematical models of phenomena in electrical contacts" published last year in Novosibirsk.

The mathematical community, many his friends and colleagues and the Editorial Board of the Eurasian Mathematical Journal cordially congratulate Stanislav Nikolayevich on the occasion of his 80th birthday and wish him good health, happiness and new achievements in mathematics and mathematical education.

Short communications

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KOLMOGOROV WIDTHS OF WEIGHTED SOBOLEV CLASSES WITH “SMALL” SINGULARITY SETS

A.A. Vasil’eva

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Abstract. Sharp order estimates are stated for the Kolmogorov widths of weighted Sobolev classes on h-sets foe sertain limiting cases of the parameters

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In [12] order estimates for the Kolmogorov widths of weighted Sobolev classes on a John domain were obtained, where weights are functions of the distance from the given subset of the boundary. This subset is an h-set [3] with $h(t) = t^\theta |\log t|^\gamma \tau(|\log t|)$, where τ is a “slowly varying” function (all definitions will be given below; as examples of h-sets we can take Lipschitz manifolds and some fractal sets: the Cantor set, the Koch curve). For $\theta = 0$ and some limiting conditions for parameters the method in [12] does not give the desired order estimates of the Kolmogorov widths. Here we obtain order estimates for such “limiting” cases.

By the Kolmogorov n-width of a set $M \subset X$ in the space X, we mean the quantity

$$d_n(M,\, X) = \inf_{L \in \mathcal{L}_n(X)} \sup_{x \in M} \inf_{y \in L} \|x - y\|_X,$$

where $\mathcal{L}_n(X)$ is the family of subspaces of X of dimension at most n (see [6, 9]).

The estimates for widths and entropy numbers of weighted Sobolev classes were investigated by Triebel [10, 11], Boykov [1, 2], Mieth [4, 5] and other authors (for details, see [12]).

Let $\Omega \subset \mathbb{R}^d$ be a bounded domain, and let $a > 0$. We say that $\Omega \in \mathbf{FC}(a)$ if there exists a point $x_* \in \Omega$ such that, for any $x \in \Omega$, there exist a number $T(x) > 0$ and a curve $\gamma_x : [0,\, T(x)] \to \Omega$ with the following properties:

1. γ_x has the natural parametrization,

2. $\gamma_x(0) = x$, $\gamma_x(T(x)) = x_*$,

3. $B_{at}(\gamma_x(t)) \subset \Omega$ for all $t \in [0,\, T(x)]$.

We say that Ω satisfies the John condition (and call Ω a John domain) if $\Omega \in \mathbf{FC}(a)$ for some $a > 0$.

Domains with Lipschitz boundary, the Koch's snowflake are examples of John domains. Domains with zero angles do not satisfy the John condition. For John domains the Sobolev embedding condition is the same as for a cube [7, 8].

We define h-sets according to [3]. Let $\mathbb{H}$ be the set of all nondecreasing positive functions defined on $(0, 1]$. Let $\Gamma \subset \mathbb{R}^d$ be a nonempty compact set and $h \in \mathbb{H}$. We say that Γ is an h-set if there are a constant $\hat{c} \ge 1$ and a finite countably additive measure μ on $\mathbb{R}^d$ such that $\operatorname{supp} \mu = \Gamma$ and $\hat{c}^{-1} h(t) \le \mu(B_t(x)) \le \hat{c} h(t)$ for any $x \in \Gamma$ and $t \in (0, 1]$.

Let $\Omega \subset \mathbb{R}^d$, $\Omega \in \mathbf{FC}(a)$, and let $\Gamma \subset \partial\Omega$ be an h-set. We denote $\operatorname{dist}(x, \Gamma) = \inf_{y\in\Gamma} |x - y|$, $x \in \mathbb{R}^d$.

At first we consider the following case. Let the function $h \in \mathbb{H}$ near zero be given by formula

$$h(t) = |\log t|^{-\gamma} |\log|\log t||^{-\kappa} \tau(|\log|\log t||); \tag{1}$$

here $\gamma > 0$, $\kappa \in \mathbb{R}$, $\tau : (0, \infty) \to (0, \infty)$ is an absolutely continuous function such that $\lim\limits_{y\to\infty} y\tau'(y)/\tau(y) = 0$. Let $g(x) = \varphi_g(\operatorname{dist}(x, \Gamma))$, $v(x) = \varphi_v(\operatorname{dist}(x, \Gamma))$, and let the continuous functions φ_g and $\varphi_v : (0, \infty) \to (0, \infty)$ near zero be given by

$$\begin{aligned} \varphi_g(t) &= t^{-\beta_g} |\log t|^{-\alpha_g} |\log|\log t||^{-\sigma_g} \rho_g(|\log|\log t||), \\ \varphi_v(t) &= t^{-\beta_v} |\log t|^{-\alpha_v} |\log|\log t||^{-\sigma_v} \rho_v(|\log|\log t||); \end{aligned} \tag{2}$$

here ρ_g, $\rho_v : (0, \infty) \to (0, \infty)$ are absolutely continuous functions such that

$$\lim_{y\to\infty} y\rho_g'(y)/\rho_g(y) = \lim_{y\to\infty} y\rho_v'(y)/\rho_v(y) = 0.$$

We set

$$\alpha = \alpha_g + \alpha_v, \quad \rho(y) = \rho_g(y)\rho_v(y).$$

Let $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$ be sequences. We write $x_n \asymp y_n$ if there exist $c \ge 1$ and $n_0 \in \mathbb{N}$ such that $c^{-1}x_n \le y_n \le cx_n$ for all $n \ge n_0$.

Theorem 1. Let $1 < p \le q < \infty$, $\delta := r + \frac{d}{q} - \frac{d}{p} > 0$, let conditions (1) and (2) hold, $\beta_v = \frac{d}{q}$, $\beta_g = r - \frac{d}{p}$, $\alpha > \frac{1}{p'} + \frac{1}{q}$, $\alpha_v > \frac{1+\gamma}{q}$.

1. Let $p = q$ or $q \le 2$. We set

$$\theta_1 = \frac{\delta}{d}, \quad \theta_2 = \frac{\alpha}{\gamma+1}, \quad \theta_3 = \frac{\alpha - \frac{1}{p'} - \frac{1}{q}}{\gamma},$$

$$\varphi_1(t) \equiv 1, \quad \varphi_2(t) = |\log t|^{\frac{\kappa\alpha}{\gamma+1} - \sigma} (\tau(|\log t|))^{-\frac{\alpha}{\gamma+1}} \rho(|\log t|),$$

$$\varphi_3(t) = |\log t|^{\frac{\kappa}{\gamma}\left(\alpha - \frac{1}{q} - \frac{1}{p'}\right) - \sigma} (\tau(|\log t|))^{-\frac{1}{\gamma}\left(\alpha - \frac{1}{q} - \frac{1}{p'}\right)} \rho(|\log t|).$$

Suppose that there is $j_* \in \{1, 2, 3\}$ such that $\theta_{j_*} < \min_{j\ne j_*} \theta_j$. Then

$$d_n(W^r_{p,g}(\Omega), L_{q,v}(\Omega)) \asymp n^{-\theta_{j_*}} \varphi_{j_*}(n).$$

2. Let $p < q$, $q > 2$. We set

$$\theta_1 = \frac{\delta}{d} + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\}, \quad \theta_2 = \frac{\alpha}{\gamma+1} + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\},$$

$$\theta_3 = \frac{\alpha - \frac{1}{p'} - \frac{1}{q}}{\gamma} + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\},$$

$$\theta_4 = \frac{q\delta}{2d}, \quad \theta_5 = \frac{q\left(\alpha - \frac{1}{p'} - \frac{1}{q}\right)}{2\gamma},$$

$$\varphi_1(t) = \varphi_4(t) \equiv 1, \quad \varphi_2(t) = |\log t|^{\frac{\kappa\alpha}{\gamma+1} - \sigma}(\tau(|\log t|))^{-\frac{\alpha}{\gamma+1}}\rho(|\log t|),$$

$$\varphi_3(t) = |\log t|^{\frac{\kappa}{\gamma}\left(\alpha - \frac{1}{q} - \frac{1}{p'}\right) - \sigma}(\tau(|\log t|))^{-\frac{1}{\gamma}\left(\alpha - \frac{1}{q} - \frac{1}{p'}\right)}\rho(|\log t|), \quad \varphi_5(t) = \varphi_3(t^{q/2}).$$

Suppose that there is $j_* \in \{1, 2, 3, 4, 5\}$ such that $\theta_{j_*} < \min_{j \neq j_*} \theta_j$. Then

$$d_n(W^r_{p,g}(\Omega), L_{q,v}(\Omega)) \asymp n^{-\theta_{j_*}}\varphi_{j_*}(n).$$

Consider one more example. Let $0 \in \partial\Omega$, $g(x) = \varphi_g(|x|)$, $v(x) = \varphi_v(|x|)$, $0 < \mu < 1$, $c > 0$, the continuous functions φ_g, $\varphi_v : (0, \infty) \to (0, \infty)$ near zero are given by formula

$$\varphi_g(t) = t^{-\beta_g} \cdot 2^{c|\log t|^\mu}|\log t|^{-\alpha_g}, \quad \varphi_v(t) = t^{-\beta_v} \cdot 2^{-c|\log t|^\mu}|\log t|^{-\alpha_v}. \tag{3}$$

We denote $\alpha = \alpha_g + \alpha_v$.

Theorem 2. Let $1 < p \le q < \infty$, $\delta := r + \frac{d}{q} - \frac{d}{p} > 0$, let condition (3) hold, $\beta_g = r - \frac{d}{p}$, $\beta_v = \frac{d}{q}$, $\alpha > (1-\mu)\left(\frac{1}{p'} + \frac{1}{q}\right)$.

1. Let $p = q$ or $q \le 2$. We set

$$\theta_1 = \frac{\delta}{d}, \quad \theta_2 = \alpha, \quad \theta_3 = \frac{1}{\mu}\left(\alpha - (1-\mu)\left(\frac{1}{p'} + \frac{1}{q}\right)\right).$$

Suppose that there is $j_* \in \{1, 2, 3\}$ such that $\theta_{j_*} < \min_{j \neq j_*} \theta_j$. Then

$$d_n(W^r_{p,g}(\Omega), L_{q,v}(\Omega)) \asymp n^{-\theta_{j_*}}.$$

2. Let $p < q$ and $q > 2$. We set

$$\theta_1 = \frac{\delta}{d} + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\}, \quad \theta_2 = \alpha + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\},$$

$$\theta_3 = \frac{1}{\mu}\left(\alpha - (1-\mu)\left(\frac{1}{p'} + \frac{1}{q}\right)\right) + \min\left\{\frac{1}{2} - \frac{1}{q}, \frac{1}{p} - \frac{1}{q}\right\},$$

$$\theta_4 = \frac{q\delta}{2d}, \quad \theta_5 = \frac{q}{2\mu}\left(\alpha - (1-\mu)\left(\frac{1}{p'} + \frac{1}{q}\right)\right).$$

Suppose that there is $j_* \in \{1, 2, 3, 4, 5\}$ such that $\theta_{j_*} < \min_{j \neq j_*} \theta_j$. Then

$$d_n(W^r_{p,g}(\Omega), L_{q,v}(\Omega)) \asymp n^{-\theta_{j_*}}.$$

Remark 1. In this article we stated results for the Kolmogorov widths. Similar estimates (with appropriate changes in formulas) can be stated for linear and Gelfand's widths.

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