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SULTANAEV YAUDAT TALGATOVICH

(to the 70th birthday)

On 19th July 2018 was 70th birthday of Yaudat Talgatovich Sultanaev, doctor of science (1990), professor (1991), honorary scientist of the Russian Federation, laureate of State award of the Republic of Bashkortostan in the field of science and technology, professor of the Bashkir State Pedagogical University, member of the Editorial Board of the Eurasian Mathematical Journal.

Ya.T. Sultanaev was born in the sity of Orsk. In 1971 he graduated from the Bashkir State University and then completed his postgraduate studies in the Moscow State University. Ya.T. Sultanaev's scientific supervisors were distinguished mathematicians A.G. Kostyuchenko and B.M. Levitan.

Ya.T. Sultanaev is a famous specialist in the spectral theory of differential operators and the qualitative theory of ordinary differential equations.

He obtained bilateral Tauberian theorems of Keldysh type, completely solved the problem on spectral assymptotics for semi-bounded ordinary differential operators, suggested a new method of investigation of assymptotic behaviour of solutions to singular differential equations which allowed him to essentially weaken the conditions on coefficients.

Jointly with V.A. Sadovnichii and A.M. Akhtyamov, he investigated inverse spectral problems with non-separated boundary conditions.

He published more than 70 papers in leading mathematical journals.

Among pupils of Ya.T. Sultanaev there are more than 20 candidates of science and one doctor of science.

The Editorial Board of the Eurasian Mathematical Journal congratulates Yaudat Talgatovich on the occasion of his 70th birthday and wishes him good health and new achievements in mathematics and mathematical education.

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BABENKO'S WORK ON SPHERICAL LEBESGUE CONSTANTS

E. Liflyand

Communicated by E.D. Nursultanov

Key words: Lebesgue constants, Fourier transform, integrability.

AMS Mathematics Subject Classification: 42B15, 42B10.

Abstract. The idea of this note is two-fold. On the one hand, this is a preface to the publication of English translation of the celebrated K.I. Babenko' preprint. On the other hand, we give a brief background of the topic at that time and in the subsequent years.

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This is a sort of preface to the publication of the legendary preprint by K.I. Babenko [1]. Everyone who works or ever worked on multidimensional Lebesgue constants knows about this preprint and its place and importance for the topic. However, very few saw it, even those who read Russian. Though it was written long ago, I strongly believe that it still is of great importance and interest, moreover the main open problem posed in this work (see also [8]) yet remains unsolved. In fact, I also believe that Babenko's work speaks for itself, therefore the only goal of this brief preface is to introduce the reader the part of the area the preprint concerns and point out what this and other works by Babenko have contributed to it.

In problems of summability of multiple Fourier series spherical summation plays a special role. Besides spherical partial sums, Riesz type means are intensively studied. The norms of corresponding operators are called the Lebesgue constants:

$$\Lambda(N,\alpha)=\sup_{\|f\|\le 1}\|S_N^\alpha(\cdot,f)\|=\int_{\mathbb{T}^m}\left|\sum_{|k|\le N}\lambda(\frac{k}{N})e^{ik\cdot x}\right|dx,$$

with $\mathbb{T}^m=[-\pi,\pi)^m$ and

$$\lambda(x)=\begin{cases}(1-|x|^2)^\alpha, & |x|<1,\\ 0, & |x|\ge 1,\end{cases}$$

the Bochner-Riesz means of order α. The norms are taken either in L^1 or in C. The most intriguing is the case $0\le\alpha<\frac{m-1}{2}$, the Bochner-Riesz means of order lower than the critical one $\alpha=\frac{m-1}{2}$ and spherical partial Fourier sums if $\alpha=0$. The reason is that for $\alpha>\frac{m-1}{2}$ these norms are uniformly bounded. The critical case is also well studied: the Lebesgue constants behave like those for the one-dimensional partial sums. Their logarithmic asymptotics is proved in [9], a sharp constant is found in [6]. As for the case below critical, what we have is bilateral power order: there are positive constants A and B such that

$$AN^{\frac{m-1}{2}-\alpha}\le\Lambda(N,\alpha)\le BN^{\frac{m-1}{2}-\alpha}. \tag{3.1}$$

The lower estimate is known from [3], the upper one was obtained in [4] and simultaneously and independently in Babenko's preprint. Publication of [4] was apparently the reason why Babenko never converted his preprint in a regular paper.

Many ways are known today how to prove (3.1), the reader can find a comprehensive survey in [7]. Much can also be found in recent books [10] and [5]. However, there are many reasons why the preprint in question is still of interest. First of all, the method used by Babenko, where the theory of zeta-function and number theory were involved, is very special and promising. Promising as an attempt to prove the main open question: *Prove or disprove that there is an asymptotics in (3.1), or, in other words, whether there is or not the limit as* $N \to \infty$ *of*

$$\frac{\Lambda(N,\alpha)}{N^{\frac{m-1}{2}-\alpha}}\,?$$

This question was posed just in this preprint (see also [8]). Other known methods leave even less hope for solving this problem. Mention another interesting attempt by Babenko in the same spirit: [2]. Last but not least there is a hope that publication of the translation of Babenko's preprint will awake interest of new researchers in these specific problems and in the topic in whole.

We mostly preserve Babenko's notation and use it in this note, though it somewhat differs from that nowadays. We also try to express his style. This is seen in some not very traditional expressions, sectioning, numbering. We cannot guarantee that there are no misprints in the author's text. Some of them were corrected without indicating the fact itself and place. The reader must understand that this is just a preprint that never received editorial work and proofreading. The literature is given in the order chosen by the author, somewhere references to translations are given additionally. Despite of the form and passed time, this preprint is still of interest to harmonic analysts.

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