ISSN (Print): 2077–9879
ISSN (Online): 2617-2658

Eurasian Mathematical Journal

2018, Volume 9, Number 3

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia (RUDN University)
the University of Padua

Starting with 2018 co-funded
by the L.N. Gumilyov Eurasian National University
and
the Peoples' Friendship University of Russia (RUDN University)

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Astana, Kazakhstan

Aims and Scope

The Eurasian Mathematical Journal (EMJ) publishes carefully selected original research papers in all areas of mathematics written by mathematicians, principally from Europe and Asia. However papers by mathematicians from other continents are also welcome.

From time to time the EMJ publishes survey papers.

The EMJ publishes 4 issues in a year.

The language of the paper must be English only.

The contents of EMJ are indexed in Scopus, Web of Science (ESCI), Mathematical Reviews, MathSciNet, Zentralblatt Math (ZMATH), Referativnyi Zhurnal – Matematika, Math-Net.Ru.

The EMJ is included in the list of journals recommended by the Committee for Control of Education and Science (Ministry of Education and Science of the Republic of Kazakhstan) and in the list of journals recommended by the Higher Attestation Commission (Ministry of Education and Science of the Russian Federation).

Information for the Authors

Submission. Manuscripts should be written in LaTeX and should be submitted electronically in DVI, PostScript or PDF format to the EMJ Editorial Office via e-mail (eurasianmj@yandex.kz).

When the paper is accepted, the authors will be asked to send the tex-file of the paper to the Editorial Office.

The author who submitted an article for publication will be considered as a corresponding author. Authors may nominate a member of the Editorial Board whom they consider appropriate for the article. However, assignment to that particular editor is not guaranteed.

Copyright. When the paper is accepted, the copyright is automatically transferred to the EMJ. Manuscripts are accepted for review on the understanding that the same work has not been already published (except in the form of an abstract), that it is not under consideration for publication elsewhere, and that it has been approved by all authors.

Title page. The title page should start with the title of the paper and authors' names (no degrees). It should contain the Keywords (no more than 10), the Subject Classification (AMS Mathematics Subject Classification (2010) with primary (and secondary) subject classification codes), and the Abstract (no more than 150 words with minimal use of mathematical symbols).

Figures. Figures should be prepared in a digital form which is suitable for direct reproduction.

References. Bibliographical references should be listed alphabetically at the end of the article. The authors should consult the Mathematical Reviews for the standard abbreviations of journals' names.

Authors' data. The authors' affiliations, addresses and e-mail addresses should be placed after the References.

Proofs. The authors will receive proofs only once. The late return of proofs may result in the paper being published in a later issue.

Offprints. The authors will receive offprints in electronic form.

Publication Ethics and Publication Malpractice

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the EMJ implies that the work described has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The EMJ follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http : //publicationethics.org/files/u2/New$_C$ode.pdf). To verify originality, your article may be checked by the originality detection service CrossCheck http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the EMJ.

The Editorial Board of the EMJ will monitor and safeguard publishing ethics.

The procedure of reviewing a manuscript, established by the Editorial Board of the Eurasian Mathematical Journal

1. **Reviewing procedure**

1.1. All research papers received by the Eurasian Mathematical Journal (EMJ) are subject to mandatory reviewing.

1.2. The Managing Editor of the journal determines whether a paper fits to the scope of the EMJ and satisfies the rules of writing papers for the EMJ, and directs it for a preliminary review to one of the Editors-in-chief who checks the scientific content of the manuscript and assigns a specialist for reviewing the manuscript.

1.3. Reviewers of manuscripts are selected from highly qualified scientists and specialists of the L.N. Gumilyov Eurasian National University (doctors of sciences, professors), other universities of the Republic of Kazakhstan and foreign countries. An author of a paper cannot be its reviewer.

1.4. Duration of reviewing in each case is determined by the Managing Editor aiming at creating conditions for the most rapid publication of the paper.

1.5. Reviewing is confidential. Information about a reviewer is anonymous to the authors and is available only for the Editorial Board and the Control Committee in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (CCFES). The author has the right to read the text of the review.

1.6. If required, the review is sent to the author by e-mail.

1.7. A positive review is not a sufficient basis for publication of the paper.

1.8. If a reviewer overall approves the paper, but has observations, the review is confidentially sent to the author. A revised version of the paper in which the comments of the reviewer are taken into account is sent to the same reviewer for additional reviewing.

1.9. In the case of a negative review the text of the review is confidentially sent to the author.

1.10. If the author sends a well reasoned response to the comments of the reviewer, the paper should be considered by a commission, consisting of three members of the Editorial Board.

1.11. The final decision on publication of the paper is made by the Editorial Board and is recorded in the minutes of the meeting of the Editorial Board.

1.12. After the paper is accepted for publication by the Editorial Board the Managing Editor informs the author about this and about the date of publication.

1.13. Originals reviews are stored in the Editorial Office for three years from the date of publication and are provided on request of the CCFES.

1.14. No fee for reviewing papers will be charged.

2. **Requirements for the content of a review**

2.1. In the title of a review there should be indicated the author(s) and the title of a paper.

2.2. A review should include a qualified analysis of the material of a paper, objective assessment and reasoned recommendations.

2.3. A review should cover the following topics:

- compliance of the paper with the scope of the EMJ;

- compliance of the title of the paper to its content;

- compliance of the paper to the rules of writing papers for the EMJ (abstract, key words and phrases, bibliography etc.);

- a general description and assessment of the content of the paper (subject, focus, actuality of the topic, importance and actuality of the obtained results, possible applications);

- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

- exposition of the paper (clarity, conciseness, completeness of proofs, completeness of bibliographic references, typographical quality of the text);

- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

2.4. The final part of the review should contain an overall opinion of a reviewer on the paper and a clear recommendation on whether the paper can be published in the Eurasian Mathematical Journal, should be sent back to the author for revision or cannot be published.

Web-page

The web-page of EMJ is www.emj.enu.kz. One can enter the web-page by typing Eurasian Mathematical Journal in any search engine (Google, Yandex, etc.). The archive of the web-page contains all papers published in EMJ (free access).

Subscription

For Institutions

- US$ 200 (or equivalent) for one volume (4 issues)
- US$ 60 (or equivalent) for one issue

For Individuals

- US$ 160 (or equivalent) for one volume (4 issues)
- US$ 50 (or equivalent) for one issue.

The price includes handling and postage.

The Subscription Form for subscribers can be obtained by e-mail:

eurasianmj@yandex.kz

The Eurasian Mathematical Journal (EMJ)
The Astana Editorial Office
The L.N. Gumilyov Eurasian National University
Building no. 3
Room 306a
Tel.: +7-7172-709500 extension 33312
13 Kazhymukan St
010008 Astana, Kazakhstan

The Moscow Editorial Office
The Peoples' Friendship University of Russia
(RUDN University)
Room 515
Tel.: +7-495-9550968
3 Ordzonikidze St
117198 Moscow, Russia

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 9, Number 3 (2018), 14 – 24

ON THE UNIQUE SOLVABILITY OF NONLOCAL PROBLEMS WITH INTEGRAL CONDITIONS FOR A HYBRID SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS

A.T. Assanova, A.P. Sabalakhova

Communicated by V.I. Korzyuk

Key words: hybrid system, nonlocal problem, integral condition, solvability, algorithm.

AMS Mathematics Subject Classification: 34A38, 34B10, 34K34, 35G45, 35L50.

Abstract. A nonlocal problem with integral conditions for a hybrid system of partial differential equations is investigated. Based on the results for nonlocal problems for a system of hyperbolic equations coefficient conditions are established ensuring the existence of classical solutions to a nonlocal problem with integral conditions for a hybrid system of partial differential equations, and algorithms of finding such solutions are suggested.

DOI: https://doi.org/10.32523/2077-9879-2018-9-3-14-24

1 Introduction

In this paper, in the domain $\Omega = [0,T] \times [0,\omega]$ we consider the following nonlocal problem with integral conditions for a hybrid system of partial differential equations

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = A_1(t,x)\dfrac{\partial u}{\partial t} + B_1(t,x)\dfrac{\partial v}{\partial x} + C_1(t,x)u + D_1(t,x)v + f_1(t,x) \\ \dfrac{\partial^2 v}{\partial t \partial x} = A_2(t,x)\dfrac{\partial v}{\partial x} + B_2(t,x)\dfrac{\partial u}{\partial t} + C_2(t,x)v + D_2(t,x)u + f_2(t,x) \end{cases}, \tag{1.1}$$

$$u(0,x) = \varphi_0(x), \qquad x \in [0,\omega], \tag{1.2}$$

$$P_1(x)\frac{\partial u(t,x)}{\partial t}|_{t=0} + S_1(x)\frac{\partial u(t,x)}{\partial t}|_{t=a} + \int_0^a K_1(\tau,x)\frac{\partial u(\tau,x)}{\partial \tau}d\tau = \varphi_1(x), \quad x \in [0,\omega], \tag{1.3}$$

$$v(t,0) = \psi(t), \qquad t \in [0,T], \tag{1.4}$$

$$P_2(x)v(0,x) + S_2(x)v(b,x) + \int_0^b K_2(\tau,x)v(\tau,x)d\tau = \varphi_2(x), \quad x \in [0,\omega], \tag{1.5}$$

where $u(t,x)$ and $v(t,x)$ are the unknown functions, the functions $A_1(t,x)$, $A_2(t,x)$, $B_1(t,x)$, $B_2(t,x)$, $C_1(t,x)$, $C_2(t,x)$, $D_1(t,x)$, $D_2(t,x)$, $f_1(t,x)$, $f_2(t,x)$ are continuous on Ω, the functions $P_1(x)$, $S_1(x)$, $\varphi_1(x)$ are continuous on $[0,\omega]$, the functions $P_2(x)$, $S_2(x)$, $\varphi_0(x)$, $\varphi_2(x)$ are continuously differentiable on $[0,\omega]$, the function $K_1(t,x)$ is continuous on Ω, the function $K_2(t,x)$ is continuous and continuously differentiable in x on Ω, and the function $\psi(t)$ is continuously differentiable on $[0,T]$, $0 < a, b \le T$. Moreover, it is assumed that the following compatibility condition is satisfied:

$$P_2(0)\psi(0) + S_2(0)\psi(b) + \int_0^b K_2(\tau,0)\psi(\tau)d\tau = \varphi_2(0).$$

In recent years nonlocal problems for different classes of hybrid systems are of great interest to specialists [11, 14, 15, 18, 19, 22-24, 26]. Mathematical modelling of various processes in physics, chemistry and biology leads to nonlocal problems with integral conditions for hybrid system of partial differential equations of different orders. Sufficient conditions for the existence and uniqueness of solutions to problems for some classes of hybrid systems have been obtained by various methods [11, 14, 15, 18, 19, 22-24, 26].

In [9], a linear boundary value problem with an integral condition for a system of hyperbolic equations was investigated. With the new approach, proposed in [1-8] for boundary value problems with the data on characteristics without integral terms, we established necessary and sufficient conditions for the well-posedness of a linear boundary value problem with an integral condition for a system of hyperbolic equations with mixed derivatives.

In this paper, we study the existence problems of classical solutions to a nonlocal problem with integral conditions for hybrid system (1.1)–(1.5) and methods of constructing their approximate solutions. The results and methods of [9] are extended to a new class of problems - nonlocal problem with integral conditions for the one class of hybrid systems. We establish sufficient conditions for the unique solvability of a nonlocal problem with integral conditions (1.1)–(1.5) in terms of the right-hand side of the system, the boundary functions and the kernels of the integral terms. An algorithm for finding the solution of the considered problem is constructed and the convergence of successive approximations is shown. The results can be used in the numerical solving of applied problems.

A family of problems with integral conditions for a system of ordinary differential equations was investigated in [9] for $a = T$ and $b = T$. Sufficient conditions for the unique solvability were obtained and the ways of finding solutions to considered problems were proposed. In this paper, the results of the paper [9] are extended to the case $0 < a, b < T$.

In this paper we apply the parametrization method [10] to the family of problems with an integral conditions for ordinary differential equations (3.1)–(3.3) without partitioning domain Ω.

2 Reduction of problem (1.1)–(1.5) to an equivalent problem

Let $C(\Omega, R)$ $(C([0,\omega], R))$ be a space of continuous functions $u : \Omega \to R$ $(\varphi : [0,\omega] \to R)$ on Ω ($[0,\omega]$)with norm

$$||u||_0 = \max_{(t,x)\in\Omega} |u(t,x)| \quad (||\varphi||_0 = \max_{x\in[0,\omega]} |\varphi(x)|).$$

The system of functions $(u(t,x), v(t,x))$, where $u(t,x) \in C(\Omega,R)$, $v(t,x) \in C(\Omega,R)$, have partial derivatives $\frac{\partial u(t,x)}{\partial t} \in C(\Omega,R)$, $\frac{\partial^2 u(t,x)}{\partial t^2} \in C(\Omega,R)$, $\frac{\partial v(t,x)}{\partial x} \in C(\Omega,R)$, $\frac{\partial v(t,x)}{\partial t} \in C(\Omega,R)$, $\frac{\partial^2 v(t,x)}{\partial t\partial x} \in C(\Omega,R)$ is called a classical solution to problem (1.1)–(1.5) if it satisfies hybrid system (1.1) for all $(t,x) \in \Omega$ and meets boundary conditions (1.2), (1.3), (1.4) and (1.5).

In this section, we introduce new unknown functions and reduce problem (1.1)–(1.5) to an equivalent problem consisting of the family of problems with integral conditions for a system of partial differential equations and integral relations. The algorithm is suggested for finding approximate solutions to the considered problem, their convergence is proved. Conditions, ensuring the classical solvability of problem (1.1)–(1.5), are established.

We introduce the new unknown functions $z_1(t,x) = \frac{\partial u(t,x)}{\partial t}$, and $z_2(t,x) = \frac{\partial v(t,x)}{\partial x}$, and reduce problem (1.1)–(1.5) to the equivalent problem

$$\begin{cases} \frac{\partial z_1}{\partial t} = A_1(t,x)z_1 + B_1(t,x)z_2 + C_1(t,x)u + D_1(t,x)v + f_1(t,x) \\ \frac{\partial z_2}{\partial t} = A_2(t,x)z_2 + B_2(t,x)z_1 + C_2(t,x)v + D_2(t,x)u + f_2(t,x) \end{cases}, \tag{2.1}$$

$$P_1(x)z_1(0,x)+S_1(x)z_1(a,x)+\int_0^a K_1(\tau,x)z_1(\tau,x)d\tau=\varphi_1(x), \quad x\in[0,\omega], \tag{2.2}$$

$$P_2(x)z_2(0,x)+S_2(x)z_2(b,x)+\int_0^b K_2(\tau,x)z_2(\tau,x)d\tau=\varphi_2'(x)-$$

$$-P_2'(x)v(0,x)-S_2'(x)v(b,x)-\int_0^b \frac{\partial K_2(\tau,x)}{\partial x}v(\tau,x)d\tau, \quad x\in[0,\omega], \tag{2.3}$$

$$u(t,x)=\varphi_0(x)+\int_0^t z_1(\tau,x)d\tau, \qquad v(t,x)=\psi(t)+\int_0^x z_2(t,\xi)d\xi, \qquad (t,x)\in\Omega. \tag{2.4}$$

In problem (2.1)–(2.4) conditions (1.2) and (1.4) are taken into account in relations (2.4).

A quadruple $\{z_1(t,x),z_2(t,x),u(t,x),v(t,x)\}$ of functions continuous on Ω is called a solution to problem (2.1)–(2.4) if the functions $z_1(t,x)$ and $z_2(t,x))$ belong to $C(\Omega,R)$, have continuous derivatives with respect to t on Ω and satisfy system of ordinary differential equations (2.1) and integral conditions (2.2), (2.3), where the functions $u(t,x)$, and $v(t,x)$ are expressed via $z_1(t,x)$ and $z_2(t,x)$ by functional relations (2.4).

The problems (1.1)–(1.5) and (2.1)–(2.4) are equivalent in the following sense. Let a pair $(u^*(t,x),v^*(t,x))$ be a classical solution to problem (1.1)–(1.5). Then the quadruple $\{z)_1^*(t,x),z_2^*(t,x),\ u^*(t,x),\ v^*(t,x)$, where $z_1^*(t,x)=\dfrac{\partial u^*(t,x)}{\partial t}$, $z_2^*(t,x)=\dfrac{\partial v^*(t,x)}{\partial x}$, is a solution to problem (2.1)–(2.4). Conversely, if a quadruple $\{\widetilde{z}_1(t,x),\widetilde{z}_2(t,x),\widetilde{u}(t,x),\widetilde{v}(t,x)\}$ is a solution to problem (2.1)–(2.4), then the pair $(\widetilde{u}(t,x),\widetilde{v}(t,x))$ is a classical solution to problem (1.1)–(1.5).

Under the fixed $u(t,x)$, $v(t,x)$ we may consider system of equations (2.1) with conditions (2.2), (2.3) as a one-parametered family of problems with integral conditions for system of ordinary differential equations. Integral conditions (2.4) allow us to determine the functions $u(t,x)$, $v(t,x)$ via the solution to the family of problems with integral conditions for a system of ordinary differential equations.

Thus, the solution to a problem with integral conditions for a hybrid system (1.1)–(1.5) depends on the solutions to a family of problems with integral conditions for a system of ordinary differential equations.

3 Family of problems with integral conditions for system of ordinary differential equations

Consider the following one-parametered family of problems with integral conditions for a system of ordinary differential equations

$$\begin{cases} \dfrac{\partial z_1}{\partial t}=A_1(t,x)z_1+B_1(t,x)z_2+F_1(t,x) \\ \dfrac{\partial z_2}{\partial t}=A_2(t,x)z_2+B_2(t,x)z_1+F_2(t,x) \end{cases}, \tag{3.1}$$

$$P_1(x)z_1(0,x)+S_1(x)z_1(a,x)+\int_0^a K_1(\tau,x)z_1(\tau,x)d\tau=\Phi_1(x), \qquad x\in[0,\omega], \tag{3.2}$$

$$P_2(x)z_2(0,x)+S_2(x)z_2(b,x)+\int_0^b K_2(\tau,x)z_2(\tau,x)d\tau=\Phi_2(x), \qquad x\in[0,\omega], \tag{3.3}$$

where $F_i(t,x)\in C(\Omega,R)$, and $\Phi_i(x)\in C([0,\omega],R)$, $i=1,2$.

A pair of functions $(z_1(t,x),z_2(t,x))$, where $z_1:\Omega\to R$, $z_2:\Omega\to R$, continuous on Ω and continuously differentiable with respect to t on Ω, is called a solution to the one-parametered family of problems with integral conditions (3.2), (3.3), if given any $(t,x)\in\Omega$ it satisfies the system (3.1) and given any $x\in[0,\omega]$ it satisfies the conditions (3.2), (3.3).

For a fixed $x\in[0,\omega]$ problem (3.1)–(3.3) is a linear problem with integral condition for a system of ordinary differential equations. The different types of problems with integral condition for differential equations have been investigated by various methods [12, 13, 16, 17, 20, 25]. Suppose a variable x is changed on $[0,\omega]$; then we obtain a family of problems with an integral conditions for ordinary differential equations.

The scheme of parametrization method. Let a pair of functions $(z_1(t,x),z_2(t,x))$ be a solution to problem (3.1)–(3.3). By $\lambda_1(x)$ and $\lambda_2(x)$ denote the values of $z_1(t,x)$ and $z_2(t,x)$ under $t=0$, respectively. We replace $z_1(t,x)$ by $\widetilde{z}_1(t,x)+\lambda_1(x)$ and $z_2(t,x)$ by $\widetilde{z}_2(t,x)+\lambda_2(x)$ in the domain Ω.

Problem (3.1)–(3.3) is equivalent to a problem with unknown functions $\lambda_1(x)$, $\lambda_2(x)$:

$$\begin{cases} \dfrac{\partial\widetilde{z}_1}{\partial t}=A_1(t,x)\widetilde{z}_1+A_1(t,x)\lambda_1(x)+B_1(t,x)\widetilde{z}_2+B_1(t,x)\lambda_2(x)+F_1(t,x)\\ \dfrac{\partial\widetilde{z}_2}{\partial t}=A_2(t,x)\widetilde{z}_2+A_2(t,x)\lambda_2(x)+B_2(t,x)\widetilde{z}_1+B_2(t,x)\lambda_1(x)+F_2(t,x)\end{cases}, \tag{3.4}$$

$$\widetilde{z}_1(0,x)=0,\qquad \widetilde{z}_2(0,x)=0,\qquad x\in[0,\omega], \tag{3.5}$$

$$\left[P_1(x)+S_1(x)+\int_0^a K_1(\tau,x)d\tau\right]\lambda_1(x)+S_1(x)\widetilde{z}_1(a,x)+$$
$$+\int_0^a K_1(\tau,x)\widetilde{z}_1(\tau,x)d\tau=\Phi_1(x),\qquad x\in[0,\omega], \tag{3.6}$$

$$\left[P_2(x)+S_2(x)+\int_0^b K_2(\tau,x)d\tau\right]\lambda_2(x)+S_2(x)\widetilde{z}_2(b,x)+$$
$$+\int_0^b K_2(\tau,x)\widetilde{z}_2(\tau,x)d\tau=\Phi_2(x),\qquad x\in[0,\omega], \tag{3.7}$$

The quadruple $(\lambda_1^*(x),\lambda_2^*(x),\widetilde{z}_1^*(t,x),\widetilde{z}_2^*(t,x))$, where $\lambda_1^*(x)\in C([0,\omega],R)$, $\lambda_2^*(x)\in C([0,\omega],R)$, and $\widetilde{z}_1^*(t,x)\in C(\Omega,R)$, $\widetilde{z}_2^*(t,x)\in C(\Omega,R)$, is a solution to problem (3.4)–(3.7) if the functions $\widetilde{z}_1^*(t,x)$, $\widetilde{z}_2^*(t,x)$ are continuously differentiable on Ω, satisfies the Cauchy problem (3.4), (3.5) with $\lambda_1(x)=\lambda_1^*(x)$, $\lambda_2(x)=\lambda_2^*(x)$ for all $(t,x)\in\Omega$ and conditions (3.6), (3.7) for all $x\in[0,\omega]$.

Problems (3.1)–(3.3) and (3.4)–(3.7) are equivalent in the following sense. If a pair of functions $(z_1(t,x),z_2(t,x))$ is a solution to problem (3.1)–(3.3), then the quadruple $(\lambda_1(x),\lambda_2(x),\widetilde{z}_1(t,x),\widetilde{z}_2(t,x))$ with components $\lambda_1(x)=z_1(0,x)$, $\lambda_2(x)=z_2(0,x)$, and $\widetilde{z}_1(t,x)=z_1(t,x)-z_1(0,x)$, $\widetilde{z}_2(t,x)=z_2(t,x)-z_2(0,x)$, and $(t,x)\in\Omega$, is a solution to problem (3.4)–(3.7). Conversely, if a quadruple $(\lambda_1^*(x),\lambda_2^*(x),\widetilde{z}_1^*(t,x),\widetilde{z}_2^*(t,x))$ is a solution to problem (3.4)–(3.7), then the pair of functions $(z_1^*(t,x),z_2^*(t,x))$ defined by the equalities $z_1^*(t,x)=\lambda_1^*(x)+\widetilde{z}_1^*(t,x)$, $z_2^*(t,x)=\lambda_2^*(x)+\widetilde{z}_2^*(t,x)$ for all $(t,x)\in\Omega$ is a solution to problem (3.1)–(3.3).

In problem (3.4)–(3.7), we have the initial conditions $\widetilde{z}_1(0,x)=0$, $\widetilde{z}_2(0,x)=0$. Cauchy problem (3.4), (3.5) is equivalent to the following family of systems of integral equations on $[0,T]$ with $\lambda_1(x)$ and $\lambda_2(x)$

$$\widetilde{z}_1(t,x)=\int_0^t A_1(\tau,x)\widetilde{z}_1(\tau,x)d\tau+\int_0^t B_1(\tau,x)\widetilde{z}_2(\tau,x)d\tau+$$
$$+\int_0^t A_1(\tau,x)d\tau\lambda_1(x)+\int_0^t B_1(\tau,x)d\tau\lambda_2(x)+\int_0^t F_1(\tau,x)d\tau,\quad t\in[0,T], \tag{3.8}$$

$$\widetilde{z}_2(t,x) = \int_0^t A_2(\tau,x)\widetilde{z}_2(\tau,x)d\tau + \int_0^t B_2(\tau,x)\widetilde{z}_1(\tau,x)d\tau +$$
$$+ \int_0^t A_2(\tau,x)d\tau\lambda_2(x) + \int_0^t B_2(\tau,x)d\tau\lambda_1(x) + \int_0^t F_2(\tau,x)d\tau, \quad t \in [0,T]. \tag{3.9}$$

From (3.8) we find $\widetilde{z}_1(a,x)$, $\widetilde{z}_1(\tau,x)$ for all $x \in [0,\omega]$, and from (3.9) we find $\widetilde{z}_2(b,x)$, $\widetilde{z}_2(\tau,x)$ for all $x \in [0,\omega]$. Then, substituting them in (3.6) and (3.7), we obtain the following system of equations with respect to functional parameters $\lambda_1(x)$ and $\lambda_2(x)$:

$$Q(x)\lambda(x) = -G(x,\widetilde{z}) - H(x), \qquad x \in [0,\omega], \tag{3.10}$$

where

$$\lambda(x) = \begin{pmatrix} \lambda_1(x) \\ \lambda_2(x) \end{pmatrix}, \quad Q(x) = \begin{bmatrix} Q_{11}(x) & Q_{12}(x) \\ Q_{21}(x) & Q_{22}(x) \end{bmatrix}, \quad \widetilde{z}(t,x) = \begin{pmatrix} \widetilde{z}_1(t,x) \\ \widetilde{z}_2(t,x) \end{pmatrix},$$

$$G(x,\widetilde{z}) = \begin{pmatrix} G_1(x,\widetilde{z}) \\ G_2(x,\widetilde{z}) \end{pmatrix}, \qquad H(x) = \begin{pmatrix} H_1(x) \\ H_2(x) \end{pmatrix},$$

$$Q_{11}(x) = P_1(x) + S_1(x)\Big(1 + \int_0^a A_1(\tau,x)d\tau\Big) + \int_0^a K_1(\tau,x)\Big(1 + \int_0^\tau A_1(\tau_1,x)d\tau_1\Big)d\tau,$$
$$Q_{12}(x) = S_1(x)\int_0^a B_1(\tau,x)d\tau + \int_0^a K_1(\tau,x)\int_0^\tau B_1(\tau_1,x)d\tau_1 d\tau,$$
$$Q_{21}(x) = S_2(x)\int_0^b B_2(\tau,x)d\tau + \int_0^b K_2(\tau,x)\int_0^\tau B_2(\tau_1,x)d\tau_1 d\tau,$$
$$Q_{22}(x) = P_2(x) + S_2(x)\Big(1 + \int_0^b A_2(\tau,x)d\tau\Big) + \int_0^b K_2(\tau,x)\Big(1 + \int_0^\tau A_2(\tau_1,x)d\tau_1\Big)d\tau,$$
$$G_1(x,\widetilde{z}) = S_1(x)\int_0^a \Big[A_1(\tau,x)\widetilde{z}_1(\tau,x) + B_1(\tau,x)\widetilde{z}_2(\tau,x)\Big]d\tau +$$
$$+ \int_0^a K_1(\tau,x)\int_0^\tau \Big[A_1(\tau_1,x)\widetilde{z}_1(\tau_1,x) + B_1(\tau_1,x)\widetilde{z}_2(\tau_1,x)\Big]d\tau_1 d\tau,$$
$$G_2(x,\widetilde{z}) = S_2(x)\int_0^b \Big[B_2(\tau,x)\widetilde{z}_1(\tau,x) + A_2(\tau,x)\widetilde{z}_2(\tau,x)\Big]d\tau +$$
$$+ \int_0^b K_2(\tau,x)\int_0^\tau \Big[B_2(\tau_1,x)\widetilde{z}_1(\tau_1,x) + A_2(\tau_1,x)\widetilde{z}_2(\tau_1,x)\Big]d\tau_1 d\tau,$$
$$H_1(x) = S_1(x)\int_0^a F_1(\tau,x)d\tau + \int_0^a K_1(\tau,x)\int_0^\tau F_1(\tau_1,x)d\tau_1 d\tau - \Phi_1(x),$$
$$H_2(x) = S_2(x)\int_0^b F_2(\tau,x)d\tau + \int_0^b K_2(\tau,x)\int_0^\tau F_2(\tau_1,x)d\tau_1 d\tau - \Phi_2(x).$$

If we know $\widetilde{z}_1(t,x) \in C(\Omega,R)$ and $\widetilde{z}_2(t,x) \in C(\Omega,R)$, then from (3.10) we find $\lambda(x) = (\lambda_1(x),\lambda_2(x))' \in C([0,\omega],R^2)$. Conversely, if we know $\lambda(x) \in C([0,\omega],R^2)$, then from (3.8) and (3.9) we can find $\widetilde{z}_1(t,x) \in C(\Omega,R)$ and $\widetilde{z}_2(t,x) \in C(\Omega,R)$. Since the $\widetilde{z}_1(t,x)$, $\widetilde{z}_2(t,x)$ and $\lambda_1(x)$, $\lambda_2(x)$ are unknown, to find a solution to problem (3.4)–(3.7) we use the iterative method. A quadruple $(\lambda_1^*(x),\lambda_2^*(x),\widetilde{z}_1^*(t,x),\widetilde{z}_2^*(t,x))$ we determine as the limit of the sequence $(\lambda_1^{(m)}(x),\lambda_2^{(m)}(x),\widetilde{z}_1^{(m)}(t,x),\widetilde{z}_2^{(m)}(t,x))$, $m = 0,1,2,...$, constructed by the following algorithm:

Step 0. We assume that the (2×2) matrix $Q(x)$ is invertible for all $x \in [0,\omega]$. The zero approximations with respect to the functional parameter $\lambda^{(0)}(x) \in C([0,\omega],R^2)$ we define from system of linear equations (3.10) with $\widetilde{z}_1(t,x) = 0$, $\widetilde{z}_2(t,x) = 0$ for all $(t,x) \in \Omega$. Further, solving family of Cauchy problems (3.4), (3.5) for $\lambda_1(x) = \lambda_1^{(0)}(x)$, $\lambda_2(x) = \lambda_2^{(0)}(x)$ on Ω, we find $\widetilde{z}_1^{(0)}(t,x) \in C(\Omega,R)$ and $\widetilde{z}_2^{(0)}(t,x) \in C(\Omega,R)$.

Step 1. Replacing the functions $\widetilde{z}_1(t,x)$ and $\widetilde{z}_2(t,x)$ by $\widetilde{z}_1^{(0)}(t,x)$ and $\widetilde{z}_2^{(0)}(t,x)$ in system (3.10), we determine $\lambda^{(1)}(x) \in C([0,\omega],R^2)$. From family of Cauchy problems (3.4), (3.5) for $\lambda_1(x) = \lambda_1^{(1)}(x)$ and $\lambda_2(x) = \lambda_2^{(1)}(x)$ on Ω we find $\widetilde{z}_1^{(1)}(t,x) \in C(\Omega,R)$ and $\widetilde{z}_2^{(1)}(t,x) \in C(\Omega,R)$. And so on.

Step m. Substituting $\widetilde{z}_1^{(m-1)}(t,x)$ and $\widetilde{z}_2^{(m-1)}(t,x)$ for $\widetilde{z}_1(t,x)$ and $\widetilde{z}_2(t,x)$ in system (3.10), we determine $\lambda^{(m)}(x) \in C([0,\omega],R^2)$. From family of Cauchy problems (3.4), (3.5) for $\lambda_1(x) = \lambda_1^{(m)}(x)$ and $\lambda_2(x) = \lambda_2^{(m)}(x)$ on Ω we find $\widetilde{z}_1^{(m)}(t,x) \in C(\Omega,R)$ and $\widetilde{z}_2^{(m)}(t,x) \in C(\Omega,R)$, $m = 1,2,\dots$.

The method of parametrization divide the process of finding unknown functions into two parts: 1) from system of functional equations (3.10) we find the introducing parameters $\lambda_1(x)$ and $\lambda_2(x)$. 2) from family of Cauchy problems for ordinary differential equations (3.4), (3.5) we find the unknown functions $\widetilde{z}_1(t,x)$ and $\widetilde{z}_2(t,x)$.

Let $\alpha(x) = \max\Big(\max\limits_{t\in[0,T]}|A_1(t,x)| + \max\limits_{t\in[0,T]}|B_1(t,x)|, \max\limits_{t\in[0,T]}|A_2(t,x)| + \max\limits_{t\in[0,T]}|B_2(t,x)|\Big)$.

Now we state the main theorem on the realization and convergence of the proposed algorithm. This assertion also provides sufficient conditions for the unique solvability of problem (3.1)–(3.3).

Theorem 3.1. *Suppose that the* (2×2) *matrix* $Q(x)$ *is invertible for all* $x \in [0,\omega]$ *and the following inequalities hold:*

(a) $||Q^{-1}(x)|| \le \gamma(x)$;

(b)
$$q(x) = \gamma(x)\max\Bigg(\{|S_1(x)| + a\max_{t\in[0,a]}|K_1(t,x)|\}\Big[e^{\alpha(x)a} - 1 - \alpha(x)a\Big],$$
$$\{|S_2(x)| + b\max_{t\in[0,b]}|K_2(t,x)|\}\Big[e^{\alpha(x)b} - 1 - \alpha(x)b\Big]\Bigg) \le \chi < 1,$$
where $\gamma(x)$ *is a positive continuous function in* $x \in [0,\omega]$, *and* $\chi - const$.

Then problem (3.1)–(3.3) has a unique solution $(z_1^*(t,x), z_2^*(t,x)) \in C(\Omega,R)\times C(\Omega,R)$, *and the following estimates hold*

$$\max_{t\in[0,T]}||z^*(t,x)|| = \max_{t\in[0,T]}\max\Big(|z_1^*(t,x)|, |z_2^*(t,x)|\Big) \le$$

$$\le [k_1(x) + k_2(x)]\max\Big(\max_{t\in[0,T]}|F_1(t,x)|, \max_{t\in[0,T]}|F_2(t,x)|, |\Phi_1(x)|, |\Phi_2(x)|\Big), \qquad (3.11)$$

where

$$k_1(x) = \frac{\gamma(x)}{1-q(x)}\max\Big(\big[|S_1(x)| + a\max_{t\in[0,a]}|K_1(t,x)|\big]a, \big[|S_2(x)| + b\max_{t\in[0,b]}|K_2(t,x)|\big]b\Big)\alpha(x)\cdot k_0(x)$$
$$+\gamma(x)\max\Big(a\Big\{1 + |S_1(x)| + \max_{t\in[0,a]}|K_1(t,x)|a\Big\}, b\Big\{1 + |S_2(x)| + \max_{t\in[0,b]}|K_2(t,x)|b\Big\}\Big),$$
$$k_2(x) = \max\Big(\Big\{[e^{\alpha(x)a} - 1]\frac{\gamma(x)}{1-q(x)}\big[|S_1(x)| + \max_{t\in[0,a]}|K_1(t,x)|a\big]\alpha(x)a + 1\Big\},$$
$$\Big\{[e^{\alpha(x)b} - 1]\frac{\gamma(x)}{1-q(x)}\big[|S_2(x)| + \max_{t\in[0,b]}|K_2(t,x)|b\big]\alpha(x)b + 1\Big\}\Big)k_0(x),$$
$$k_0(x) = \max\Big(e^{\alpha(x)a} - 1, e^{\alpha_2(x)b} - 1\Big)\gamma(x)\Big[1+$$
$$+\max\Big(|S_1(x)| + \max_{t\in[0,a]}|K_1(t,x)|a, |S_2(x)| + \max_{t\in[0,a]}|K_2(t,x)|b\Big)\Big] + \max\Big(ae^{\alpha(x)a}, be^{\alpha(x)b}\Big).$$

This theorem is proved similarly to the proof of Theorem 2 in [9].

4 Unique solvability of problem (2.1)–(2.4). Main result

Consider problem (2.1)–(2.4) which is equivalent to (1.1)–(1.5).

Assume that the (2×2) matrix $Q(x)$ is invertible for all $x \in [0,\omega]$.

If we know that $z_1(t,x)$ and $z_2(t,x)$ are a classical solution to problem (2.1)–(2.3), then from integral relations (2.4) we find $u(t,x)$ and $v(t,x)$. Conversely, if we know $u(t,x)$ and $v(t,x)$, then from nonlocal problem (2.1)–(2.3) we can find $z_1(t,x)$ and $z_2(t,x)$. Since the $z_1(t,x)$, $z_2(t,x)$ and $u(t,x)$, $v(t,x)$ are unknown, to find a solution to problem (2.1)–(2.4) we use the iterative method. A quadruple $\{z_1^*(t,x), z_2^*(t,x), u^*(t,x), v^*(t,x)\}$, we determine as the limit of the sequence $\{z_1^{(k)}(t,x), z_2^{(k)}(t,x), u^{(k)}(t,x), v^{(k)}(t,x)\}$, $k=0,1,2,...$ constructed by the following algorithm:

Step 0. 1) From boundary value problem with integral condition (2.1)–(2.3) for $u(t,x)=\varphi_0(x)$, $v(t,x)=\psi(t)$ on Ω we find $z_1^{(0)}(t,x)\in C(\Omega,R)$ and $z_2^{(0)}(t,x)\in C(\Omega,R)$. 2) From integral relations (2.4) for $z_1(t,x)=z_1^{(0)}(t,x)$, $z_2(t,x)=z_2^{(0)}(t,x)$ we find $u^{(0)}(t,x)\in C(\Omega,R)$, $v^{(0)}(t,x)\in C(\Omega,R)$.

Step 1. 1) From boundary value problem with integral condition (2.1)–(2.3) for $u(t,x)=u^{(0)}(t,x)$, $v(t,x)=v^{(0)}(t,x)$ on Ω we find $z_1^{(1)}(t,x)\in C(\Omega,R)$ and $z_2^{(1)}(t,x)\in C(\Omega,R)$. 2) From integral relations (2.4) for $z_1(t,x)=z_1^{(1)}(t,x)$, $z_2(t,x)=z_2^{(1)}(t,x)$, we find $u^{(1)}(t,x)\in C(\Omega,R)$, $v^{(1)}(t,x)\in C(\Omega,R)$.And so on.

Step k. 1) From boundary value problem with integral condition (2.1)–(2.3) for $u(t,x)=u^{(k-1)}(t,x)$, $v(t,x)=v^{(k-1)}(t,x)$ on Ω we find $z_1^{(k)}(t,x)\in C(\Omega,R)$ and $z_2^{(k)}(t,x)\in C(\Omega,R)$. 2) From integral relations (2.4) for $z_1(t,x)=z_1^{(k)}(t,x)$, $z_2(t,x)=z_2^{(k)}(t,x)$, we find $u^{(k)}(t,x)\in C(\Omega,R)$, $v^{(k)}(t,x)\in C(\Omega,R)$, $k=1,2,...$.

Let

$$K=\max_{x\in[0,\omega]}[k_1(x)+k_2(x)],\quad c_1(x)=\max_{t\in[0,T]}|C_1(t,x)|,\quad c_2(x)=\max_{t\in[0,T]}|C_2(t,x)|,$$

$$d_1(x)=\max_{t\in[0,T]}|D_1(t,x)|,\quad d_2(x)=\max_{t\in[0,T]}|D_2(t,x)|,$$

$$p(x)=|P_2'(x)|+|S_2'(x)|+b\max_{t\in[0,b]}\left|\frac{\partial K_2(t,x)}{\partial x}\right|.$$

The following statement provides conditions of the realization and convergence of the proposed algorithm. This assertion also defines sufficient conditions for the unique solvability of problem (2.1)–(2.4).

Theorem 4.1 *Suppose that*
i) the (2×2) *matrix* $Q(x)$ *is invertible for all* $x\in[0,\omega]$*;*
ii) the inequalities a) and b) of Theorem 3.1 hold;
iii) for some $0<\chi_1<1$ *for all* $x\in[0,\omega]$ *the inequality* $q_1(x)=K\max\big(c_1(x),d_2(x)\big)T\le\chi_1$ *is valid.*

Then problem (2.1)–(2.4) has a unique solution $\{z_1^*(t,x), z_2^*(t,x), u^*(t,x), v^*(t,x)\}$.

Proof. The proof will be carried out according to the above algorithm. Let conditions i)-ii) of Theorem 4.1 hold. On kth step of the algorithm we will solve the following problems

$$\begin{cases}\dfrac{\partial z_1^{(k)}}{\partial t}=A_1(t,x)z_1^{(k)}+B_1(t,x)z_2^{(k)}+C_1(t,x)u^{(k-1)}+D_1(t,x)v^{(k-1)}+f_1(t,x)\\ \dfrac{\partial z_2^{(k)}}{\partial t}=A_2(t,x)z_2^{(k)}+B_2(t,x)z_1^{(k)}+C_2(t,x)v^{(k-1)}+D_2(t,x)u^{(k-1)}+f_2(t,x)\end{cases},\tag{4.1}$$

$$P_1(x)z_1^{(k)}(0,x)+S_1(x)z_1^{(k)}(a,x)+\int_0^a K_1(\tau,x)z_1^{(k)}(\tau,x)d\tau=\varphi_1(x),\quad x\in[0,\omega],\tag{4.2}$$

$$P_2(x)z_2^{(k)}(0,x)+S_2(x)z_2^{(k)}(b,x)+\int_0^b K_2(\tau,x)z_2^{(k)}(\tau,x)d\tau=\varphi_2'(x)-$$

$$-P_2'(x)v^{(k-1)}(0,x)-S_2'(x)v^{(k-1)}(b,x)-\int_0^b\frac{\partial K_2(\tau,x)}{\partial x}v^{(k-1)}(\tau,x)d\tau,\quad x\in[0,\omega],\tag{4.3}$$

$$u^{(k)}(t,x)=\varphi_0(x)+\int_0^t z_1^{(k)}(\tau,x)d\tau, \quad v^{(k)}(t,x)=\psi(t)+\int_0^x z_2^{(k)}(t,\xi)d\xi, \quad (t,x)\in\Omega. \tag{4.4}$$

For the successive approximations $z_1^{(k)}(t,x)$, $z_2^{(k)}(t,x)$, $u^{(k)}(t,x)$, $v^{(k)}(t,x)$ we obtain the following estimates

$$\max_{t\in[0,T]}\max\Big(|z_1^{(k)}(t,x)|,|z_2^{(k)}(t,x)|\Big)\le K\cdot\max\Big(\max(c_1(x),d_2(x))\max_{t\in[0,T]}|u^{(k-1)}(t,x)|+$$

$$+\max(d_1(x),c_2(x))\max_{t\in[0,T]}|v^{(k-1)}(t,x)|+\max\Big(\max_{t\in[0,T]}|f_1(t,x)|,\max_{t\in[0,T]}|f_2(t,x)|\Big),$$

$$p(x)\max_{t\in[0,T]}|v^{(k-1)}(t,x)|+\max\Big(|\varphi_1(x)|,|\varphi_2'(x)|\Big)\Big),$$

$$|u^{(k)}(t,x)|\le|\varphi_0(x)|+\int_0^t|z_1^{(k)}(\tau,x)|d\tau,$$

$$|v^{(k)}(t,x)|\le|\psi(t)|+\int_0^x|z_2^{(k)}(t,\xi)|d\xi.$$

Further, from $(k+1)$th step of the algorithm we find the successive approximations $z_1^{(k+1)}(t,x)$, $z_2^{(k+1)}(t,x)$, $u^{(k+1)}(t,x)$, $v^{(k+1)}(t,x)$.

We construct the differences $\Delta z_1^{(k+1)}(t,x)=z_1^{(k+1)}(t,x)-z_1^{(k)}(t,x)$, $\Delta z_2^{(k+1)}(t,x)=z_2^{(k+1)}(t,x)-z_2^{(k)}(t,x)$, $\Delta u^{(k+1)}(t,x)=u^{(k+1)}(t,x)-u^{(k)}(t,x)$, $\Delta v^{(k+1)}(t,x)=v^{(k+1)}(t,x)-v^{(k)}(t,x)$.

For these differences, analogously to the above, we obtain the following estimates

$$\max_{t\in[0,T]}\max\Big\{|\Delta z_1^{(k+1)}(t,x)|,|\Delta z_2^{(k+1)}(t,x)|\Big\}\le K\Big(\max(c_1(x),d_2(x))\max_{t\in[0,T]}|\Delta u^{(k)}(t,x)|+$$

$$+[\max(d_1(x),c_2(x))+p(x)]\max_{t\in[0,T]}|\Delta v^{(k)}(t,x)|\Big), \tag{4.5}$$

$$|\Delta u^{(k+1)}(t,x)|\le\int_0^t|\Delta z_1^{(k+1)}(\tau,x)|d\tau, \tag{4.6}$$

$$|\Delta v^{(k+1)}(t,x)|\le\int_0^x|\Delta z_2^{(k+1)}(t,\xi)|d\xi. \tag{4.7}$$

This implies the main inequality

$$\max\Big\{\max_{t\in[0,T]}|\Delta z_1^{(k+1)}(t,x)|,\max_{t\in[0,T]}|\Delta z_2^{(k+1)}(t,x)|\Big\}\le$$

$$\le q_1(x)\Big\{\max_{t\in[0,T]}|\Delta z_1^{(k)}(t,x)|,\max_{t\in[0,T]}|\Delta z_2^{(k)}(t,x)|\Big\}+$$

$$+K[\max(d_1(x),c_2(x))+p(x)]\int_0^x\Big\{\max_{t\in[0,T]}|\Delta z_1^{(k+1)}(t,\xi)|,\max_{t\in[0,T]}|\Delta z_2^{(k+1)}(t,\xi)|\Big\}d\xi. \tag{4.8}$$

From (4.8) and condition iii) of Theorem 2 it follows that the sequences $\{z_1^{(k)}(t,x)\}$ and $\{z_2^{(k)}(t,x)\}$ converges to $\{z_1^*(t,x)\}$ and $\{z_2^*(t,x)\}$ as $k\to\infty$ for all $(t,x)\in\Omega$.

Then from (4.6), (4.7) it follows that the sequences $\{u^{(k)}(t,x)\}$ and $\{v^{(k)}(t,x)\}$ are convergent in the space $C(\Omega,R)$ as $k\to\infty$. In this case, the limit functions $u^*(t,x)$ and $v^*(t,x)$ are continuous on Ω.

Passing to the limit in relations (4.1)–(4.4) as $k \to \infty$ we obtain that the quadruple $\{z_1^*(t,x), z_2^*(t,x),\ u^*(t,x),\ v^*(t,x)$ is a solution to problem (2.1)–(2.4).

Finally we show that the uniqueness of a solution to problem (2.1)–(2.4). Let the quadruples $\{z_1^*(t,x), z_1^*(t,x),\ u^*(t,x),\ v^*(t,x)\}$ and $\{z_1^{**}(t,x), z_2^{**}(t,x),\ u^{**}(t,x),\ v^{**}(t,x)\}$ be solutions to problem (2.1)–(2.4).

Using inequality (4.8) for the differences $z_1^*(t,x) - z_1^{**}(t,x)$, $z_2^*(t,x) - z_2^{**}(t,x)$, we obtain

$$\max\Big\{\max_{t\in[0,T]} |z_1^*(t,x) - z_1^{**}(t,x)|, \max_{t\in[0,T]} |z_2^*(t,x) - z_2^{**}(t,x)|\Big\} \leq$$

$$\leq \frac{1}{1-q_1(x)} K[\max(d_1(x), c_2(x)) + p(x)] \times$$

$$\times \int_0^x \max\Big\{\max_{t\in[0,T]} |z_1^*(t,\xi) - z_1^{**}(t,\xi)|, \max_{t\in[0,T]} |z_2^*(t,\xi) - z_2^{**}(t,\xi|\Big\} d\xi. \tag{4.9}$$

By applying the Gronwall - Bellman inequality in integral equations (4.9), we get

$$\max\Big\{\max_{t\in[0,T]} |z_1^*(t,x) - z_1^{**}(t,x)|, \max_{t\in[0,T]} |z_2^*(t,x) - z_2^{**}(t,x)|\Big\} \leq$$

$$\leq \exp\Big\{\frac{1}{1-q_1(x)} K[\max(d_1(x), c_2(x)) + p(x)]x\Big\} \cdot 0. \tag{4.10}$$

From (4.10) it follows $z_1^*(t,x) \equiv z_1^{**}(t,x)$ and $z_2^*(t,x) \equiv z_2^{**}(t,x)$ for all $(t,x) \in \Omega$. Then from

$$|u^*(t,x) - u^{**}(t,x)| \leq \int_0^t |z_1^*(\tau,x) - z_1^{**}(\tau,x)| d\tau,$$

$$|v^*(t,x) - v^{**}(t,x)| \leq \int_0^x |z_2^*(t,\xi) - z_2^{**}(t,\xi)| d\xi,$$

we have $u^*(t,x) \equiv u^{**}(t,x)$, $v^*(t,x) \equiv v^{**}(t,x)$. This contradicts with our assumption that problem (2.1)–(2.4) has two solution, i.e. the quadruples $\{z_1^*(t,x), z_2^*(t,x),\ u^*(t,x),\ v^*(t,x)\}$ and $\{z_1^{**}(t,x), z_2^{**}(t,x),\ u^{**}(t,x),\ v^{**}(t,x)\}$. Therefore, solution to problem (2.1)–(2.4) is unique. Theorem 4.1 is proved.

Thus, from the equivalence of problems (1.1)–(1.5) and (2.1)–(2.4) we obtain

Theorem 4.2. *Suppose that the conditions i)- iii) of Theorem 4.1 are valid. Then problem (1.1)–(1.5) has a unique solution.*

References

[1] A.T. Asanova, D.S. Dzhumabaev, *Unique solvability of the boundary value problem for systems of hyperbolic equations with data on the characteristics,* Computational Mathematics and Mathematical Physics, 42 (2002), no. 11., 1609–1621.

[2] A.T. Asanova, D.S. Dzhumabaev, *Unique solvability of nonlocal boundary value problems for systems of hyperbolic equations,* Differential Equations, 39 (2003), no. 10., 1414–1427.

[3] A.T. Asanova, D.S. Dzhumabaev, *Criteria of well-posed solvability of boundary value problem for system of hyperbolic equations,* Izvestia NAN Respubl. Kazakhstan. Ser. phyz.-mathem. (2002), no. 3., 20–26 (in Russian).

[4] A.T. Asanova, D.S. Dzhumabaev, *Correct solvability of a nonlocal boundary value problem for systems of hyperbolic equations,* Doklady Mathematics, 68 (2003), no. 1., 46–49.

[5] A.T. Asanova, D.S. Dzhumabaev, *Well-posed solvability of nonlocal boundary value problems for systems of hyperbolic equations,* Differential Equations, 41 (2005), no. 3., 352–363.

[6] A.T. Asanova, *A Nonlocal boundary value problem for systems of quasilinear hyperbolic equations,* Doklady Mathematics. 74 (2006), no. 3., 787–791.

[7] A.T. Asanova, *On the unique solvability of a family of two-point boundary-value problems for systems of ordinary differential equations,* Journal of Mathematical Sciences, 150 (2008), no.-5., 2302–2316.

[8] D.S. Dzhumabaev, A.T. Asanova, *Well-posed solvability of a linear nonlocal boundary value problem for systems of hyperbolic equations.* Dopovidi NAN Ukraine, (2010), no. 4., 7–11 (in Russian).

[9] A.T. Asanova, D.S. Dzhumabaev, *Well-posedness of nonlocal boundary value problems with integral condition for the system of hyperbolic equations,* Journal of Mathematical Analysis and Applications, 402 (2013), no. 1, 167–178.

[10] D.S. Dzhumabaev, *Conditions the unique solvability of a linear boundary value problem for a ordinary differential equations,* USSR Computational Mathematics and Mathematical Physics, 29 (1989), no. 1., 34–46.

[11] R. Goebel, R.G. Sanfelice, A.R. Teel, *Hybrid dynamical systems,* Princeton University Press, Princeton, NJ, USA, 2012

[12] N.D. Golubeva, L.S. Pul'kina, *A nonlocal problem with integral conditions,* Mathematical Notes, 59 (1996), no. 3, 326–328

[13] T.I. Kiguradze, *Some boundary value problems for systems of linear partial differential equations of hyperbolic type.* Mem. Differential Equations Math. Phys., 1(1994), 1–144

[14] A.B. Kurzhanskii, P.A. Tochilin, *Weakly invariant sets of hybrid systems,* Differential Equations, 44 (2008), no. 11, 1585-1594

[15] R. Margellos, J. Lygeros, *Viable set computation for hybrid systems,* Nonlinear Analysis: Hybrid Systems, 10 (2013), no. 1, 45–52

[16] A.M. Nakhushev, *Problems with shift for a partial differential equations,* Nauka, Moskow, 2006 (in Russian)

[17] B.I. Ptashnyck, *Ill-posed boundary value problems for partial differential equations,* Naukova Dumka, Kiev, Ukraine, 1984 (in Russian).

[18] E. Santis, M.D. Benedetto, G. Pola, *Digital idle speed control of automotive engines a safety problem for hybrid systems,* Nonlinear Analysis, 65 (2006), no. 9, 1705–1724

[19] Yu.B. Senichenkov, *Numerical modeling of hybrid systems,* Izd-vo Politechn. un-ta, Saint-Petersburg, Russia, 2004 (in Russian)

[20] B.P. Tkach, L.B. Urmancheva, *Numerical-analytical method for finding solutions of systems with distributed parameters and integral condition,* Nonlinear Oscillations, 12 (2009), no 1, 110–119.

[21] A. Van der Schaft, H. Schumacher, *An Introduction to hybrid dynamical systems*, Lecture Notes in Control and Information Sciences, no. 251, Berlin, 2000.

[22] L. Yao, J. Li, *Stability and stabilization of a class of nonlinear impulsive hybrid systems based on FSM with MDADT,* Nonlinear Analysis: Hybrid Systems. 15 (2015), no. 1, 1–10

[23] C. Yuan, F. Wu, *Analysis and synthesis of linear impulsive hybrid systems with state-triggered jumps,* Nonlinear Analysis: Hybrid Systems. 14 (2014), no. 1, 47-60

[24] S.V. Zhestkov, *The Goursat problem with integral boundary conditions,* Ukranian Mathematical Journal. 42 (1990), no. 1, 119–122

[25] A.L. Zuyev, *Partial asymptotic stabilization of nonlinear distributed parameter systems,* Automatica. 41 (2005), no. 1, 1–10

Anar T. Assanova
Department of Differential Equations
Institute of Mathematics and Mathematical Modeling MES RK
125 Pushkin St,
050010 Almaty, Kazakhstan
E-mails: anarasanova@list.ru, anartasan@gmail.com

Aigul P. Sabalakhova
Faculty of Physics and Mathematics
M. Auezov South-Kazakhstan State University
5 Tauke Khan St,
160012 Shymkent, Kazakhstan
E-mail: sabalahova@mail.ru

Received: 28.10.2016