

STABILITY OF A PROGRAMME MANIFOLD OF INDIRECT CONTROL SYSTEMS WITH DIFFERENT FEEDBACKS IN THE PRESENCE OF AN EXTERNAL LOAD

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Abstract. The problem of constructing indirect control systems of a special type for a given manifold is considered. This problem comes down to studying the stability of the programme manifold of the system under construction with respect to a given function. The stability of a given manifold of an automatic indirect control system with rigid and tachometric feedback is investigated, taking into account external load. By constructing Lyapunov functions, sufficient conditions for the absolute stability of the programme manifold are obtained. The obtained results can be used to construct stable automatic indirect control systems of a special type.

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1 Introduction and statement of problem

Inverse problems of ordinary differential equations began to develop intensively in the second half of the 20th century. The initial work in this direction has been done in article [2] published by Erugin, where he formulated and gave a solution method for constructing the whole set of systems of differential equations having a given integral curve. This article became fundamental in the research of inverse problems in the theory of ordinary differential equations on the construction of a set of systems of differential equations by given partial integrals. This problem is further developed in the following works [4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18] of many authors. The problems of stability of programme motion, solution of inverse problems of dynamics, and construction of programme motion systems were considered by Galiullin in [4, 5, 6, 7]. Mukhametzyanov in [6, 7, 13, 14] investigated the stability of the programme manifold of control systems with scalar nonlinearity. Mukharlyamov examined problems of the construction of differential equations of motion for mechanical systems. Control and reduction of dynamics equations with connections to a given structure were discussed in works [6, 7, 8, 15, 16, 17, 18]. Works [3, 20, 21, 24, 19, 22, 23] are dedicated to the study of inverse problems of stochastic differential equations. Basically, because the programme manifold has a smaller size than the equation under construction, there is no uniqueness when solving problems of constructing differential equations [1]. This fact is used for inverse problems of differential equations in combination with the stability of a given programme manifold. In works [6, 13, 14, 12, 25, 26, 27] the stability of a program manifold in the presence of various perturbations was studied. In [28] for the system in canonical form was constructed Lyapunov's functions. Over the years, an overview of the inverse problems of ordinary differential equations was given in works [6, 25, 11, 27, 29]. Currently, the study of the stability of the programme manifold with respect to certain functions has become a separate theory.

In the space $\mathbb{R}^n$ for $\rho > 0$, we consider a simple connected closed domain $G(\rho) \subset \mathbb{R}^n$:

$$G(\rho) = \{(t, x) : t \in I \text{ and } \|\omega(t, x)\| \leq \rho < \infty\}. \quad (1.1)$$

Here $I = [0, \infty)$, $\omega \in \mathbb{R}^s$, $\|\omega(t, x)\| = \sqrt{\omega_1^2(t, x) + \dots + \omega_s^2(t, x)}$.

In this domain, we consider the problem of constructing the following system of differential equations for a given smooth programme manifold $\Omega(t)$

$$\dot{x} = f(t, x), \quad (1.2)$$

where f, x are n -dimensional vectors, f is continuous in all variables and the existence conditions of the solution $x(t) = 0$ are satisfied. Moreover, the programme manifold $\Omega(t)$ is defined by the following system of equations:

$$\Omega(t) = \{x \in \mathbb{R}^n : \omega(t, x) = 0\}, \quad (1.3)$$

where an s -dimensional vector ω ($s \leq n$) is continuous in domain (1.1), including the manifold $\Omega(t)$, with its partial derivatives of the first order. The Jacobian rank $H = \frac{\partial \omega}{\partial x}$ is equal to the rank $H = s$ at all points of $\Omega(t)$.

For the initial condition $x(t_0) = x_0$, the programme manifold $\Omega(t)$ is exactly realized if for the corresponding solution $x(t)$ of system (1.2) holds the equality $\omega(t, x(t)) = 0$, $t \geq t_0$.

This requires, first, that the initial point belong to the programme manifold, $\omega(t_0, x_0) = 0$.

Moreover, the vector field of system (1.2) must be tangent to the manifold $\Omega(t)$. Equivalently, for every solution of (1.2),

$$\dot{\omega}(t, x(t)) = \frac{\partial \omega(t, x)}{\partial t} + H(t, x)f(t, x) = 0 \quad \text{whenever} \quad \omega(t, x) = 0.$$

Following Erugin's construction [25], this invariance condition can be expressed in the form:

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + H(t, x)f(t, x) = F(t, x, \omega), \quad (1.4)$$

where F is an s -dimensional Erugin vector-function satisfying $F(t, x, 0) = 0$.

In practice, perturbing forces may cause the state to leave the programme manifold. Therefore, in designing programme-motion control systems, it is required to investigate the stability of $\Omega(t)$ and to ensure that deviations measured by $\omega(t, x)$ tend to zero or are sufficiently small.

Together with equation (1.2), we consider the following indirect automatic control system with rigid feedback, taking into account the external load

$$\begin{aligned} \dot{x} &= f(t, x) - b_1 \xi, \quad t \in I = [0, \infty), \\ \dot{\xi} &= \varphi(\sigma) \psi(\nu), \quad \sigma = p^T \omega - q \xi, \end{aligned} \quad (1.5)$$

where the coefficients $b_1 \in \mathbb{R}^n$, $p \in \mathbb{R}^s$ are constant vectors, q is a constant coefficient of rigid feedback, σ is a total impulse-signal and the differentiable function $\varphi(\sigma)$ satisfies the following conditions:

$$\varphi(0) = 0 \text{ and } \varphi(\sigma)\sigma > 0 \quad \forall \sigma \neq 0, \quad (1.6)$$

the multiplier $\psi(\nu)$ accounts for the influence of the external load on the actuator velocity. Here, ν is a complex discontinuous function of the automatic control system. In the general case, the function ν has the following form [10]:

$$\nu = 1 - (a\ddot{\xi} + b\dot{\xi} + c\xi) \text{ sign } \sigma. \quad (1.7)$$

In the simplest case, it looks like this:

$$\nu = 1 - c\xi \operatorname{sign} \sigma. \quad (1.8)$$

Here a, b, c are real numbers, and $\operatorname{sign} \sigma$ is the Kroneker function:

$$\operatorname{sign} \sigma = \begin{cases} +1 & \text{at } \sigma > 0, \\ 0 & \text{at } \sigma = 0, \\ -1 & \text{at } \sigma < 0. \end{cases} \quad (1.9)$$

The dynamics of the servo-motor output coordinates $\xi = \xi(t)$ are described by the equation [10]:

$$\dot{\xi} = \varphi(\sigma) \cdot \psi(\nu), \quad (1.10)$$

where σ is the control signal, $\varphi(\sigma)$ characterizes the velocity of the unloaded actuator, and the multiplier $\psi(\nu)$ takes in account the influence of the external load.

Equation (1.10) is a general representation of the hydraulic-actuator model proposed by Khokhlov [9]:

$$\dot{\xi} = \mu \sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F} \sqrt{p_0 - \Delta p \operatorname{sign} \sigma} \cdot \sigma. \quad (1.11)$$

Here $\mu \sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F}$ is the constructive constant, μ is the flow coefficient, σ is the spool displacement, $p_0 = p_k - p_\sigma$, p_k is the pressure in the supply line, p_σ is the pressure at the drain, Δp is the pressure difference in the cameras of the actuator, determined by the load.

The function φ function replaces the expression $\sqrt{\frac{gp_0}{\gamma}} \cdot \frac{l}{F}$, which determines the speed of an unloaded executor, and the multiplier $\psi(\nu) = \sqrt{1 - \frac{\Delta p}{p_0} \operatorname{sign} \sigma}$ takes into account the influence of the load.

From a physical point of view, the throttle control of a hydraulic actuator is always $p_0 > \Delta p \operatorname{sign} \sigma$ in the presence of a positional load.

The multiplier $\psi(\nu)$, if ν is determined by (1.8), is presented as follows:

$$\psi(\nu) = \begin{cases} 1 & \text{at } \nu \geq 1, \\ \sqrt{\nu} & \text{at } 0 < \nu < 1, \\ 0 & \text{at } \nu \leq 0. \end{cases} \quad (1.12)$$

For the manifold $\Omega(t)$ to be an integral manifold and for system (1.5)–(1.7) with $\omega = 0$ the condition $\xi = 0$ should be satisfied. This condition is satisfied if and only if $q \neq 0$.

Definition 1. The programme manifold $\Omega(t)$ of an indirect control system, taking into account the external load, is called *absolutely stable* if it is globally stable on solutions of system (1.5) for any $\omega(t_0, x_0)$ and $\varphi(\sigma)$, $\psi(\nu)$ satisfying conditions (1.6), (1.7).

Statement of the problem. Find the absolute stability condition for the programme manifold $\Omega(t)$ of the indirect control system, taking into account the external load with respect to the vector function ω under conditions (1.6), (1.7).

The reviews of the works devoted to the construction of autonomous and non-autonomous automatic control systems on the given programme manifold possessing quality properties and solving various inverse problems of dynamics were given in [5, 6, 7, 13, 14, 15, 16, 17].

2 Stability of control system with respect to external load

Considering that the manifold $\Omega(t)$ is an integral manifold also for system (1.5)–(1.7) and assuming the Erugin function to be linear with respect to the vector function ω :

$$F(t, x, \omega) = -A\omega, \quad (2.1)$$

we arrive at the following system with respect to ω :

$$\begin{aligned} \dot{\omega} &= -A\omega - \bar{b}\xi, & t \in I = [0, \infty), \\ \dot{\xi} &= \varphi(\sigma)\psi(\nu), & \sigma = p^T\omega - q\xi, \end{aligned} \quad (2.2)$$

where $\bar{b} = Hb_1$, $H = \frac{\partial \omega}{\partial x}$, and $-A(s \times s)$ is a constant Hurwitz matrix, the nonlinearity φ satisfies conditions (1.6), and the multiplier ψ is determined by formula (1.12). System (2.2) is reduced to the canonical form [9]:

$$\begin{aligned} \dot{\eta} &= -\rho\eta + \varphi(\sigma) \cdot \psi(\nu), \\ \sigma &= c^T\eta - g\xi, \\ \dot{\sigma} &= \gamma^T - q\varphi(\sigma) \cdot \psi(\nu), \end{aligned} \quad (2.3)$$

where $\rho = \text{diag}(\rho_1, \dots, \rho_s)$, c, g, q, γ are constants.

It should be noted that $\xi = 0$ in the region of a complete stop of the mechanism, and we come from system (2.3) to the following form:

$$\begin{aligned} \dot{\eta} &= -\rho\eta, \\ \sigma &= c^T\eta - g\xi. \end{aligned} \quad (2.4)$$

Consider the case in which the load is positional, i.e. (1.8) is satisfied. For system (14), we construct the Lyapunov function as follows:

$$V = \eta^T H \eta + \frac{1}{2} \eta^T L \eta + \int_0^\sigma \varphi(\sigma) \cdot \psi(\nu) d\sigma. \quad (2.5)$$

Here, $H = \left\| \frac{l_i l_j}{\rho_i + \rho_j} \right\|_1^s$, l_i, l_j are real numbers and $L = \text{diag}\|L_1, \dots, L_s\| > 0$.

The condition for V to be definitely positive if the function $\xi \neq 0$ is always satisfied, i.e. when the actuator is running. We find the total derivative of the function V with respect to the variable t using system (2.3):

$$-\dot{V} = \left[\tilde{H}\eta + \sqrt{q}\varphi(\sigma) \cdot \psi(\nu) \right]^2 + \eta^T \rho L \eta - \left[\int_0^\sigma \frac{\partial \psi}{\partial \xi} \cdot \varphi(\sigma) d\sigma \right] \varphi(\sigma) \cdot \psi(\nu) > 0, \quad (2.6)$$

where $\tilde{H} = \text{diag}(l_1, \dots, l_s)$, ρ is also a diagonal matrix.

In order that the inequality $-\dot{V} > 0$ be satisfied the following equality:

$$L + 2\sqrt{q}\tilde{H}\varphi\psi + 2H + \gamma = 0, \quad \gamma = \text{diag}(\gamma_1, \dots, \gamma_s), \quad (2.7)$$

should hold.

It should be noted that the last sum of derivative (2.6) has a positive sign, for any $\sigma \neq 0$, i.e. $\frac{\partial \psi}{\partial \xi} < 0$ if $\sigma > 0$ and $\frac{\partial \psi}{\partial \xi} > 0$ if $\sigma < 0$. Therefore, when the actuator is running, and the inequality $-\dot{V} > 0$ is satisfied.

Theorem 2.1. *Let the Erugin function be linear with respect to ω , the nonlinearity φ satisfy condition (1.6), the function ψ be determined by formula (1.7), and $\tilde{H} = \text{diag}(l_1, \dots, l_s)$, $L = \text{diag}\|L_1, \dots, L_s\| > 0$.*

Then, in order for the programme manifold of the automatic system of indirect control, taking into account the external load, to be absolutely stable with respect to the vector-function ω , it suffices that equalities (2.7) are valid.

3 Stability of control system with rigid and tachometric feedback, taking into account external load

Together with equation (1.2), we now consider the following system of indirect automatic control with rigid and tachometric feedback, taking into account the external load

$$\begin{aligned} \dot{x} &= f(t, x) - b_1 \xi, & t \in I = [0, \infty), \\ \dot{\xi} &= \varphi(\sigma) \psi(\nu), & \sigma = p^T \omega - q\xi - N\dot{\xi}, \end{aligned} \quad (3.1)$$

where $b_1 \in \mathbb{R}^n$, $p \in \mathbb{R}^s$, are constant vectors, q, N are constant coefficients of rigid and tachometric feedback, σ is the total control impulse-signal, and φ is a differentiable function satisfying the following conditions:

$$\varphi(0) = 0 \quad \text{and} \quad \varphi(\sigma)\sigma > 0 \quad \forall \sigma \neq 0, \quad 0 < \left. \frac{\partial \varphi}{\partial \sigma} \right|_{\sigma=0} < \chi, \quad \chi > 0, \quad (3.2)$$

and the function ψ , taking into account the actions of the external load, is defined by formula (1.12).

For the manifold $\Omega(t)$ to be an integral also for system (3.1)–(3.2) on the manifold $\omega = 0$ the condition $\xi = 0$ must be satisfied. This condition is satisfied for $q \neq 0$.

Taking into account that the manifold $\Omega(t)$ is also an integral manifold for system (3.1)–(3.2) and choosing the Erugin function in form (2.2), we arrive at the following system for ω :

$$\begin{aligned} \dot{\omega} &= -A\omega - \bar{b}\xi, & t \in I = [0, \infty), \\ \dot{\xi} &= \varphi(\sigma) \psi(\nu), \\ \sigma &= p^T \omega - q\xi - N\dot{\xi}, \end{aligned} \quad (3.3)$$

where $\bar{b} = Hb_1$, $H = \frac{\partial \omega}{\partial x}$, and $-A(s \times s)$ is a constant Hurwitz matrix, the nonlinearity φ satisfies conditions (3.3), and the multiplier ψ is determined by formula (1.12).

Statement of the problem. Find a condition for the absolute stability of programme manifold $\Omega(t)$ for indirect control systems (3.2)–(3.2) with rigid and tachometric feedback, taking into account the external load with respect to the vector function ω satisfying conditions (1.8), (3.2).

In the operating area of the hydraulic actuator ($\dot{\xi} \neq 0$), we introduce the following notation:

$$\dot{\xi} = z. \quad (3.4)$$

Then, due to (1.8), (3.2), from the third equation of system (3.3) we obtain

$$z = \varphi(\sigma) \sqrt{1 - ((a\dot{z} + bz + c\xi) \text{ sign } \sigma)}. \quad (3.5)$$

Solving equation (3.5) with respect to $\dot{z}$, under the condition $\sigma \neq 0$, and considering notation (3.4), we obtain the following system instead of the second equation of system (3.3) we obtain the following system:

$$\begin{aligned} \dot{\xi} &= z, \\ \dot{z} &= \lambda_1 \xi + \lambda_2 z + \phi(\sigma, z) \text{ sign } \sigma, \end{aligned} \quad (3.6)$$

where

$$\lambda_1 = -\frac{c}{a}, \lambda_2 = -\frac{b}{a}, \quad (3.7)$$

$$\phi(\sigma, z) = \frac{\varphi^2(\sigma) - z^2}{a\varphi^2(\sigma)}. \quad (3.8)$$

Here, the constants λ_1 and λ_2 are negative numbers, and the function $\psi(\sigma, z)$ in the work area for all $\sigma \neq 0$, due to the second equation in (3.3) and (3.2), (3.8) satisfies the condition

$$\sigma \leq \phi(\sigma, z) \leq \frac{1}{\sigma}, \quad (3.9)$$

and in the stooping region ($\dot{\xi} = 0$) we have $\xi = \text{const}$.

In the domain of hydraulic actuator operation due to (3.6) system of equations (3.3) can be written as:

$$\begin{aligned} \dot{\omega} &= -A\omega - \bar{b}\xi, \quad t \in I = [0, \infty), \\ \dot{\xi} &= z, \\ \dot{z} &= \lambda_1\xi + \lambda_2z + \phi(\sigma, z) \text{ sign } \sigma, \\ \sigma &= p^T\omega - q\xi - N\dot{\xi}, \end{aligned} \quad (3.10)$$

In order to investigate the stability of the system (3.10), it is necessary to reduce it to the canonical form.

Assume that $q \neq 0$ and $N \neq 0$. Taking into account the second equation of (3.10), we write the last equation as follows

$$\sigma = p^T\omega - q\xi - Nz. \quad (3.11)$$

Introducing the notation

$$\begin{aligned} \omega_{s+1} &= \xi, \quad p_{s+1} = -q, \quad m_{s+1} = 1, \quad \varepsilon_{s+1} = \frac{c}{b}N, \\ \begin{pmatrix} a_{1s+1} \\ \dots \\ \dots \\ a_{ss+1} \end{pmatrix} &= \begin{pmatrix} \bar{b}_1 \\ \dots \\ \dots \\ \bar{b}_s \end{pmatrix}, \quad \begin{pmatrix} a_{s+11} \\ \dots \\ a_{s+1s} \\ a_{s+1s+1} \end{pmatrix}^T &= \begin{pmatrix} p_1/N \\ \dots \\ p_s/N \\ -q/N \end{pmatrix}^T, \\ g &= p^T A_1 \omega + \frac{b}{a}(p + \varepsilon), \quad M = \frac{b}{a} + \frac{1}{N}p^T m, \end{aligned}$$

and excluding the variable z in (3.11) and differentiating σ entering the second and third equations of (3.10), we obtain the following system of equations:

$$\begin{aligned} \dot{\omega} &= -A_1\omega - \frac{m_k}{N}\sigma, \quad t \in I = [0, \infty), \\ \dot{\sigma} &= g^T\omega - M\sigma - N\phi(\sigma, z) \text{ sign } \sigma, \end{aligned} \quad (3.12)$$

where $m = (0, \dots, 0, 1)^T$, $\varepsilon = (0, \dots, 0, \frac{c}{b}N)^T$ and g, M are expressed in terms of the coefficients of the initial equation,

$$A_1 = \left\| \begin{array}{cccc} a_{11} & \dots & a_{1s} & \bar{b}_1 \\ \dots & \ddots & \dots & \dots \\ a_{s1} & \dots & a_{ss} & \bar{b}_s \\ p_1/N & \dots & p_s/N & -q/N \end{array} \right\|.$$

Suppose that the matrix $-A_1$ is also Hurwitzian, then there is a non-singular transformation that reduces system (3.12) to the canonical form [2]:

$$\begin{aligned}\dot{\eta} &= -\rho\eta + \sigma, \\ \dot{\sigma} &= \beta^T\eta - M\xi - N\phi(\sigma, z) \operatorname{sign} \sigma,\end{aligned}\tag{3.13}$$

where $\rho = \operatorname{diag}(\rho_1, \dots, \rho_s)$, $\beta = \operatorname{diag}(\beta_1, \dots, \beta_s)$ is a constant vector.

For system (3.13), we construct the Lyapunov function of the form

$$V = \eta^T H \eta + \frac{1}{2} \eta^T L \eta + \theta C \vartheta + \frac{1}{2} l_{s+2} \sigma^2,\tag{3.14}$$

where

$$C = \operatorname{diag}(C_1, C_3, \dots, C_{s-m}), \quad \theta = \|\eta_{m+1}, \eta_{m+2}, \dots, \eta_s\|^T, \quad \vartheta = \|\eta_{m+2}, \eta_{m+3}, \dots, \eta_{s+1}\|^T,$$

$l_1, \dots, l_m$ are real and $l_{m+1}, \dots, l_{m+s+1}$ are complex pairwise conjugate numbers.

Differentiating (3.14) with respect to the time t , we find the derivative $\dot{V}$, taking into account (3.13):

$$\begin{aligned}-\dot{V} &= [l^T] - 2\sigma H \eta + \eta^T L \rho \eta - \sigma L \eta - \sigma l_{s+2} \beta \eta + l_{s+2} R \sigma^2 + \\ &\quad + N l_{s+2} \sigma \psi \cdot \operatorname{sign} \sigma + \theta C \Lambda \vartheta - \sigma C(\theta + \vartheta),\end{aligned}\tag{3.15}$$

where $\Lambda = \operatorname{diag}(\rho_{m+1} + \rho_{m+2}, \dots, \rho_s + \rho_{s+1})$.

By introducing the notation

$$\bar{\eta} = (\eta_1, \dots, \eta_m), \quad \tilde{\eta} = (\eta_{m+1}, \dots, \eta_{s-m+1}), \quad \bar{\beta} = \operatorname{diag}(\beta_1, \dots, \beta_m),$$

$$\tilde{\beta} = \operatorname{diag}(\beta_{m+1}, \dots, \beta_{s-m+1}), \quad \bar{H} = \sum_{k=1}^m \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k}, \quad \tilde{H} = \sum_{k=m+1}^{s-m+1} \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k},$$

we rewrite (3.15) in the following form

$$\begin{aligned}-\dot{V} &= [l^T] + \eta^T L \rho \eta + l_{s+2} R \sigma^2 + N l_{s+2} \sigma \psi \cdot \operatorname{sign} \sigma + \\ &\quad + \theta C \Lambda \vartheta - \sigma(2\bar{H} - l_{s+2}\bar{\beta} - L)\bar{\eta} - \sigma(C - 2\tilde{H} - l_{s+2}\tilde{\beta})\tilde{\eta}.\end{aligned}$$

It should be noted that we have adopted the following notation for the convenience of writing in the last brackets $C_1 = C_2$, $C_3 = C_4, \dots$, $C_{s-m} = C_{s-m+1}$.

Provided that $R > 0$, $N > 0$, in order for $-\dot{V} > 0$ to hold, it suffices that the following equalities are satisfied:

$$L + l_{s+2}\bar{\beta} + 2\bar{H} = 0,\tag{3.16}$$

$$C + l_{s+2}\tilde{\beta} + 2\tilde{H} = 0.\tag{3.17}$$

Theorem 3.1. *Let the Erugin function be linear with respect to ω , a matrix $-A_1$ be Hurwitzian, L , C , be positive real matrices, and $l_{s+2} > 0$. Moreover, let a non-linear function φ satisfy conditions (3.2) and a function ψ be defined by (1.7), (1.12).*

Then, in order that the programme manifold of the control system with feedback taking into account external load, be absolutely stable with respect to the vector-function ω , it suffices that equalities (3.16) and (3.17) hold and also

$$\dot{V} = - \sum_{i=1}^{s+1} l_k l_i \eta_i \eta_k + 2\sigma \sum_{i=1}^{s+1} \frac{l_k l_i}{\rho_i + \rho_k} - \sum_{i=1}^m \rho_k A_k \eta_k^2 + \sigma \sum_{k=1}^m A_k \eta_k + l_{s+2} \sigma \sum_{k=1}^{s+1} \beta_k \eta_k -$$

$$-l_{s+2}R\sigma^2 - Nl_{s+2}\sigma\psi(\sigma, \eta)\text{sign } \sigma - \sum_{i=1}^{s-m} C_i(\rho_{m+1} + \rho_{m+i+1})\eta_{m+1}\eta_{m+i+1}$$

$$L_k + l_{s+2}\beta_k + 2l_k \sum_{i=1}^{n+1} \frac{l_i}{\rho_i + \rho_k} = 0 \quad \forall k = \overline{1, \dots, m}, \quad (3.18)$$

$$C_j + l_{s+2}\beta_{m+j} + 2l_{m+j} \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k} = 0 \quad \forall j = \overline{1, \dots, s-m+1}, \quad (3.19)$$

where $\bar{\beta} = \text{diag}(\beta_1, \dots, \beta_m)$, $\tilde{\beta} = \text{diag}(\beta_{m+1}, \dots, \beta_{s-m+1})$,

$$\bar{H} = \sum_{k=1}^m \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k}, \quad \tilde{H} = \sum_{k=m+1}^{s-m+1} \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k}, \quad a_{k,s+1} = \bar{b}, \quad a_{s+1,j} = \frac{p_j}{N}.$$

Conclusion. In the paper, we establish new results for the problem of constructing indirect control systems of special type for a given manifold. This problem comes down to studying the stability of the programme manifold of the system under construction with respect to some given function. The stability of a given manifold of automatic indirect control systems rigid and tachometric feedback is investigated taking into account external load. By constructing the Lyapunov functions, sufficient conditions for the absolute stability of the programme manifold are obtained. The obtained results can be used to construct stable automatic indirect control systems of a special type.

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