ISSN 2077–9879

Eurasian Mathematical Journal

2018, Volume 9, Number 1

Founded in 2010 by
the L.N. Gumilyov Eurasian National University

in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia
the University of Padua

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Astana, Kazakhstan

EURASIAN MATHEMATICAL JOURNAL

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Short communications

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 9, Number 1 (2018), 83 – 87

ON CONTINUOUS SELECTIONS OF FINITE-VALUED SET-VALUED MAPPINGS

S.E. Zhukovskiy

Communicated by V.I. Burenkov

Key words: set-valued mapping, continuous selection.

AMS Mathematics Subject Classification: 54C60.

Abstract. Set-valued mappings with finite images are considered. For these mappings, a theorem on the existence of continuous selections is proved.

1 Introduction

This paper is devoted to continuous selections of a specific type of continuous set-valued mappings. Given topological spaces X and Y, recall that a mapping $F : X \rightrightarrows Y$ is called a **set-valued mapping** if it corresponds a closed nonempty set $F(x) \subset Y$ to each $x \in X$. A set-valued mapping $F : X \rightrightarrows Y$ is called **lower semicontinuous** if for each $x \in X$, for each open set $V \subset Y$ intersecting $F(x)$ there exists a neighbourhood $U \subset X$ of x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$. A mapping $f : X \to Y$ is called a **selection** of a set-valued mapping $F : X \rightrightarrows Y$ if $f(x) \in F(x)$ for each $x \in X$. Here and below the notation $f : X \to Y$ means that f is a single-valued mapping.

Theorems on the existence and properties of selections play important role in control theory, variational analysis and differential inclusions theory. One of the classical result in this area is Michael selection theorem. It states that *if X is a paracompact space, Y is a Banach space, a set-valued mapping $F : X \rightrightarrows Y$ is lower semicontinuous, and $F(x)$ is convex for each $x \in X$, then F has a continuous selection* (see [3]).

In this paper, we obtain a continuous selection theorem for a very specific class of set-valued mappings, namely, for mappings $F : X \rightrightarrows Y$ satisfying the following condition: there exists a positive integer number k such that

$(\star)$ $\text{card}(F(x)) = k \ \forall \ x \in X$.

Here $\text{card}(F(x))$ is the cardinality of the set $F(x)$.

Recall some definitions and auxiliary propositions. A set-valued mapping $F : X \rightrightarrows Y$ is called **upper semicontinuous** if for each $x \in X$, for each neighbourhood $V \subset Y$ of the set $F(x)$ there exists a neighbourhood $U \subset X$ of x such that $F(U) \subset V$. A set-valued mapping is called **continuous** if it is both upper and lower semicontinuous.

It is a straightforward task to ensure that *if Y is a Hausdorff space, then for any mapping $F : X \rightrightarrows Y$ satisfying assumption $(\star)$ for some k properties of continuity and lower semicontinuity are equivalent.* For a single-valued mapping this proposition is valid even if the topological space Y is not a Hausdorff space.

2 Main result

Theorem 2.1. *Let Y be a Hausdorff space, X be a simply-connected topological space (i.e. X is path-connected and its fundamental group is trivial), a set-valued mapping $F : X \rightrightarrows Y$ be continuous and satisfy ($\star$) for some positive integer k. Then there exist continuous mappings $f_j : X \to Y$, $j = \overline{1,k}$, such that*

$$F(x) = \{f_1(x), ..., f_k(x)\} \quad \forall\, x \in X.$$

We precede the proof of this theorem by auxiliary propositions.

Lemma 2.1. *Let Y be a Hausdorff space, a set-valued mapping F be continuous and satisfy ($\star$) for some positive integer k. Then for each $\bar{x} \in X$ there exists a neighbourhood $\overline{U}$ of $\bar{x}$ and continuous mappings $f_j : \overline{U} \to Y$, $j = \overline{1,k}$, such that*

$$F(x) = \{f_1(x), ..., f_k(x)\} \quad \forall\, x \in \overline{U}.$$

Proof. Denote the pairwise different points of $F(\bar{x})$ by $y_1, ...y_k$. Since Y is a Hausdorff space, there exist pairwise disjoint neighbourhoods $V_1, ..., V_k \subset Y$ of the points $y_1, ..., y_k$, respectively. Since F is lower semicontinuous, for each $j = \overline{1,k}$ there exists a neighbourhood $U_j \subset X$ of $\bar{x}$ such that $F(x) \cap V_j \neq \emptyset$ for each $x \in U_j$. Set $\overline{U} = \bigcap_{j=1}^{k} U_j$. Obviously, $\overline{U}$ is a neighbourhood of $\bar{x}$ and

$$F(x) \cap V_j \neq \emptyset \quad \forall\, x \in \overline{U}, \quad \forall\, j = \overline{1,k}.$$

Thus, since V_j are pairwise disjoint, ($\star$) implies that $\mathrm{card}(F(x) \cap V_j) = 1$ for each $j = \overline{1,k}$. Therefore, for each $j = \overline{1,k}$ a mapping

$$f_j : \overline{U} \to Y, \quad \{f_j(x)\} = F(x) \cap V_j \quad \forall\, x \in \overline{U}$$

is well defined. We have $F(x) = \{f_1(x), ..., f_k(x)\}$ for each $x \in \overline{U}$, since V_j are pairwise disjoint, the union of V_j contains $F(x)$, and each V_j contains the only point of $F(x)$. Moreover, the inclusion $y_j \in V_j$ implies $f_j(\bar{x}) = y_j$, $j = \overline{1,k}$.

Let us prove that f_j is continuous for each $j = \overline{1,k}$. Take an arbitrary $x \in \overline{U}$ and a neighbourhood $V \subset Y$ of $f_j(x)$. Since F is upper semicontinuous, there exists a neighbourhood $U \subset \overline{U}$ of x such that

$$F(U) \subset (V \cap V_j) \cup \left(\bigcup_{i \neq j} V_i \right).$$

It follows from the relations $f_j(U) \subset V_j$ and $V_j \cap V_i = \emptyset$ for each $i \neq j$ that $f_j(U) \subset V \cap V_j \subset V$. Therefore, f_j is continuous. □

Lemma 2.2. *Let Y be a Hausdorff space, a mapping $F : [0,1] \rightrightarrows Y$ be continuous and satisfy ($\star$) for some positive integer k. Then there exist continuous mappings $f_j : [0,1] \to Y$, $j = \overline{1,k}$, such that*

$$F(x) = \{f_1(x), ..., f_k(x)\} \quad \forall\, x \in [0,1].$$

Proof. It follows by Lemma 2.1 that for each $x \in [0,1]$ there exists $\varepsilon(x) > 0$ such that the restriction of F to $U(x) := (x - \varepsilon(x), x + \varepsilon(x)) \cap [0,1]$ is the union of k continuous selections, whose graphs are pairwise disjoint. Since $[0,1]$ is compact, there exist a finite subset $D_0 \subset [0,1]$ such that

$$[0,1] = \bigcup_{x \in D_0} U(x).$$

Let us prove that there exist $x_0, x_1, ..., x_n \in D_0$, $a_0, a_1, ..., a_n \in [0,1]$ such that

$$0 = a_0 \le x_1 < a_1 < ... < a_{n-1} < x_n \le a_n = 1, \quad [a_{i-1}, a_i] \subset U(x_i), \quad i = \overline{1,n}.$$

Set $a_0 := 0$. As x_1 take any point that provides maximum to the function $x \mapsto x+\varepsilon(x)$ on the set $\{x \in D_0 : x-\varepsilon(x) < a_0\}$. This point exists, since D_0 is finite and intervals $(x-\varepsilon(x), x+\varepsilon(x))$, $x \in D_0$, cover the segment $[a_0, 1]$. Set $D_1 := \{x \in D_0 : x - \varepsilon(x) \ge a_0\}$. By construction, intervals $(x - \varepsilon(x), x + \varepsilon(x))$, $x \in D_1$, cover the segment $[x_1 + \varepsilon(x_1), 1]$.

Set $a_1 := x_1 + \varepsilon(x_1)$. For x_2 take any point that provides maximum to the function $x \mapsto x + \varepsilon(x)$ on the set $\{x \in D_1 : x - \varepsilon(x) < a_1\}$. This point exists since D_1 is finite and intervals $(x - \varepsilon(x), x + \varepsilon(x))$, $x \in D_1$, cover the segment $[a_1, 1]$. Set $D_2 := \{x \in D_1 : x - \varepsilon(x) \ge a_1\}$. By construction, intervals $(x - \varepsilon(x), x + \varepsilon(x))$, $x \in D_2$, cover the segment $[x_2 + \varepsilon(x_2), 1]$.

Repeating this procedure we obtain that for some positive integer n at the n-th step the desired points a_j, x_j, $j = \overline{1,n}$, are constructed, since the initial set D_0 is finite and intervals $(x - \varepsilon(x), x + \varepsilon(x))$ cover the segment $[0,1]$.

Let us proceed to the construction of selections. Let $f_j : U(x_1) \to Y$, $j = \overline{1,k}$, be continuous selections of the restriction of F to the interval $U(x_1)$ such that

$$F(x) = \{f_1(x), ..., f_k(x)\} \quad \forall\, x \in U(x_1).$$

Mappings f_j are defined on the segment $[a_0, a_1] \subset U(x_1)$. Let us construct their extensions to the segment $[a_0, a_2]$.

Let $g_i : U(x_2) \to Y$ be continuous selections of the restriction of F to the interval $U(x_2)$ such that

$$F(x) = \{g_1(x), ..., g_k(x)\} \quad \forall\, x \in U(x_2).$$

It follows from relations $a_1 \in U(x_1) \cap U(x_2)$, $f_i(a_1) \ne f_j(a_1)$ and $g_i(a_1) \ne g_j(a_1)$ for $i \ne j$ that

$$\{f_1(a_1), ..., f_k(a_1)\} = F(a_1) = \{g_1(a_1), ..., g_k(a_1)\}.$$

In virtue of condition $(\star)$ the points $f_j(a_j)$, $j = \overline{1,k}$, are pairwise different and the points $g_j(a_j)$, $j = \overline{1,k}$, are pairwise different. So, the above equality implies that the mappings g_j, $j = \overline{1,k}$, can be renumerated in such a way that the equalities $f_j(a_1) = g_j(a_1)$, $j = \overline{1,k}$, hold. So, putting $f_j(x) := g_j(x)$ for $x \in (a_1, a_2]$ we extend the mappings f_j to the larger domain $[a_0, a_2]$, preserving continuity property and the relation

$$F(x) = \{g_1(x), ..., g_k(x)\} \quad \forall\, x \in [0, a_2].$$

Repeating this procedure we extend the mappings f_j, $j = \overline{1,k}$ to each of the segments $[0, a_j]$, $j = \overline{2,k}$, preserving continuity property and the relation

$$F(x) = \{g_1(x), ..., g_k(x)\} \quad \forall\, x \in [0, a_j].$$

Obviously, the mappings f_j, $j = \overline{1,k}$, defined on the segment $[0, a_k] = [0,1]$ are the desired ones. □

Proof of Theorem 2.1. Take an arbitrary point $\bar{x} \in X$. Denote the pairwise different points of $F(\bar{x})$ by $y_1, ... y_k$. Set

$$\mathrm{gph}(F) := \{(x, y) : \; x \in X, \;\; y \in F(x)\}.$$

Take an arbitrary $i \in \{1, ..., k\}$, denote by $\mathfrak{G}_i$ a path-component of the set $\mathrm{gph}(F)$ that contains $(\bar{x}, y_i)$. Set

$$P_i : \mathfrak{G}_i \to X, \quad P_i(x, y) := x \quad \forall\, (x, y) \in \mathfrak{G}_i.$$

Take an arbitrary $i \in \{1, ..., k\}$. Let us show that P_i is a covering for each $i = \overline{1,k}$.

First, let us prove that $P_i(\mathfrak{G}_i) = X$. Take an arbitrary $x \in X$. Since X is path-connected, there exists a continuous mapping $u : [0,1] \to X$ such that $u(0) = \bar{x}$, $u(1) = x$. Set

$$G : [0,1] \to Y, \quad G(t) := F(u(t)) \quad \forall\, t \in [0,1].$$

Lemma 2.2 implies that there exists a continuous mapping $v : [0,1] \to Y$ such that $v(t) \in G(t)$ for each $t \in [0,1]$, $v(0) = y_i$. Obviously, $(u(t), v(t)) \in \operatorname{gph}(F)$ for every $t \in [0,1]$. Since $\mathfrak{G}_i$ is a path-component, we obtain $(x, v(1)) \in \mathfrak{G}_i$. Thus, the equality $P_i(x, v(1)) = x$ implies $x \in P_i(\mathfrak{G}_i)$. Therefore, $P_i(\mathfrak{G}_i) = X$.

Let us show now that for any $x \in X$ there exists a neighbourhood $U \subset X$, a set of indices J, and disjoint open sets $W_j \subset \mathfrak{G}$, $j \in J$, such that $P_i^{-1}(U)$ is the union of W_j, $j \in J$, each of which is mapped homeomorphically onto U by P_i.

It follows by Lemma 2.1 that there exists a neighbourhood $U \subset X$ and a continuous mapping $f_j : U \to Y$, $j = \overline{1,k}$, such that $F(x) = \{f_1(x), ..., f_k(x)\}$ for each $x \in U$. Set

$$J := \{j : f_j(\bar{x}) \in \mathfrak{G}_i\}.$$

The graphs of f_j, $j = \overline{1,k}$, are pairwise disjoint, since $\operatorname{card}(F(x)) \equiv k$. Thus, the sets

$$W_j := \{(x, f_j(x)) :\ x \in U\}, \quad j \in J$$

are pairwise disjoint. Obviously the sets W_j are open, $P_i^{-1}(U)$ is the disjoint union of the open sets $W_j \subset \mathfrak{G}_i$, $j \in J$, the restriction of P_i to each W_j, $j \in J$ is continuous and bijective. Moreover, the inverse mappings to these restrictions $P_i\big|_{W_j}^{-1}: U \to W_i$, $j \in J$, are continuous, since $P_i\big|_{W_j}^{-1}(x) = (x, f_j(x))$ for each $x \in U$. Hence, each W_j is mapped homeomorphically onto U by P_i. Thus, the mapping P_i is a covering.

So, each $P_i : \mathfrak{G}_i \to X$ is a covering mapping, the spaces $\mathfrak{G}_i$ are path-connected, the space X is simply-connected. Thus, P_i is a homeomorphism (see, for example, [2], Chapter 18). Set

$$Q_i : \mathfrak{G}_i \to Y, \quad Q_i(x,y) \equiv y.$$

Since $f_i(x) \equiv Q_i(P_i^{-1}(x))$, Q_i are continuous and P_i are homeomorphisms, we obtain that the mappings f_i are continuous. Obviously $F(x) = \{f_1(x), ..., f_k(x)\}$ for each $x \in X$. $\square$

Remark 1. Under the assumption that both X and Y are compact Hausdorff spaces this result was proved in [4].

Remark 2. Assumption $(\star)$ is essential and cannot be omitted. The set-valued mapping

$$F : \mathbb{C} \rightrightarrows \mathbb{C}, \ F(x) = \{y \in \mathbb{C} :\ y^2 = x\} \text{ for each } x \in \mathbb{C}$$

provides P° corresponding example. For this mapping, assumption $(\star)$ does not hold, since $\operatorname{card}(F(x)) = 2$ for each $x \neq 0$ and $\operatorname{card}(F(0)) = 1$, while the rest of the assumptions of Theorem 2.1 hold. At the same time it follows from Brouwer domain invariance theorem that F has no continuous selections (for more details see [1], Lemma 1 and Example 2).

Let us eliminate zero from the domain, i.e. consider F as a self-mapping of $\mathbb{C} \setminus \{0\}$. In this case, assumption $(\star)$ holds for $k = 2$, but the space $X = \mathbb{C} \setminus \{0\}$ is not simply-connected. Obviously, the mapping F has no continuous selections. This example shows that in Theorem 2.1 the assumption of simple-connectedness is essential.

Acknowledgments

The author's research was supported by the grant of the Russian Science Foundation (Project no. 17-11-01168).

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Received: 09.09.2017