Short communications

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ON INTERPOLATION OF LOCAL MORREY-TYPE SPACES

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Abstract. In this paper, we study the interpolation properties of local Morrey-type spaces related to the interpolation method for anisotropic spaces. We define approximation local Morrey spaces $\overline{LM_{pr}^{\lambda q}}$ and approximation spaces $\widetilde{LM_{pr}^{\lambda q}}$, and in terms of these spaces we obtain a description of interpolation spaces for pairs of local Morrey-type spaces $\left(LM_{p_0,q_0}^{\lambda_0}, LM_{p_1,q_1}^{\lambda_1}\right)$ in the case $p_0 \neq p_1$.

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1 Introduction

Morrey spaces [22] and their generalizations have been widely applied in various problems of function theory and partial differential equations (see, for example, survey papers [10, 11]).

Let $0 < p \leq \infty$ and $0 \leq \lambda \leq n/p$. The Morrey spaces M_p^λ were defined in [22] as the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ such that

$$\|f\|_{M_p^\lambda} = \sup_{x \in \mathbb{R}^n} \sup_{r>0} r^{-\lambda} \|f\|_{L_p(B(x,r))} < \infty,$$

where $B(x,r)$ is the open ball of radius $r > 0$ centered at $x \in \mathbb{R}^n$. If $\lambda = 0$, then $M_p^0 = L_p(\mathbb{R}^n)$, while if $\lambda = n/p$, then $M_p^{n/p} = L_\infty(\mathbb{R}^n)$. If $\lambda < 0$ or $\lambda > n/p$, then $M_p^\lambda = \Theta$, where Θ is the set of all functions that are equivalent to zero on $\mathbb{R}^n$.

Interpolation of these spaces was considered in [27, 18, 25]. According to the results of [25], we have

$$\left(M_p^{\lambda_0}, M_p^{\lambda_1}\right)_{\theta,\infty} \hookrightarrow M_p^\lambda,$$

where $1 \leq p \leq \infty$, $0 \leq \lambda_0 \neq \lambda_1 \leq n/p$, $0 < \theta < 1$, $\lambda = (1-\theta)\lambda_0 + \theta\lambda_1$, and the symbol $\hookrightarrow$ denotes continuous embedding. In [26, 9] it was established that this inclusion is strict, which raised the problem of giving a complete description of the interpolation spaces.

In [15] a similar problem was considered for a local variant of the Morrey spaces and for their generalizations involving an additional parameter.

Definition 1. Let $0 < p, q \leq \infty$ and $0 < \lambda < \infty$ if $q < \infty$ and $0 \leq \lambda < \infty$ if $q = \infty$. The local Morrey-type spaces $LM_{p,q}^\lambda$ are defined as the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ such that for $q < \infty$

$$\|f\|_{LM_{p,q}^\lambda} = \left(\int_0^\infty \left(t^{-\lambda} \|f\|_{L_p(B(0,t))}\right)^q \frac{dt}{t}\right)^{1/q} < \infty,$$

and for $q = \infty$

$$\|f\|_{LM^{\lambda}_{p,q}} = \sup_{t>0} t^{-\lambda} \|f\|_{L_p(B(0,t))} < \infty.$$

Unlike the Morrey spaces M^{λ}_{p}, the scale of local Morrey-type spaces $LM^{\lambda}_{p,q}$ remains closed under interpolation when $p_0 = p_1$. Specifically, the following statement was proved in [15].

Theorem 1.1 ([15]). *Let $0 < p, q_0, q_1, q \leq \infty$ and $0 < \theta < 1$. Suppose, in addition, that $\lambda_0 \neq \lambda_1$ and $0 < \lambda_0, \lambda_1 < n/p$ if $p < \infty$ and at least one of the parameters q_0, q_1 and q is finite, and $0 \leq \lambda_0, \lambda_1 \leq n/p$ if $q_0 = q_1 = q = \infty$. Then*

$$\left(LM^{\lambda_0}_{p,q_0}, LM^{\lambda_1}_{p,q_1}\right)_{\theta,q} = LM^{\lambda}_{p,q},$$

where $\lambda = (1-\theta)\lambda_0 + \theta\lambda_1$.

Later, in works [13, 17, 12, 16, 24, 14] generalizations of this result were obtained, and their applications to the study of various problems in analysis were explored.

In [5, 6, 7, 8] further modifications and generalizations of local and global Morrey spaces were defined, based on the use of norms of general symmetric spaces X and l (instead of Lebesgue space norms L_p and l_q). In these works there were investigated problems related to the interpolation of such spaces. Below, we provide some of the definitions from these works.

A Banach space X of measurable functions on $\mathbb{R}^n$ is called an ideal space [21] if, for every function $f \in X$, any measurable function g satisfying $|g(t)| \leq |f(t)|$ for almost all $t \in \mathbb{R}^n$ also belongs to X and satisfies $\|g\|_X \leq \|f\|_X$ (the symbol $\|f\|_X$ denotes the norm of an element f in the space X).

For $f : \mathbb{R}^n \to \mathbb{R}$ we denote by $m(f, \gamma)$ the distribution function of f, namely,

$$m(f, \gamma) = \mu(\{t \in \mathbb{R}^n : |f(t)| > \gamma\}),$$

where μ is the Lebesgue measure on $\mathbb{R}^n$.

An ideal space X is said to be symmetric if from the condition $f \in X$, the measurability of g follows, and also the validity of the inequality $m(g, \gamma) \leq m(f, \gamma)$ for all $\gamma \in \mathbb{R}_+$ that $g \in X$ and $\|g\|_X \leq \|f\|_X$.

Along with ideal function spaces, we need to define ideal sequence spaces. Let $e_i = \{\ldots, 0, 1, 0, \ldots\}$ (1 stands in the ith place, $i \in \mathbb{Z}$) be the standard basis in the space of two-sided sequences. We denote by l an ideal sequence space consisting of sequences $x = \sum_{i=-\infty}^{\infty} x_i e_i$ with the norm $\|\cdot\|_l$. By definition, l is an ideal sequence space if, for every sequence $x = \{x_i\}_{i\in\mathbb{Z}} \in l$ and every sequence $y = \{y_i\}_{i\in\mathbb{Z}}$ satisfying $|y_i| \leq |x_i|$ for all $i \in \mathbb{Z}$, we have $y \in l$ and $\|y\|_l \leq \|x\|_l$.

Let $U(0,1) \subset \mathbb{R}^n$ be such that $0 \in U(0,1)$ and $\mu(U(0,1)) \in (0, \infty)$. We also assume that $U(0,1)$ is star-shaped with respect to the point 0, that is, if $t \in U(0,1)$, then $\nu t \in U(0,1)$ for $\nu \in (0,1)$.

Let $U(0,r)$ be the homothetic set to the set $U(0,1)$ with a coefficient $r > 0$. We denote by Υ the set of all non-negative number sequences $\tau = \{\tau_i\}$ each of which satisfies the conditions

$$\forall i : \tau_i < \tau_{i+1}, \quad \bigcup_i (\tau_i, \tau_{i+1}] = \mathbb{R}_+.$$

If $\tau_{i+1} = \infty$, we assume that $(\tau_i, \infty] = (\tau_i, \infty)$. For every sequence $\tau = \{\tau_i\}$ we construct a family of sets $U(0, \tau_i)$ and a family of disjoint annuli $R(0, \tau_{i-1}, \tau_i) = U(0, \tau_i) \setminus U(0, \tau_{i-1})$.

Definition 2. Let an ideal space X on $\mathbb{R}^n$, an ideal space l of two-sided sequences with the standard basis $\{e_i\}$ and a sequence $\tau \in \Upsilon$ be given. The discrete local Morrey spaces $M^{\tau}_{l,X}$ are defined as the spaces of all functions $f \in L^{loc}_1(\mathbb{R}^n)$ such that

$$\|f\|_{M^{\tau}_{l,X}} = \left\| \sum_{i=-\infty}^{\infty} e_i \, \|f\chi(U(0,\tau_i))\|_X \right\|_l < \infty.$$

The approximation local Morrey space $\overline{M^{\tau}_{l,X}}$ is defined as the space of all functions $f \in L_1^{loc}(\mathbb{R}^n)$ such that

$$\|f\|_{\overline{M^{\tau}_{l,X}}} = \left\| \sum_{i=-\infty}^{\infty} e_i \left\| f\chi(R(0,\tau_{i-1},\tau_i))\right\|_X \right\|_l < \infty.$$

Here $\chi(D)$ is the characteristic function of a set D.

In [5], conditions under which the equality $\overline{M^{\tau}_{l,X}} = M^{\tau}_{l,X}$ holds were studied. It was also shown that the approximation local Morrey space $\overline{M^{\tau}_{l,X}}$ is a retract of the space of vector-valued sequences $l(X)$.

Note that in the framework of the classical interpolation method, it is only possible to describe the interpolation result for pairs $\left(LM^{\lambda_0}_{p,q_0}, LM^{\lambda_1}_{p,q_1}\right)$ with the same parameter p. We are interested in the problem of describing interpolation spaces for pairs $\left(LM^{\lambda_0}_{p_0,q_0}, LM^{\lambda_1}_{p_1,q_1}\right)$ when $p_0 \neq p_1$.

2 Anisotropic interpolation method and spaces of vector-valued sequences

Let us consider the interpolation method for anisotropic spaces proposed by E.D. Nursultanov [23] (see also [1, 2, 19]).

Let A_1 be a Banach space, and A_2 be a functional Banach lattice (see [20]). We denote by $\mathbf{A} = (A_1, A_2)$ the space of A_1-valued measurable functions such that $\|f\|_{A_1} \in A_2$, with the norm $\|f\|_{\mathbf{A}} = \left\| \|f\|_{A_1} \right\|_{A_2}$. This space is called a mixed-norm space.

Let $\mathbf{A}_0 = (A_1^0, A_2^0)$, $\mathbf{A}_1 = (A_1^1, A_2^1)$ be two mixed-norm spaces, and let $\varepsilon = (\varepsilon_1, \varepsilon_2) \in E = \{0,1\}^2$. We define the space $\mathbf{A}_\varepsilon = (A_1^{\varepsilon_1}, A_2^{\varepsilon_2})$ with the norm

$$\|a\|_{\mathbf{A}_\varepsilon} = \left\| \|a\|_{A_1^{\varepsilon_1}} \right\|_{A_2^{\varepsilon_2}}.$$

A pair of mixed-norm spaces $\mathbf{A_0} = (A_1^0, A_2^0)$, $\mathbf{A_1} = (A_1^1, A_2^1)$ is called *compatible* if there exists a linear topological Hausdorff space $\mathcal{A}$, containing the spaces $\mathbf{A}_\varepsilon$ as subspaces for all $\varepsilon \in E$.

We define a functional for $a \in \sum_{\varepsilon \in E} \mathbf{A}_\varepsilon$ as follows:

$$K(\mathbf{t}, a; \mathbf{A}) = \inf_{a = \sum_{\varepsilon\in E} a_\varepsilon} \sum_{\varepsilon \in E} \mathbf{t}^{\varepsilon} \|a_\varepsilon\|_{\mathbf{A}_\varepsilon}.$$

Let $\mathbf{0} < \theta = (\theta_1, \theta_2) < \mathbf{1}$, $\mathbf{0} < \mathbf{r} = (r_1, r_2) \leq \infty$. For any rearrangement $\star = (j_1, j_2)$ of the set $\{1,2\}$ let $\mathbf{r}^\star = (r_{j_1}, r_{j_2})$. We denote by $\mathbf{A}_{\theta\mathbf{r}^\star} = (\mathbf{A}_0, \mathbf{A}_1)_{\theta\mathbf{r}^\star}$ the linear subset of $\sum_{\varepsilon\in E} \mathbf{A}_\varepsilon$ for all elements of which the following quasi-norm (norm if $\mathbf{r} \geq \mathbf{1}$) is finite:

$$\|a\|_{\mathbf{A}_{\theta\mathbf{r}^\star}} = \left(\int_0^\infty \left(\int_0^\infty \left(t_{j_1}^{-\theta_{j_1}} t_{j_2}^{-\theta_{j_2}} K(\mathbf{t}, a; \mathbf{A}_0, \mathbf{A}_1) \right)^{r_{j_1}} \frac{dt_{j_1}}{t_{j_1}} \right)^{r_{j_2}/r_{j_1}} \frac{dt_{j_2}}{t_{j_2}} \right)^{1/r_{j_2}} < \infty.$$

Lemma 2.1 ([23]). *Let $\mathbf{0} < \theta < \mathbf{1}$, $\mathbf{0} < \mathbf{r} \leq \infty$, $\star = (j_1, j_2)$ be some rearrangement of the set $\{1,2\}$, $\mathbf{A} = \{A_\varepsilon\}_{\varepsilon\in E}$ and $\mathbf{B} = \{B_\varepsilon\}_{\varepsilon\in E}$ be two compatible families of Banach spaces. If a linear operator $T : A_\varepsilon \to B_\varepsilon$ with the quasi-norm M_ε for any $\varepsilon \in E$, then*

$$T : \mathbf{A}_{\theta\mathbf{r}^\star} \to \mathbf{B}_{\theta\mathbf{r}^\star},$$

with the quasi-norm

$$\|T\|_{\mathbf{A}_{\theta\mathbf{r}^\star} \to \mathbf{B}_{\theta\mathbf{r}^\star}} \lesssim \max_{\varepsilon\in E} M_\varepsilon.$$

Here, $A \lesssim B$ means that there exists a constant $c > 0$ such that $A \leq cB$ for all A, B under consideration.

Below we give the definition of the Lorentz space $L_{pr}(\mathbb{R}^n)$, and spaces of vector-valued sequences $l_q^\sigma(L_{pr})(\mathbb{R}^n)$ and $\widetilde{L_{pr}(l_q^\sigma)}(\mathbb{R}^n)$.

Let $0 < p, r \leq \infty$ and $0 < p < \infty$ if $r < \infty$ and $0 < p \leq \infty$ if $r = \infty$. The Lorentz spaces $L_{pr}(\mathbb{R}^n)$ are defined as the spaces of all functions f measurable on $\mathbb{R}^n$ such that for $r < \infty$

$$\|f\|_{L_{pr}(\mathbb{R}^n)} = \left(\int_0^\infty \left(t^{1/p} f^*(t) \right)^r \frac{dt}{t} \right)^{1/r} < \infty,$$

and for $r = \infty$

$$\|f\|_{L_{pr}(\mathbb{R}^n)} = \sup_{t>0} t^{1/p} f^*(t) < \infty,$$

where f^* is the nonincreasing rearrangement of f.

Let $-\infty < \sigma < \infty$, $1 \leq q, p, r \leq \infty$, we define spaces $l_q^\sigma(L_{pr})(\mathbb{R}^n)$ and $\widetilde{L_{pr}(l_q^\sigma)}(\mathbb{R}^n)$, as the set of sequences $a = \{a_k(x)\}_{k=-\infty}^\infty$, where $a_k(x)$ are functions, measurable on $\mathbb{R}^n$, for which the following norms are respectively finite:

$$\|a\|_{l_q^\sigma(L_{pr})(\mathbb{R}^n)} = \left(\sum_{k=-\infty}^\infty \left(2^{\sigma k} \|a_k\|_{L_{pr}(\mathbb{R}^n)} \right)^q \right)^{1/q},$$

$$\|a\|_{\widetilde{L_{pr}(l_q^\sigma)}(\mathbb{R}^n)} = \left\| \left(\sum_{k=-\infty}^\infty \left(2^{\sigma k} a_k^* \right)^q \right)^{1/q} \right\|_{L_{pr}(\mathbb{R}^n)},$$

with the standard modification for $q = \infty$.

3 Main result

We use the scheme for constructing local Morrey spaces from Definition 2.

Let $U(0,1) \subset \mathbb{R}^n$ satisfy the conditions imposed earlier in Introduction. Set $\tau = \{\tau_i\}$, where $\tau_i = 2^i$, $i \in \mathbb{Z}$. Let $1 \leq p, r \leq \infty$ and $1 \leq p < \infty$ if $r < \infty$ and $1 \leq p \leq \infty$ if $r = \infty$, $1 \leq q \leq \infty$ and $0 < \lambda < \infty$ if $q < \infty$ and $0 \leq \lambda < \infty$ if $q = \infty$. As the space X, we take the Lorentz space $L_{pr}(\mathbb{R}^n)$, and as the space l, we take the space $l_q^{-\lambda}$. The thus-defined discrete local Morrey spaces $M_{l,X}^\tau$ and approximation local Morrey spaces $\overline{M_{l,X}^\tau}$ will be denoted by the symbols $LM_{pr}^{\lambda q}$ and $\overline{LM_{pr}^{\lambda q}}$, respectively.

Remark 1. From the results of [5], it follows that in this case, the equality $LM_{pr}^{\lambda q} = \overline{LM_{pr}^{\lambda q}}$ holds. It can also be shown that when $p = r$ the space $LM_{pr}^{\lambda q}$ coincides with the local Morrey-type spaces $LM_{p,q}^\lambda$ (with norms equivalence).

The norm in the approximation local Morrey space $\overline{LM_{pr}^{\lambda q}}$ can be written as follows

$$\|f\|_{LM_{pr}^{\lambda q}} = \left\| \{ f\chi(R(0, 2^{i-1}, 2^i)) \} \right\|_{l_q^{-\lambda}(L_{pr})(\mathbb{R}^n)}.$$

We also define the approximation space $\widetilde{LM_{pr}^{\lambda q}}$ as the space of all functions $f \in L_1^{loc}(\mathbb{R}^n)$ such that

$$\|f\|_{\widetilde{LM_{pr}^{\lambda q}}} = \left\| \{ f\chi(R(0, 2^{i-1}, 2^i)) \} \right\|_{\widetilde{L_{pr}(l_q^{-\lambda})}(\mathbb{R}^n)} < \infty.$$

We have obtained the following theorem.

Theorem 3.1. *Let* $1 \leq p_0 \neq p_1 \leq \infty$, $1 \leq r \leq \infty$ *and* $1 \leq q_0, q_1, q \leq \infty$. *Suppose, in addition, that* $\lambda_0 \neq \lambda_1$ *and* $0 < \lambda_i < n/p_i$ *if* $p_i < \infty$ $(i = 0, 1)$ *and at least one of the parameters* q_0, q_1 *and* q *is finite, and* $0 \leq \lambda_i \leq n/p_i$ $(i = 0, 1)$ *if* $q_0 = q_1 = q = \infty$. *Then, for* $0 < \theta_1, \theta_2 < 1$, *the following equalities hold:*

a) *if* $\star_1 = (1, 2)$, *then*

$$\left(LM_{p_0,q_0}^{\lambda_0}, LM_{p_1,q_1}^{\lambda_1}\right)_{(\theta_1,\theta_2),(r,q)^{\star_1}} = \overline{LM_{pr}^{\lambda q}},$$

b) *if* $\star_2 = (2, 1)$, *then*

$$\left(LM_{p_0,q_0}^{\lambda_0}, LM_{p_1,q_1}^{\lambda_1}\right)_{(\theta_1,\theta_2),(r,q)^{\star_2}} = \widetilde{LM_{pr}^{\lambda q}},$$

where $\lambda = (1 - \theta_2)\lambda_0 + \theta_2\lambda_1$, $1/p = (1 - \theta_1)/p_0 + \theta_1/p_1$.

Remark 2. This approach was previously used by us to study the interpolation properties of Nikol'skii-Besov and Lizorkin-Triebel type spaces by applying the interpolation method for anisotropic spaces (see [3, 4]).

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