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MUKHTARBAY OTELBAEV

(to the 75th birthday)

On October 3, 2017 was the 75th birthday of Mukhtarbay Otelbaev, Doctor of Physical and Mathematical Sciences (1978), Professor (1983), academician of the National Academy of Sciences of the Republic of Kazakhstan (2004), Honored Worker of the Republic of Kazakhstan (2012), laureate of the State Prize of the Republic of Kazakhstan in the field of science and technology (2007), Director of the Eurasian Mathematical Institute (since 2001), Professor of the Department вЂњFundamental MathematicsвЂќ of the L.N. Gumilyov Eurasian National University, the editor-in-chief of the Eurasian Mathematical Journal (together with V.I. Burenkov and V.A. Sadovnichy).

M. Otelbaev was born in the village of Karakemer of the Kurdai district, Zhambyl region. He graduated from the M.V. Lomonosov Moscow State University (1969) and then completed his postgraduate studies at the same university (1972). There he defended his doctor of sciences thesis (1978).

Professor Otelbaev's scientific interests are related to functional analysis, differential equations, computational mathematics, and theoretical physics.

He introduced the q-averaging, which is now called the Otelbaev function; using it he obtained a number of fundamental results. For embedding of the Sobolev weighted spaces and the resolvent of the Schrödinger operator, he established criterions for the compactness and finiteness of the type, as well as estimates of the eigenvalues of the Schrödinger and Dirac operators that are exact in order. He was the first to establish that there is no universal asymptotic formula for the distribution function of the Sturm-Liouville operator. He obtained effective conditions for the separation of the differential operators with nonsmooth and oscillating coefficients, he developed an abstract theory of extension and contraction of operators which are not necessarily linear in linear topological spaces. M. Otelbaev proposed a new numerical method for solving boundary value problems, and a method for approximate calculation of eigenvalues and eigenvectors of compact operators. He obtained the fundamental results in the theory of nonlinear evolution equations and in theoretical physics.

He has published more than 70 scientific papers in leading international journals entering the rating lists of Thomson Reuters and Scopus. Under his supervision 70 postgraduate students have defended their candidate of sciences theses, 9 of them became doctors of sciences. In 2006 and 2011 he was awarded the state grant "The best university teacher".

The Editorial Board of the Eurasian Mathematical Journal congratulates Mukhtarbay Otelbaev on the occasion of his 75th birthday and wishes him good health and new achievements in mathematics and mathematical education.

Award for the Eurasian Mathematical Journal

Dear readers, authors, reviewers and members of the Editorial Board of the Eurasian Mathematical Journal,

we are happy to inform you that in November 2017 the Eurasian Mathematical Journal was awarded the title "Leader of Science 2017" by the National Center of State Scientific-Technical Expertise of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan in the nomination "Leader of Kazakhstan Scientific Publications" for the high level of publication activities and high level of citations in Web of Science Core Collection in 2014-2016.

Recall that the Eurasian Mathematical Journal was founded by the L.N. Gumilyov Eurasian National University in 2010 in co-operation with the M.V. Lomonosov Moscow State University, the Peoples' Friendship University of Russia and the University of Padua (see [1]).

The journal pulishes carefully selected original research papers in all areas of mathematics, survey papers, and short communications. It publishes 4 issues in a year. The language of the paper must be English only. Papers accepted for publication are edited from the point of view of English.

More than 280 papers were published written by mathematicians from more than 40 countries representing all continents.

In 2014 the journal was registered in Scopus and in September 2014 the Elsevier-Kazakhstan Research Excellence Forum was held at the L.N. Gumilyov Eurasian National University dedicated to this occasion in which the Elsevier Chairman Professor Y.S. Chi participated (see [3] for details).

In 2015 the Eurasian Mathematical Journal was included in the list of Scopus mathematical journals, quartile Q4, and it is on the way to entering quartile Q3 (see [3]).

Attached is the invitation letter to the Rector of the L.N. Gumilyov Eurasian National University Professor E.B. Sydykov to the ceremony of awarding, which took place in Almaty on November 8, 2017.

On behalf of the Editorial Board of the EMJ V.I. Burenkov, E.D. Nursultanov, T.Sh. Kalmenov, R. Oinarov, M. Otelbaev, T.V. Tararykova, A.M. Temirkhanova

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Ректору
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Уважаемый Ерлан Батташевич!

АО «Национальный центр государственной научно-технической экспертизы» (далее АО «НЦГНТЭ») и компания Clarivate Analytics имеют честь пригласить Вас на церемонию вручения независимой награды «Лидер науки-2017» за высокие показатели публикационной активности и цитируемости в Web of Science Core Collection в период 2014-2016 годы.

Ваш журнал *«Eurasian Mathematical Journal»* награждается в номинации «**Лидер казахстанских научных изданий**».

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Президент **Ибраев А.Ж.**

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ON NONEXISTENCE OF NONNEGATIVE SOLUTIONS FOR SOME QUASILINEAR ELLIPTIC INEQUALITIES IN A BOUNDED DOMAIN

O. A. Salieva

Communicated by V. I. Burenkov

Key words: quasilinear elliptic inequalities; nonexistence of solutions.

AMS Mathematics Subject Classification: 35A01, 35J87, 35J92.

Abstract. Using the test function method, we study sufficient conditions of nonexistence of nonnegative solutions for a quasilinear elliptic inequality of the form $-\mathrm{div}(u^r|Du|^{p-2}Du) \geq a(x)u^q|Du|^s$ in a bounded domain and its generalizations.

1 Introduction

Sufficient conditions of nonexistence of solutions to nonlinear elliptic inequalities with singular coefficients and their systems were studied by many authors.

For inequalities with the Laplacian in the principal part and with a coefficient in the nonlinear term possessing a point singularity the first results in this direction were obtained by H. Brezis and X. Cabré [1] by using comparison principles.

For higher order inequalities that do not obey comparison principles S. Pohozaev [9] suggested the so-called test function method. Later it was developed in joint papers with E. Mitidieri and other authors (see, in particular, the monograph [8] and references therein). This method allowed to obtain a series of new sharp results on nonexistence of solutions to elliptic and other differential inequalities in different functional classes. The method is based on asymptotically optimal a priori estimates obtained by algebraic analysis of the integral form of the inequalities under consideration with a special choice of test functions. Applications of this method to different types of elliptic inequalities and systems with degeneration, point singularities, gradient terms etc., can be found, e.g., in [2, 3, 4, 7].

In this paper a modification of the test function method is used in order to obtain dimension-independent sufficient conditions of nonexistence of solutions to some quasilinear elliptic inequalities and their systems in a bounded domain with coefficients possessing singularities near the boundary. This distinguishes the formulation of the problem given here from the above-mentioned papers where singularities appeared at single points or at infinity. Note that [7] contained some results in the case of boundary singularities, but they were dimension-dependent.

To prove the results on nonexistence of solutions by the test function method, test functions with a different geometrical structure of the support were constructed, in order to take into account the specific nature of the problems under consideration. First results in this direction were published in [5, 6].

The rest of the paper consists of two sections. In Section 2 we establish results on nonexistence of solutions to scalar quasilinear elliptic inequalities, and in Section 3 – to systems of such inequalities.

The work was supported by the Russian Foundation of Fundamental Research (project 14-01-00736).

2 Scalar quasilinear inequalities

Consider the problem

$$\begin{cases} -\mathrm{div}(u^r|Du|^{p-2}Du) \geq a(x)u^q|Du|^s, & x \in \Omega, \\ u(x) \geq 0, & x \in \Omega, \end{cases} \tag{2.1}$$

where Ω is a bounded domain with a smooth boundary, and $a \in C(\Omega)$ is a positive function.

Introduce the notation $\rho(x) = \mathrm{dist}(x, \partial\Omega)$,

$$\Omega_{k\eta} = \{x \in \Omega : \ \rho(x) \geq k\eta\} \quad (\eta > 0, k = 1, 2).$$

There holds

Theorem 2.1. *Let $\alpha \in \mathbb{R}$, $p > 1$, $q > p - 1$, $s > r - 1$,*

$$\frac{pq - s(r-1)}{p + r - 1} > 1,$$

$$\frac{(\alpha + s)r + (\alpha - q - 1)(p - 1)}{q + s - r - p + 1} > 0.$$

If, for some constant $c > 0$, $a(x) \geq c\rho^{-\alpha}(x)$, $x \in \Omega$, then problem (2.1) *has no nontrivial (distinct from a constant a.e.) nonnegative solutions.*

Proof. Let u be a nontrivial solution of inequality (2.1), and let $\varphi_\eta \in C_0^\infty(\Omega; [0, 1])$ be a test function of the form

$$\varphi_\eta(x) = \begin{cases} 1 & (x \in \Omega_{2\eta}), \\ 0 & (x \in \Omega_\eta), \end{cases} \tag{2.2}$$

$$|D\varphi_\eta(x)| \leq c\eta^{-1} \quad (x \in \Omega) \tag{2.3}$$

and $\lambda > 0$ be sufficiently large (to be specified below). Then we get

$$\begin{aligned} \int\limits_\Omega a(x)u^{q+\gamma}|Du|^s\varphi_\eta\,dx &\leq \int\limits_\Omega (u^r|Du|^{p-2}Du, D(u^\gamma\varphi_\eta))\,dx = \\ &= \gamma \int\limits_\Omega u^{r+\gamma-1}|Du|^p\varphi_\eta\,dx + \int\limits_\Omega u^{r+\gamma}|Du|^{p-2}(Du, D\varphi_\eta)\,dx \leq \\ &\leq \gamma \int\limits_\Omega u^{r+\gamma-1}|Du|^p\varphi_\eta\,dx + \int\limits_\Omega u^{r+\gamma}|Du|^{p-1}|D\varphi_\eta|\,dx, \end{aligned}$$

whence

$$\int\limits_\Omega a(x)u^{q+\gamma}|Du|^s\varphi_\eta\,dx + |\gamma| \int\limits_\Omega u^{r+\gamma-1}|Du|^p\varphi_\eta\,dx \leq \int\limits_\Omega u^{r+\gamma}|Du|^{p-1}|D\varphi_\eta|\,dx.$$

Representing the integrand in the right-hand side of the obtained inequality in the form

$$2^{-\frac{y}{s}}u^{\frac{(q+\gamma)y}{s}}|Du|^y a^{\frac{y}{s}}\varphi_\eta^{\frac{y}{s}} \times 2^{\frac{y}{s}}u^{\frac{(r+\gamma)s-(q+\gamma)y}{s}}|Du|^{p-1-y}|D\varphi_\eta| \cdot a^{-\frac{y}{s}}\varphi_\eta^{-\frac{y}{s}},$$

where y will be chosen below, and applying the parametric Young inequality with the exponent s/y, we obtain

$$\frac{1}{2}\int\limits_\Omega a(x)u^{q+\gamma}|Du|^s\varphi_\eta\,dx+|\gamma|\int\limits_\Omega u^{r+\gamma-1}|Du|^p\varphi_\eta\,dx\le$$
$$\le c\int\limits_\Omega u^{\frac{(r+\gamma)s-(q+\gamma)y}{s-y}}|Du|^{\frac{(p-1-y)s}{s-y}}|D\varphi_\eta|^{\frac{s}{s-y}}\cdot a^{-\frac{y}{s-y}}\varphi_\eta^{-\frac{y}{s-y}}\,dx.$$

We apply again the Young inequality with the exponent z:

$$c\int\limits_\Omega u^{\frac{(r+\gamma)s-(q+\gamma)y}{s-y}}|Du|^{\frac{(p-1-y)s}{s-y}}|D\varphi_\eta|^{\frac{s}{s-y}}\cdot a^{-\frac{y}{s-y}}\varphi_\eta^{-\frac{y}{s-y}}\,dx\le$$
$$\le\frac{|\gamma|}{2}\int\limits_\Omega u^{\frac{((r+\gamma)s-(q+\gamma)y)z}{s-y}}|Du|^{\frac{(p-1-y)sz}{s-y}}\varphi_\eta\,dx+c\int\limits_\Omega|D\varphi_\eta|^{\frac{sz'}{s-y}}\cdot a^{-\frac{yz'}{s-y}}\varphi_\eta^{1-\frac{sz'}{s-y}}\,dx,$$

where $\frac{1}{z}+\frac{1}{z'}=1$.

Choose y and z so that

$$\begin{cases}(p-1-y)sz=p(s-y),\\[2mm] \dfrac{(r+\gamma)s-(q+\gamma)y}{s-y}\cdot z=r+\gamma-1,\end{cases}$$

i.e.,

$$\begin{cases}y=\dfrac{s(p+r+\gamma-1)}{p(q+\gamma)-s(r+\gamma-1)},\\[2mm] z=\dfrac{p[p(q+\gamma)-s(r+\gamma-1)-(p+r+\gamma-1)]}{(p-1)(p(q+\gamma)-s(r+\gamma-1))-s(p+r+\gamma-1)}.\end{cases}$$

Then due to the choice of φ_η with properties (2.2), (2.3) and for sufficiently large $\lambda>0$ this implies

$$\frac{1}{2}\int\limits_\Omega a(x)u^{q+\gamma}|Du|^s\varphi_\eta\,dx+\frac{|\gamma|}{2}\int\limits_\Omega u^{\gamma-1}|Du|^p\varphi_\eta\,dx\le c\eta^{\frac{\alpha(r+\gamma+p-1)-p(q+\gamma)+s(r+\gamma)+q-r-p+1}{q+s-r-p+1}}.$$

Taking $\eta\to+0$, for sufficiently small $\gamma<0$ we obtain a contradiction, which proves the claim. □

Remark 1. By a similar argument one can prove an analogous result for the problem

$$\begin{cases}-\mathrm{div}(u^{r(x)}|Du|^{p(x)-2}Du)\ge a(x)u^{q(x)}|Du|^{s(x)}, & x\in\Omega,\\[2mm] u(x)\ge 0, & x\in\Omega,\end{cases}\tag{2.4}$$

where Ω is a bounded domain with a smooth boundary and $p, q, r, s \in C(\Omega)$,

$$\begin{aligned}
&\inf_{x\in\Omega} p(x) > 1,\\
&\inf_{x\in\Omega} (p(x) - q(x)) > -1,\\
&\inf_{x\in\Omega} s(x) > 0,\\
&\inf_{x\in\Omega} (s(x) + q(x) - p(x) - r(x)) > 1,\\
&\inf_{x\in\Omega} \frac{p(x)q(x) - s(x)(r(x) - 1)}{p(x) + r(x) - 1} > 1,\\
&\inf_{x\in\Omega} \frac{p(x)[p(x)(q(x) - 1) - (s(x) + 1)(r(x) - 1)]}{(p(x) - 1)(p(x)q(x) - s(x)(r(x) - 1)) - s(x)(p(x) + r(x) - 1)} > 1,
\end{aligned}$$

$a \in C(\Omega)$ is a positive function. Namely, if we denote

$$\begin{aligned}
b_\gamma(x) &= \frac{p(x)(q(x) + \gamma) - s(x)(r(x) + \gamma - 1)}{q(x) + s(x) - p(x) - r(x) + 1},\\
c_\gamma(x) &= \frac{p(x) + r(x) + \gamma - 1}{q(x) + s(x) - p(x) - r(x) + 1},\\
D(\eta) &= \int\limits_{\Omega_\eta \setminus \Omega_{2\eta}} \eta^{b_\gamma(x)} a^{c_\gamma(x)}\, dx,
\end{aligned} \tag{2.5}$$

then there holds

Theorem 2.2. *Let*

$$\lim_{\eta\to 0_+} D(\eta) = 0. \tag{2.6}$$

Then problem (2.4) *has no nontrivial solutions.*

3 Systems of quasilinear inequalities

Further we consider the system of inequalities

$$\begin{cases}
-\mathrm{div}(u^{r_1}|Du|^{p-2}Du) \geq a(x)v^{q_1}|Dv|^{q_2}, & x \in \Omega,\\
-\mathrm{div}(v^{r_2}|Dv|^{q-2}Dv) \geq b(x)u^{p_1}|Du|^{p_2}, & x \in \Omega,\\
u,\, v \geq 0, & x \in \Omega,
\end{cases} \tag{3.1}$$

where Ω is a bounded domain with a smooth boundary.

We assume that $p, q > 1$, and a, $b \in C(\Omega)$ are nonnegative functions such that for some $a_0, b_0 > 0$ and for all $x \in \Omega$ one has $a(x) \geq a_0\rho^{-\alpha}(x)$, $b(x) \geq b_0\rho^{-\beta}(x)$.

Then there holds

Theorem 3.1. *Let* $p_1 + p_2 > p + r_1 - 1$, $q_1 + q_2 > q + r_2 - 1$ *and*

$$((\alpha-1)(q+r_2-1)-q_1(q-1)+q_2r_2)(p_1+p_2)+((\beta-1)(p+r_1-1)-p_1(p-1)+p_2r_1)(q+r_2-1) > 0 \tag{3.2}$$

or

$$((\beta-1)(p+r_1-1)-p_1(p-1)+p_2r_1)(q+r_2-1)(p_1+p_2)+((\alpha-1)(q+r_2-1)-q_1(q-1)+q_2r_2)(p+r_1-1) > 0. \tag{3.3}$$

Then problem (3.1) *has no nontrivial (distinct from a pair of constants a.e.) nonnegative solutions.*

Proof. Let (u, v) be a nontrivial solution of (3.1), and let $\varphi_\eta \in C_0^\infty(\Omega; [0,1])$ be a test function of the same form as in the proof of Theorem 2.1, which satisfies assumptions (2.2) and (2.3). We will assume that $u > 0$ and $v > 0$ (otherwise instead of u^γ and v^γ with $\gamma < 0$ we will use u_ε^γ and v_ε^γ, where $u_\varepsilon = u + \varepsilon$, $v_\varepsilon = v + \varepsilon$, $\varepsilon > 0$, and pass to the limit as $\varepsilon \to +0$).

Multiplying the first inequality (3.1) by $u^\gamma \varphi_\eta$ and the second one by $v^\gamma \varphi_\eta$, where γ is a number such that $\max(p_1 + p_2 - p - r_1 + 1, q_1 + q_2 - q - r_2 + 1) < \gamma < 0$, we get

$$\int\limits_\Omega a v^{q_1} |Dv|^{q_2} u^\gamma \varphi_\eta \, dx \le \gamma \int\limits_\Omega u^{\gamma + r_1 - 1} |Du|^p \varphi_\eta \, dx + \int\limits_\Omega u^{\gamma + r_1} |Du|^{p-1} |D\varphi_\eta| \, dx, \tag{3.4}$$

$$\int\limits_\Omega b u^{p_1} |Du|^{p_2} v^\gamma \varphi_\eta \, dx \le \gamma \int\limits_\Omega v^{\gamma + r_2 - 1} |Dv|^q \varphi_\eta \, dx + \int\limits_\Omega v^{\gamma + r_2} |Dv|^{q-1} |D\varphi_\eta| \, dx. \tag{3.5}$$

We make use of the representation

$$u^{\gamma + r_1} |Du|^{p-1} = u^{a_1} |Du|^{b_1} \varphi_\eta^{\frac{1}{c_1}} u^{\gamma - a_1} |Du|^{p-1-b_1} \varphi_\eta^{-\frac{1}{c_1}}, \tag{3.6}$$

$$v^{\gamma + r_2} |Dv|^{q-1} = v^{a_2} |Du|^{b_2} \varphi_\eta^{\frac{1}{c_2}} v^{\gamma - a_2} |Dv|^{q-1-b_2} \varphi_\eta^{-\frac{1}{c_2}}, \tag{3.7}$$

in order to apply to the right-hand sides of (3.4) and (3.5) the parametric Young inequality with exponents denoted by c_1 and c_2, respectively. Here we choose the parameters so that

$$\begin{cases} a_1 c_1 = \gamma - 1 + r_1, \\ b_1 c_1 = p, \\ \dfrac{\gamma - a_1 + r_1}{p - 1 - b_1} = \dfrac{p_1}{p_2}, \end{cases} \tag{3.8}$$

$$\begin{cases} a_2 c_2 = \gamma - 1 + r_2, \\ b_2 c_2 = q, \\ \dfrac{\gamma - a_2 + r_2}{q - 1 - b_2} = \dfrac{q_1}{q_2}. \end{cases} \tag{3.9}$$

Remark 2. These conditions allow the further application of the Hölder inequality.

Solving the systems of equations (3.8) and (3.9), we arrive at

$$\begin{cases} a_1 = \dfrac{(\gamma + r_1 - 1)((p-1)p_1 - (\gamma + r_1)p_2)}{pp_1 + p_2(1 - \gamma - r_1)}, \\[2ex] b_1 = \dfrac{p((p-1)p_1 - (\gamma + r_1)p_2)}{pp_1 + p_2(1 - \gamma - r_1)}, \\[2ex] c_1 = \dfrac{pp_1 + p_2(1 - \gamma - r_1)}{(p-1)p_1 - (\gamma + r_1)p_2}, \end{cases} \tag{3.10}$$

$$\begin{cases} a_2 = \dfrac{(\gamma + r_2 - 1)((q-1)q_1 - (\gamma + r_2)q_2)}{qq_1 + q_2(1 - \gamma - r_2)}, \\[2ex] b_2 = \dfrac{q((q-1)q_1 - (\gamma + r_2)q_2)}{qq_1 + q_2(1 - \gamma - r_2)}, \\[2ex] c_2 = \dfrac{qq_1 + q_2(1 - \gamma - r_2)}{(q-1)q_1 - (\gamma + r_2)q_2}. \end{cases} \tag{3.11}$$

Substituting (3.10) and (3.11) into (3.6) and (3.7), we obtain the representations

$$u^{\gamma+r_1}|Du|^{p-1} = u^{\frac{(\gamma+r_1-1)((p-1)p_1-(\gamma+r_1)p_2)}{pp_1+p_2(1-\gamma-r_1)}}|Du|^{\frac{p((p-1)p_1-(\gamma+r_1)p_2)}{pp_1+p_2(1-\gamma-r_1)}}\varphi_\eta^{\frac{(p-1)p_1-(\gamma+r_1)p_2}{pp_1+p_2(1-\gamma-r_1)}}\times$$
$$\times u^{\frac{p_1(p+\gamma+r_1-1)}{pp_1+p_2(1-\gamma-r_1)}}|Du|^{\frac{p_2(p+\gamma+r_1-1)}{pp_1+p_2(1-\gamma-r_1)}}\varphi_\eta^{-\frac{(p-1)p_1-(\gamma+r_1)p_2}{pp_1+p_2(1-\gamma-r_1)}},$$

$$v^{\gamma+r_2}|Dv|^{q-1} = v^{\frac{(\gamma+r_2-1)((q-1)q_1-(\gamma+r_2)q_2)}{qq_1+q_2(1-\gamma-r_2)}}|Dv|^{\frac{q((q-1)q_1-(\gamma+r_2)q_2)}{qq_1+q_2(1-\gamma-r_2)}}\varphi_\eta^{\frac{(q-1)q_1-(\gamma+r_2)q_2}{qq_1+q_2(1-\gamma-r_2)}}\times$$
$$\times v^{\frac{q_1(q+\gamma+r_2-1)}{qq_1+q_2(1-\gamma-r_2)}}|Dv|^{\frac{q_2(q+\gamma+r_2-1)}{qq_1+q_2(1-\gamma-r_2)}}\varphi_\eta^{-\frac{(q-1)q_1-(\gamma+r_2)q_2}{qq_1+q_2(1-\gamma-r_2)}}.$$

Applying to the right-hand sides of (3.4) and (3.5) the parametric Young inequality with exponents c_1 and c_2 from (3.10) and (3.11) respectively, we get

$$\int_\Omega av^{q_1}|Dv|^{q_2}u^\gamma\varphi_\eta\,dx + \frac{|\gamma|}{2}\int_\Omega |Du|^p u^{\gamma-1}\varphi_\eta\,dx \le$$
$$\le c_\gamma\int_\Omega u^{\frac{p_1(p+\gamma+r_1-1)}{p_1+p_2}}|Du|^{\frac{p_2(p+\gamma+r_1-1)}{p_1+p_2}}\frac{|D\varphi_\eta|^{\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2}}}{\varphi_\eta^{1-\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2}}}\,dx,$$

$$\int_\Omega bu^{p_1}|Du|^{p_2}v^\gamma\varphi_\eta\,dx + \frac{|\gamma|}{2}\int_\Omega v^{\gamma-1}|Dv|^q\varphi_\eta\,dx \le$$
$$\le d_\gamma\int_\Omega v^{\frac{q_1(q+\gamma+r_2-1)}{q_1+q_2}}|Dv|^{\frac{q_2(q+\gamma+r_2-1)}{q_1+q_2}}\frac{|D\varphi_\eta|^{\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2}}}{\varphi_\eta^{1-\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2}}}\,dx,$$

where the constants c_γ and d_γ depend only on p, q, and γ. Applying the Hölder inequality with the exponents

$$\frac{p_1+p_2}{p+\gamma+r_1-1},\ \frac{p_1+p_2}{p_1+p_2-p-\gamma-r_1+1}$$

and

$$\frac{q_1+q_2}{q+\gamma+r_2-1},\ \frac{q_1+q_2}{q_1+q_2-q-\gamma-r_2+1}$$

respectively, we obtain

$$\int_\Omega au^\gamma v^{q_1}|Dv|^{q_2}\varphi_\eta\,dx + \frac{|\gamma|}{2}\int_\Omega |Du|^p u^{\gamma+r_1-1}\varphi_\eta\,dx \le$$
$$\le c_\gamma\left(\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx\right)^{\frac{p+\gamma+r_1-1}{p_1+p_2}}\left(\int_\Omega b^{-\frac{p+\gamma+r_1-1}{p_1+p_2-p-\gamma-r_1+1}}\frac{|D\varphi_\eta|^{\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2-p-\gamma-r_1+1}}}{\varphi_\eta^{1-\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2-p-\gamma-r_1+1}}}\,dx\right)^{\frac{p_1+p_2-p-\gamma+1}{p_1+p_2}}, \tag{3.12}$$

$$\int_\Omega bv^\gamma u^{p_1}|Du|^{p_2}\varphi_\eta\,dx + \frac{|\gamma|}{2}\int_\Omega |Dv|^q v^{\gamma+r_2-1}\varphi_\eta\,dx \le$$
$$\le d_\gamma\left(\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx\right)^{\frac{q+\gamma+r_2-1}{q_1+q_2}}\left(\int_\Omega a^{-\frac{q+\gamma+r_2-1}{q_1+q_2-q-\gamma-r_2+1}}\frac{|D\varphi_\eta|^{\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2-q-\gamma-r_2+1}}}{\varphi_\eta^{1-\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2-q-\gamma-r_2+1}}}\,dx\right)^{\frac{q_1+q_2-q-\gamma-r_2+1}{q_1+q_2}}. \tag{3.13}$$

Further, multiplying both differential inequalities (3.1) by φ_η, we integrate them by parts:

$$\int_\Omega av^{q_1}\varphi_\eta\,dx \le \int_\Omega u^{r_1}|Du|^{p-1}|D\varphi_\eta|\,dx\,, \tag{3.14}$$

$$\int\limits_{\Omega} bu^{p_1}\varphi_\eta\,dx \le \int\limits_{\Omega} v^{r_2}|Dv|^{q-1}|D\varphi_\eta|\,dx\,. \tag{3.15}$$

We make use of the representation

$$u^{r_1}|Du|^{p-1} = u^{a_3}|Du|^{b_3}\varphi_\eta^{\frac{1}{c_3}}u^{r_1-a_3}|Du|^{p-1-b_3}(b\varphi_\eta)^{\frac{1}{d_3}}b^{-\frac{1}{d_3}}\varphi_\eta)^{-\frac{1}{c_3}-\frac{1}{d_3}}, \tag{3.16}$$

$$v^{r_2}|Dv|^{q-1} = v^{a_4}|Dv|^{b_4}\varphi_\eta^{\frac{1}{c_4}}v^{r_2-a_4}|Dv|^{q-1-b_4}(a\varphi_\eta)^{\frac{1}{d_4}}a^{-\frac{1}{d_4}}\varphi_\eta)^{-\frac{1}{c_4}-\frac{1}{d_4}}, \tag{3.17}$$

in order to apply to the right-hand sides of (3.14) and (3.15) the triple Hölder inequality with the exponents denoted by c_3, d_3, e_3 and c_4, d_4, e_4 respectively. Here we choose the exponents so that

$$\begin{cases} a_3c_3 = \gamma + r_1 - 1, \\ b_3c_3 = p, \\ (r_1 - a_3)d_3 = p_1, \\ (p-1-b_3)d_3 = p_2, \\ \\ \dfrac{1}{c_3} + \dfrac{1}{d_3} + \dfrac{1}{e_3} = 1, \end{cases} \tag{3.18}$$

$$\begin{cases} a_4c_4 = \gamma + r_2 - 1, \\ b_4c_4 = q, \\ (r_2 - a_4)d_4 = q_1, \\ (p-1-b_4)d_4 = q_2, \\ \\ \dfrac{1}{c_4} + \dfrac{1}{d_4} + \dfrac{1}{e_4} = 1. \end{cases} \tag{3.19}$$

Solving the systems of equations (3.18) and (3.19), we obtain

$$\begin{cases} a_3 = \dfrac{(\gamma-1)(p_1(p-1)-p_2r_1)}{pp_1+p_2(1-\gamma)}, \\ \\ b_3 = \dfrac{p(p_1(p-1)-p_2r_1)}{pp_1+p_2(1-\gamma)}, \\ \\ c_3 = \dfrac{pp_1+p_2(1-\gamma-r_1)}{p_1(p-1)-p_2r_1}, \\ \\ d_3 = \dfrac{pp_1+p_2(1-\gamma-r_1)}{(p-1)(1-\gamma)+r_1}, \\ \\ e_3 = \dfrac{pp_1+p_2(1-\gamma-r_1)}{(1-\gamma)(p_2-p+1)+p_1-r_1}, \end{cases} \tag{3.20}$$

$$
\begin{cases}
a_4 = \dfrac{(\gamma-1)(q_1(q-1)-q_2r_2)}{qq_1+q_2(1-\gamma)}, \\[2ex]
b_4 = \dfrac{q(q_1(q-1)-q_2r_2)}{qq_1+q_2(1-\gamma)}, \\[2ex]
c_4 = \dfrac{qq_1+q_2(1-\gamma-r_2)}{q_1(q-1)-q_2r_2}, \\[2ex]
d_4 = \dfrac{qq_1+q_2(1-\gamma-r_2)}{(q-1)(1-\gamma)+r_2}, \\[2ex]
e_4 = \dfrac{qq_1+q_2(1-\gamma-r_2)}{(1-\gamma)(q_2-q+1)+q_1-r_2}.
\end{cases} \tag{3.21}
$$

Substituting (3.20) and (3.21) into (3.16) and (3.17), we get the representations

$$
\begin{aligned}
|Du|^{p-1} &= u^{\frac{(\gamma+r_1-1)(p_1(p-1)-p_2r_1)}{pp_1+p_2(1-\gamma-r_1)}}|Du|^{\frac{p(p_1(p-1)-p_2r_1)}{pp_1+p_2(1-\gamma-r_1)}}\varphi_\eta^{\frac{p_1(p-1)-p_2r_1}{pp_1+p_2(1-\gamma-r_1)}}\times \\
&\times u^{\frac{p_1((p-1)(1-\gamma)+r_1)}{pp_1+p_2(1-\gamma-r_1)}}|Du|^{\frac{p_2((p-1)(1-\gamma)+r_1)}{pp_1+p_2(1-\gamma-r_1)}}(b\varphi_\eta)^{\frac{(p-1)(1-\gamma)+r_1}{pp_1+p_2(1-\gamma-r_1)}}\times \\
&\times b^{-\frac{(p-1)(1-\gamma)+r_1}{pp_1+p_2(1-\gamma-r_1)}}\varphi_\eta^{\frac{(\gamma-p_1-1)(p-1)+(p_2-1)r_1}{pp_1+p_2(1-\gamma-r_1)}},
\end{aligned}
$$

$$
\begin{aligned}
|Dv|^{q-1} &= v^{\frac{(\gamma+r_2-1)(q_1(q-1)-q_2r_2)}{qq_1+q_2(1-\gamma-r_2)}}|Dv|^{\frac{qq_1(q-1)-q_2r_2}{qq_1+q_2(1-\gamma-r_2)}}\varphi_\eta^{\frac{q_1(q-1)-q_2r_2}{qq_1+q_2(1-\gamma-r_2)}}\times \\
&v^{\frac{q_1((q-1)(1-\gamma)+r_2)}{qq_1+q_2(1-\gamma-r_2)}}|Dv|^{\frac{q_2((q-1)(1-\gamma)+r_2)}{qq_1+q_2(1-\gamma-r_2)}}(b\varphi_\eta)^{\frac{(q-1)(1-\gamma)+r_2}{qq_1+q_2(1-\gamma-r_2)}}\times \\
&\times b^{-\frac{(q-1)(1-\gamma)+r_2}{qq_1+q_2(1-\gamma-r_2)}}\varphi_\eta^{\frac{(\gamma-q_1-1)(q-1)+(q_2-1)r_2}{qq_1+q_2(1-\gamma-r_2)}},
\end{aligned}
$$

and, applying to the right-hand sides of (3.14) and (3.15) the triple Hölder inequality with the exponents c_3, d_3, e_3, c_4, d_4, e_4 from (3.20) and (3.21) respectively, we arrive at

$$
\begin{aligned}
\int\limits_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx &\le \left(\int\limits_\Omega u^{\gamma+r_1-1}|Du|^p\varphi_\eta\,dx\right)^{\frac{p_1(p-1)-p_2r_1}{pp_1+p_2(1-\gamma-r_1)}}\times \\
&\times\left(\int\limits_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx\right)^{\frac{(p-1)(1-\gamma)+r_1}{pp_1+p_2(1-\gamma-r_1)}}\times \\
&\times\left(\int\limits_\Omega b^{-\frac{(p-1)(1-\gamma)+r_1}{p_1+(p_2-p+1)(1-\gamma)-r_1}}\frac{|D\varphi_\eta|^{\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+(1-\gamma)(p_2-p+1)-r_1}}}{\varphi_\eta^{1-\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+(1-\gamma)(p_2-p+1)-r_1}}}\,dx\right)^{\frac{p_1+(1-\gamma)(p_2-p+1)-r_1}{pp_1+p_2(1-\gamma-r_1)}},
\end{aligned} \tag{3.22}
$$

$$
\begin{aligned}
\int\limits_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx &\le \left(\int\limits_\Omega v^{\gamma+r_2-1}|Dv|^q\varphi_\eta\,dx\right)^{\frac{q_1(q-1)-q_2r_2}{qq_1+q_2(1-\gamma-r_2)}}\times \\
&\times\left(\int\limits_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx\right)^{\frac{(q-1)(1-\gamma)+r_2}{qq_1+q_2(1-\gamma-r_2)}}\times \\
&\times\left(\int\limits_\Omega b^{-\frac{(q-1)(1-\gamma)+r_2}{q_1+(q_2-q+1)(1-\gamma)-r_2}}\frac{|D\varphi_\eta|^{\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+(1-\gamma)(q_2-q+1)-r_2}}}{\varphi_\eta^{1-\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+(1-\gamma)(q_2-q+1)-r_2}}}\,dx\right)^{\frac{q_1+(1-\gamma)(q_2-q+1)-r_2}{qq_1+q_2(1-\gamma-r_2)}}.
\end{aligned} \tag{3.23}
$$

Making use of (3.12) and (3.13), from the previous estimates we obtain

$$\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx \le D_\gamma\left(\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx\right)^{\frac{(p_1(p-1)-p_2r_1)(p_1+p_2)+((p-1)(1-\gamma)+r_1)(p+\gamma+r_1-1)}{(pp_1+p_2(1-\gamma-r_1))(p_1+p_2)}}\times$$
$$\times\left(\int_\Omega \frac{b^{-\frac{p+\gamma+r_1-1}{p_1+p_2-p-\gamma-r_1+1}}|D\varphi_\eta|^{\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2-p-\gamma-r_1+1}}}{\varphi_\eta^{1-\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+p_2-p-\gamma-r_1+1}}}\,dx\right)^{\frac{(p_1(p-1)-p_2r_1)(p_1+p_2-p-\gamma-r_1+1)}{(pp_1+p_2(1-\gamma-r_1))(p_1+p_2)}}\times$$
$$\times\left(\int_\Omega b^{-\frac{(p-1)(1-\gamma)+r_1}{p_1+(1-\gamma)(p_2-p+1)-r_1}}\frac{|D\varphi_\eta|^{\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+(1-\gamma)(p_2-p+1)-r_1}}}{\varphi_\eta^{1-\frac{pp_1+p_2(1-\gamma-r_1)}{p_1+(1-\gamma)(p_2-p+1)-r_1}}}\,dx\right)^{\frac{p_1+(1-\gamma)(p_2-p+1)-r_1}{pp_1+p_2(1-\gamma-r_1)}}, \tag{3.24}$$

$$\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx \le E_\gamma\left(\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx\right)^{\frac{(q_1(q-1)-q_2r_2)(q_1+q_2)+((q-1)(1-\gamma)+r_2)(q+\gamma+r_2-1)}{(qq_1+q_2(1-\gamma-r_2))(q_1+q_2)}}\times$$
$$\times\left(\int_\Omega \frac{b^{-\frac{q+\gamma+r_2-1}{q_1+q_2-q-\gamma-r_2+1}}|D\varphi_\eta|^{\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2-q-\gamma-r_2+1}}}{\varphi_\eta^{1-\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+q_2-q-\gamma-r_2+1}}}\,dx\right)^{\frac{(q_1(q-1)-q_2r_2)(q_1+q_2-q-\gamma-r_2+1)}{(qq_1+q_2(1-\gamma-r_2))(q_1+q_2)}}\times$$
$$\times\left(\int_\Omega b^{-\frac{(q-1)(1-\gamma)+r_2}{q_1+(1-\gamma)(q_2-q+1)-r_2}}\frac{|D\varphi_\eta|^{\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+(1-\gamma)(q_2-q+1)-r_2}}}{\varphi_\eta^{1-\frac{qq_1+q_2(1-\gamma-r_2)}{q_1+(1-\gamma)(q_2-q+1)-r_2}}}\,dx\right)^{\frac{q_1+(1-\gamma)(q_2-q+1)-r_2}{qq_1+q_2(1-\gamma-r_2)}}, \tag{3.25}$$

where D_γ and $E_\gamma > 0$ depend only on p, q, and γ.

Hence due to (2.2) and (2.3) we have

$$\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx \le c\left(\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx\right)^{a_1}\eta^{b_1}, \tag{3.26}$$

$$\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx \le c\left(\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx\right)^{a_2}\eta^{b_2}, \tag{3.27}$$

where

$$a_1 = \frac{p+r_1-1}{p_1+p_2},\quad b_1 = \frac{(\beta-1)(p+r_1-1)-p_1(p-1)+p_2r_1}{p_1+p_2},$$

$$a_2 = \frac{q+r_2-1}{q_1+q_2},\quad b_2 = \frac{(\alpha-1)(q+r_2-1)-q_1(q-1)+q_2r_2}{q_1+q_2}.$$

Substituting (3.26) into (3.27) and vice versa, we get

$$\int_\Omega av^{q_1}|Dv|^{q_2}\varphi_\eta\,dx \le c\eta^{\frac{((\beta-1)(p+r_1-1)-p_1(p-1)+p_2r_1)(q+r_2-1)(p_1+p_2)+((\alpha-1)(q+r_2-1)-q_1(q-1)+q_2r_2)(p+r_1-1)}{(p_1+p_2)(q_1+q_2)-(p+r_1-1)(q+r_2-1)}},$$

$$\int_\Omega bu^{p_1}|Du|^{p_2}\varphi_\eta\,dx \le c\eta^{\frac{((\alpha-1)(q+r_2-1)-q_1(q-1)+q_2r_2)(p_1+p_2)+((\beta-1)(p+r_1-1)-p_1(p-1)+p_2r_1)(q+r_2-1)}{(p_1+p_2)(q_1+q_2)-(p+r_1-1)(q+r_2-1)}}.$$

Passing to the limit as $\eta \to 0_+$, by (3.2) and (3.3) we obtain a contradiction, which proves the claim. □

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