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- content of the paper (the originality of the material, survey of previously published studies on the topic of the paper, erroneous statements (if any), controversial issues (if any), and so on);

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- possibility of reducing the volume of the paper, without harming the content and understanding of the presented scientific results;

- description of positive aspects of the paper, as well as of drawbacks, recommendations for corrections and complements to the text.

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MUKHTARBAY OTELBAEV

(to the 75th birthday)

On October 3, 2017 was the 75th birthday of Mukhtarbay Otelbaev, Doctor of Physical and Mathematical Sciences (1978), Professor (1983), academician of the National Academy of Sciences of the Republic of Kazakhstan (2004), Honored Worker of the Republic of Kazakhstan (2012), laureate of the State Prize of the Republic of Kazakhstan in the field of science and technology (2007), Director of the Eurasian Mathematical Institute (since 2001), Professor of the Department вЂњFundamental MathematicsвЂќ of the L.N. Gumilyov Eurasian National University, the editor-in-chief of the Eurasian Mathematical Journal (together with V.I. Burenkov and V.A. Sadovnichy).

M. Otelbaev was born in the village of Karakemer of the Kurdai district, Zhambyl region. He graduated from the M.V. Lomonosov Moscow State University (1969) and then completed his postgraduate studies at the same university (1972). There he defended his doctor of sciences thesis (1978).

Professor Otelbaev's scientific interests are related to functional analysis, differential equations, computational mathematics, and theoretical physics.

He introduced the q-averaging, which is now called the Otelbaev function; using it he obtained a number of fundamental results. For embedding of the Sobolev weighted spaces and the resolvent of the Schrödinger operator, he established criterions for the compactness and finiteness of the type, as well as estimates of the eigenvalues of the Schrödinger and Dirac operators that are exact in order. He was the first to establish that there is no universal asymptotic formula for the distribution function of the Sturm-Liouville operator. He obtained effective conditions for the separation of the differential operators with nonsmooth and oscillating coefficients, he developed an abstract theory of extension and contraction of operators which are not necessarily linear in linear topological spaces. M. Otelbaev proposed a new numerical method for solving boundary value problems, and a method for approximate calculation of eigenvalues and eigenvectors of compact operators. He obtained the fundamental results in the theory of nonlinear evolution equations and in theoretical physics.

He has published more than 70 scientific papers in leading international journals entering the rating lists of Thomson Reuters and Scopus. Under his supervision 70 postgraduate students have defended their candidate of sciences theses, 9 of them became doctors of sciences. In 2006 and 2011 he was awarded the state grant "The best university teacher".

The Editorial Board of the Eurasian Mathematical Journal congratulates Mukhtarbay Otelbaev on the occasion of his 75th birthday and wishes him good health and new achievements in mathematics and mathematical education.

Award for the Eurasian Mathematical Journal

Dear readers, authors, reviewers and members of the Editorial Board of the Eurasian Mathematical Journal,

we are happy to inform you that in November 2017 the Eurasian Mathematical Journal was awarded the title "Leader of Science 2017" by the National Center of State Scientific-Technical Expertise of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan in the nomination "Leader of Kazakhstan Scientific Publications" for the high level of publication activities and high level of citations in Web of Science Core Collection in 2014-2016.

Recall that the Eurasian Mathematical Journal was founded by the L.N. Gumilyov Eurasian National University in 2010 in co-operation with the M.V. Lomonosov Moscow State University, the Peoples' Friendship University of Russia and the University of Padua (see [1]).

The journal pulishes carefully selected original research papers in all areas of mathematics, survey papers, and short communications. It publishes 4 issues in a year. The language of the paper must be English only. Papers accepted for publication are edited from the point of view of English.

More than 280 papers were published written by mathematicians from more than 40 countries representing all continents.

In 2014 the journal was registered in Scopus and in September 2014 the Elsevier-Kazakhstan Research Excellence Forum was held at the L.N. Gumilyov Eurasian National University dedicated to this occasion in which the Elsevier Chairman Professor Y.S. Chi participated (see [3] for details).

In 2015 the Eurasian Mathematical Journal was included in the list of Scopus mathematical journals, quartile Q4, and it is on the way to entering quartile Q3 (see [3]).

Attached is the invitation letter to the Rector of the L.N. Gumilyov Eurasian National University Professor E.B. Sydykov to the ceremony of awarding, which took place in Almaty on November 8, 2017.

On behalf of the Editorial Board of the EMJ V.I. Burenkov, E.D. Nursultanov, T.Sh. Kalmenov, R. Oinarov, M. Otelbaev, T.V. Tararykova, A.M. Temirkhanova

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ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
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Ректору
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Уважаемый Ерлан Батташевич!

АО «Национальный центр государственной научно-технической экспертизы» (далее АО «НЦГНТЭ») и компания Clarivate Analytics имеют честь пригласить Вас на церемонию вручения независимой награды «Лидер науки-2017» за высокие показатели публикационной активности и цитируемости в Web of Science Core Collection в период 2014-2016 годы.

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Президент **Ибраев А.Ж.**

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BI-Γ-HYPERIDEALS AND GREEN'S RELATIONS IN ORDERED Γ-SEMIHYPERGROUPS

S. Omidi, B. Davvaz

Communicated by J. Tussupov

Key words: ordered Γ-semihypergroup, bi-Γ-hyperideal, Green's relations.

AMS Mathematics Subject Classification: 16Y99.

Abstract. In this paper, we proceed with the study of the basic properties of bi-Γ-hyperideals of ordered Γ-semihypergroups. The purpose of the present paper is to study the Green's relations on ordered Γ-semihypergroups.

1 Introduction

The study of ordered semihypergroups was first undertaken by Heidari and Davvaz [12]. In [3, 4], Changphas and Davvaz gave some properties of hyperideals and bi-hyperideals of ordered semihypergroups. The concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. Many authors studied different aspects of ordered semihypergroups, for instance, Davvaz et al. [9], Gu and Tang [11], Heidari and Davvaz [12], Pibaljommee and Davvaz [30], and many others. Recall from [12], that an *ordered semihypergroup* $(S, \circ, \leq)$ is a semihypergroup $(S, \circ)$ together with a partial order $\leq$ that is compatible with the hyperoperation $\circ$, meaning that for any $x, y, z \in S$,

$$x \leq y \Rightarrow z \circ x \leq z \circ y \text{ and } x \circ z \leq y \circ z.$$

Here, $A \leq B$ means that for any $a \in A$, there exists $b \in B$ such that $a \leq b$, for all non-empty subsets A and B of S. For the information about ordered semigroups we refer the reader to [20, 22, 23, 24, 25].

For Green's relations in Γ-semigroups, we refer to [19]. The notion of a Γ-semigroup was introduced by Sen and Saha [31] as a generalization of semigroups as well as of ternary semigroups. The concept of an ordered Γ-semigroup has been introduced by Sen and Seth [32] in 1993 as follows: An *ordered* Γ*-semigroup* $(S, \Gamma, \leq)$ is a Γ-semigroup (S, Γ) together with an order relation $\leq$ such that $a \leq b$ implies that $a\gamma c \leq b\gamma c$ and $c\gamma a \leq c\gamma b$ for all $a, b, c \in S$ and $\gamma \in \Gamma$. Many authors studied different aspects of ordered Γ-semigroups, for instance, Dutta and Adhikari [10], Hila [14], Iampan [15], Kehayopulu [21], Kwon [27], Kwon and Lee [28], and many others.

The concept of hyperstructures was first introduced by Marty [29] at the Eighth Congress of Scandinavian Mathematicians in 1934. A comprehensive review of the theory of hyperstructures can be found in [5, 6, 7, 33]. Recently, Davvaz et al. [1, 2, 13] introduced the notion of Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup. Davvaz et al. in [1] introduced the relations $\mathcal{L}, \mathcal{R}, \mathcal{H}$ in a Γ-semihypergroup S, which are called the Green's relations in the Γ-semihypergroup S. The notion of a Γ-hyperideal of a Γ-semihypergroup was introduced in [1]. Davvaz et al. [2] introduced the notion of Pawlak's approximations in Γ-semihypergroups. The study of ordered semihyperrings began with the work of Davvaz and Omidi [8].

2 Definitions and notations

Given a nonempty set M and a nonempty set $\mathit{\Gamma}$ of binary operations on M, then, M is called a $\mathit{\Gamma}$-semigroup if

(1) $m_1\alpha m_2 \in M$,

(2) $(m_1\alpha m_2)\beta m_3 = m_1\alpha(m_2\beta m_3)$

for all $m_1, m_2, m_3 \in M$ and all $\alpha, \beta \in \mathit{\Gamma}$. Then, Kehayopulu in [18, 21, 17], has added the following property in the definition:

(3) If $m_1, m_2, m_3, m_4 \in M$, $\gamma_1, \gamma_2 \in \mathit{\Gamma}$ are such that $m_1 = m_3$, $\gamma_1 = \gamma_2$ and $m_2 = m_4$, then $m_1\gamma_1 m_2 = m_3\gamma_2 m_4$.

Let S be a non-empty set and $\mathcal{P}^*(S)$ be the family of all non-empty subsets of S. A mapping $\circ : S \times S \to \mathcal{P}^*(S)$ is called a *hyperoperation* on S. A *hypergroupoid* is a set S together with a (binary) hyperoperation. In the above definition, if A and B are two non-empty subsets of S and $x \in S$, then we denote

$$A \circ B = \bigcup_{\substack{a\in A\\ b\in B}} a \circ b,\ x \circ A = \{x\} \circ A \text{ and } B \circ x = B \circ \{x\}.$$

A hypergroupoid $(S, \circ)$ is called a *semihypergroup* if for every x, y, z in S,

$$x \circ (y \circ z) = (x \circ y) \circ z.$$

That is,

$$\bigcup_{u\in y\circ z} x \circ u = \bigcup_{v\in x\circ y} v \circ z.$$

A hypergroupoid $(S, \circ)$ is called a *quasihypergroup* if for every $x \in S$, $x \circ S = S = S \circ x$. This condition is called the *reproduction axiom*. The couple $(S, \circ)$ is called a *hypergroup* if it is a semihypergroup and a quasihypergroup.

Definition 1. [1, 2] Let S be a nonempty set and Γ be a nonempty set of hyperoperations on S. Then, S is called a Γ-*semihypergroup* if every $\gamma \in \Gamma$ is a hyperoperation on S, i.e., $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in S$, we have

$$x\alpha(y\beta z) = (x\alpha y)\beta z.$$

If m_1, m_2, m_3, $m_4 \in M$, $\gamma_1, \gamma_2 \in \Gamma$ are such that $m_1 = m_3$, $\gamma_1 = \gamma_2$ and $m_2 = m_4$, then $m_1\gamma_1 m_2 = m_3\gamma_2 m_4$.

If every $\gamma \in \Gamma$ is an operation, then S is a Γ-semigroup. Let A and B be two non-empty subsets of S. We define

$$A\gamma B = \cup\{a\gamma b \mid a \in A, b \in B\}.$$

Also,

$$A\Gamma B = \cup\{a\gamma b \mid a \in A, b \in B \text{ and } \gamma \in \Gamma\} = \bigcup_{\gamma\in\Gamma} A\gamma B.$$

A Γ-semihypergroup S is called *commutative* if for all $x, y \in S$ and $\gamma \in \Gamma$, we have $x\gamma y = y\gamma x$. A Γ-semihypergroup S is called a Γ-*hypergroup* if for every $\gamma \in \Gamma$, (S, γ) is a hypergroup.

Definition 2. [26] An algebraic hyperstructure $(S, \Gamma, \leq)$ is called an *ordered* Γ*-semihypergroup* if (S, Γ) is a Γ-semihypergroup and $(S, \leq)$ is a partially ordered set such that for any $x, y, z \in S$ and $\gamma \in \Gamma$, $x \leq y$ implies $z\gamma x \leq z\gamma y$ and $x\gamma z \leq y\gamma z$. Here, if A and B are two non-empty subsets of S, then we say that $A \leq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$.

An ordered Γ-semihypergroup is called *idempotent* if $a \in a\gamma a$ for every $a \in S$ and for every $\gamma \in \Gamma$.

Definition 3. Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. A non-empty subset I of S is called a *left* Γ*-hyperideal* of S if the following conditions are satisfied:

(1) $S\Gamma I \subseteq I$;

(2) When $x \in I$ and $y \in S$ such that $y \leq x$, imply that $y \in I$.

A right Γ-hyperideal of an ordered Γ-semihypergroup S is defined in a similar way. By *two-sided* Γ*-hyperideal* or simply Γ*-hyperideal*, we mean a non-empty subset of S which both left and right Γ-hyperideal of S. A Γ-hyperideal I of S is said to be *proper* if $I \neq S$.

Let K be a non-empty subset of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. We define

$$(K] := \{x \in S \mid x \leq k \text{ for some } k \in K\}.$$

For $K = \{k\}$, we write $(k]$ instead of $(\{k\}]$. Note that the condition (2) in Definition 3 is equivalent to $(I] \subseteq I$. If A and B are non-empty subsets of S, then we have

(1) $A \subseteq (A]$;

(2) $((A]] = (A]$;

(3) If $A \subseteq B$, then $(A] \subseteq (B]$;

(4) $(A]\Gamma(B] \subseteq (A\Gamma B]$;

(5) $((A]\Gamma(B]] = (A\Gamma B]$.

Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. A subset A of S is called *idempotent* if $A = (A\Gamma A]$.

Definition 4. [26] A sub Γ-semihypergroup B of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is called a *bi-*Γ*-hyperideal* of S if the following conditions hold:

(1) $B\Gamma S\Gamma B \subseteq B$;

(2) When $x \in B$ and $y \in S$ such that $y \leq x$, imply that $y \in B$.

An ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is said to be *B-simple* if S has no proper bi-Γ-hyperideals. A bi-Γ-hyperideal C of S is called a *minimal bi-*Γ*-hyperideal* of S if C does not properly contain any bi-Γ-hyperideal of S.

3 Basic properties of bi-Γ-hyperideals

Let A be a non-empty subset of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. The smallest left (resp. right, two-sided, bi-) Γ-hyperideal of S containing A is called left (resp. right, two-sided, bi-) Γ-hyperideal of S generated by A. A left (resp. right, two-sided, bi-) Γ-hyperideal that can be generated by only one element is called principal. In this section, we present some basic types of Γ-hyperideals and collect some basic properties of Γ-hyperideals.

Theorem 3.1. *The left (resp. right, two-sided, bi-) Γ-hyperideal of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$ generated by a non-empty subset A is the intersection of all left (resp. right, two-sided, bi-) Γ-hyperideals of S containing A.*

Proof. Let Θ be the set of all left (resp. right, two-sided, bi-) Γ-hyperideals of S containing A. Since $S \in \Theta$, it follows that $\Theta \neq \emptyset$. Let $\mathcal{G} = \bigcap_{C \in \Theta} C$. Since $A \subseteq C$ for all $C \in \Theta$, we obtain $A \subseteq \mathcal{G}$. Clearly, $\mathcal{G}$ is a left (resp. right, two-sided, bi-) Γ-hyperideal of S. Let C be a left (resp. right, two-sided, bi-) Γ-hyperideal of S containing A. Then, $C \in \Theta$. So, we have $\mathcal{G} \subseteq C$, which completes the proof. □

Lemma 3.1. *Let a be an element of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. We denote by $L_S(a)$ (resp. $R_S(a)$, $I_S(a)$, $B_S(a)$) the left (resp. right, two-sided, bi-) Γ-hyperideal of S generated by a. We have*

(1) $L_S(a) = (a \cup S\Gamma a]$;

(2) $R_S(a) = (a \cup a\Gamma S]$;

(3) $I_S(a) = (a \cup S\Gamma a \cup a\Gamma S \cup S\Gamma a\Gamma S]$;

(4) $B_S(a) = (a \cup a\Gamma a \cup a\Gamma S\Gamma a]$.

Theorem 3.2. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. Then the following assertions are equivalent:*

(1) principal Γ-hyperideals of S form a chain with respect to the inclusion relation.

(2) Γ-hyperideals of S form a chain with respect to the inclusion relation.

Proof. (1) ⇒ (2): Assume that (1) holds. Let I, J be two Γ-hyperideals of S. If $I \nsubseteq J$ and $J \nsubseteq I$, then we consider $x \in I \setminus J$ and $y \in J \setminus I$. Since principal Γ-hyperideals of S form a chain, either $I_S(x) \subseteq I_S(y)$ or $I_S(y) \subseteq I_S(x)$. Now, we can consider the following two cases:
Case 1. $I_S(x) \subseteq I_S(y)$. Since $y \in J$, we obtain $x \in I_S(x) \subseteq I_S(y) \subseteq J$ and this is a contradiction.
Case 2. $I_S(y) \subseteq I_S(x)$. Since $x \in I$, we obtain $y \in I_S(y) \subseteq I_S(x) \subseteq I$ and this is a contradiction. Hence, $I \subseteq J$ or $J \subseteq I$ and so Γ-hyperideals of S form a chain.
(2) ⇒ (1): This proof is straightforward. □

Theorem 3.3. *Let I be a Γ-hyperideal of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. Then, any idempotent Γ-hyperideal J of I is a Γ-hyperideal of S.*

Proof. First, we show that $(J] = J$. Let a be an arbitrary element of J. Since $(a, a) \in \leq$, it follows that $a \in (J]$. So, we have $J \subseteq (J]$. If $x \in (J]$, then $x \leq a$ for some $a \in J \subseteq I$. Since I is a Γ-hyperideal of S, it follows that $x \in I$. Since J is a Γ-hyperideal of I, we obtain $x \in J$. Hence, $(J] \subseteq J$ which implies that $(J] = J$. By hypothesis, J is a Γ-hyperideal of I such that $J = (J\Gamma J]$. We have

$$\begin{aligned} S\Gamma J &= S\Gamma(J\Gamma J] \\ &= (S]\Gamma(J\Gamma J] \\ &\subseteq (S\Gamma(J\Gamma J)] \\ &= ((S\Gamma J)\Gamma J] \\ &\subseteq ((S\Gamma I)\Gamma J] \\ &\subseteq (I\Gamma J] \\ &\subseteq (J] \\ &= J. \end{aligned}$$

Similarly, we obtain $J\Gamma S \subseteq J$. Therefore, J is a Γ-hyperideal of S. □

Iseki [16] proved that a commutative semigroup S is regular if and only if every ideal of S is idempotent.

Definition 5. An ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is called *regular* if for every $a \in S$ there exist $x \in S$, $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$. This is equivalent to saying that $a \in (a\Gamma S\Gamma a]$, for every $a \in S$ or $A \subseteq (A\Gamma S\Gamma A]$, for every $A \subseteq S$.

Theorem 3.4. *An ordered* Γ*-semihypergroup* $(S, \Gamma, \leq)$ *is regular if and only if for every bi-*Γ*-hyperideal* B *and every left* Γ*-hyperideal* L *of* S*, we have*

$$B \cap L \subseteq (B\Gamma L].$$

Proof. Let $a \in B \cap L$. Since S is regular, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that

$$a \leq a\alpha x\beta a = a\alpha(x\beta a) \subseteq B\Gamma(S\Gamma L) \subseteq B\Gamma L.$$

Hence, $a \in (B\Gamma L]$ and so $B \cap L \subseteq (B\Gamma L]$.

Conversely, suppose that $B \cap L \subseteq (B\Gamma L]$ for any bi-Γ-hyperideal B and any left Γ-hyperideal L of S. Let $a \in S$. Since $a \in B_S(a)$ and $a \in L_S(a)$, it follows that $a \in B_S(a) \cap L_S(a)$. By hypothesis, we have

$$\begin{aligned} a \in (B_S(a)\Gamma L_S(a)] &= ((a \cup a\Gamma a \cup a\Gamma S\Gamma a]\Gamma(a \cup S\Gamma a]] \\ &= ((a \cup a\Gamma a \cup a\Gamma S\Gamma a)\Gamma(a \cup S\Gamma a)] \\ &\subseteq (a\Gamma a \cup a\Gamma S\Gamma a]. \end{aligned}$$

Then, $a \leq u$ for some $u \in a\Gamma a \cup a\Gamma S\Gamma a$. If $u \in a\Gamma a$, then $a \leq a\alpha a \leq a\alpha(a\alpha a)$. So, $a \in (a\Gamma S\Gamma a]$. Therefore, S is regular. If $u \in a\Gamma S\Gamma a$, then $a \leq a\alpha x\beta a$ for some $x \in S$ and $\alpha, \beta \in \Gamma$. Thus, $a \in (a\Gamma S\Gamma a]$. Therefore, S is regular. □

Definition 6. Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. An element $a \in S$ is said to be *intra-regular* if there exist $x, y \in S$, $\alpha, \beta, \gamma \in \Gamma$ such that $a \leq x\alpha a\beta a\gamma y$. An ordered Γ-semihypergroup S is called intra-regular if all elements of S are intra-regular.

Equivalent definitions:

(1) $a \in (S\Gamma a\Gamma a\Gamma S]$, $\forall a \in S$.

(2) $A \subseteq (S\Gamma A\Gamma A\Gamma S]$, $\forall A \subseteq S$.

Theorem 3.5. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. The following statements are equivalent:*

(1) S is regular and intra-regular.

(2) Every bi-Γ-hyperideal of S is idempotent.

Proof. (1) $\Rightarrow$ (2): Assume that (1) holds. If B is a bi-Γ-hyperideal of S, then $B\Gamma S\Gamma B \subseteq B$. Since S is regular and intra-regular, $B \subseteq (B\Gamma S\Gamma B]$ and $B \subseteq (S\Gamma B\Gamma B\Gamma S]$. We have

$$\begin{aligned} B &\subseteq (B\Gamma S\Gamma B] \\ &\subseteq (B\Gamma S\Gamma(B\Gamma S\Gamma B]] \\ &\subseteq ((B\Gamma S]\Gamma(B\Gamma S\Gamma B]] \\ &= (B\Gamma S\Gamma B\Gamma S\Gamma B] \\ &\subseteq ((B\Gamma S]\Gamma(S\Gamma B\Gamma B\Gamma S]\Gamma(S\Gamma B]] \\ &\subseteq ((B\Gamma S)\Gamma(S\Gamma B\Gamma B\Gamma S)\Gamma(S\Gamma B)] \\ &\subseteq ((B\Gamma S\Gamma B)\Gamma(B\Gamma S\Gamma B)] \\ &\subseteq (B\Gamma B]. \end{aligned}$$

Since B is a bi-Γ-hyperideal of S, it follows that $(B\Gamma B] \subseteq (B] = B$. This shows that $B = (B\Gamma B]$, and the proof is completed.

(2) $\Rightarrow$ (1): Let $a \in S$. By hypothesis, we have

$$\begin{aligned} a \in B_S(a) &= (B_S(a)\Gamma B_S(a)] \\ &= ((a \cup a\Gamma a \cup a\Gamma S\Gamma a]\Gamma(a \cup a\Gamma a \cup a\Gamma S\Gamma a]] \\ &= ((a \cup a\Gamma a \cup a\Gamma S\Gamma a)\Gamma(a \cup a\Gamma a \cup a\Gamma S\Gamma a)] \\ &\subseteq (a\Gamma a \cup a\Gamma S\Gamma a]. \end{aligned}$$

Then, $a \leq t$ for some $t \in a\Gamma a \cup a\Gamma S\Gamma a$. If $t \in a\Gamma a$, then $a \leq a\alpha a \leq a\alpha(a\alpha a)$. So, $a \in (a\Gamma S\Gamma a]$. If $t \in a\Gamma S\Gamma a$, then $a \leq a\alpha x\beta a$ for some $x \in S$ and $\alpha, \beta \in \Gamma$. Thus, $a \in (a\Gamma S\Gamma a]$. Therefore, S is regular. Also, we have

$$\begin{aligned} a \in B_S(a) &= (B_S(a)\Gamma B_S(a)] \\ &= ((B_S(a)\Gamma B_S(a)]\Gamma B_S(a)] \\ &\subseteq ((a\Gamma a \cup a\Gamma S\Gamma a]\Gamma(a \cup a\Gamma a \cup a\Gamma S\Gamma a]] \\ &= ((a\Gamma a \cup a\Gamma S\Gamma a)\Gamma(a \cup a\Gamma a \cup a\Gamma S\Gamma a)] \\ &\subseteq (a\Gamma a\Gamma a \cup a\Gamma S\Gamma a\Gamma a \cup S\Gamma a\Gamma a\Gamma S]. \end{aligned}$$

Then, $a \leq u$ for some $u \in a\Gamma a\Gamma a \cup a\Gamma S\Gamma a\Gamma a \cup S\Gamma a\Gamma a\Gamma S$. If $u \in a\Gamma a\Gamma a$, then $a \leq a\alpha a\beta a$ for some $\alpha, \beta \in \Gamma$. We have

$$a \leq a\alpha a\beta a = a\alpha(a\beta a) \leq (a\alpha a\beta a)\alpha(a\beta a) \subseteq S\Gamma a\Gamma a\Gamma S,$$

and so $a \in (S\Gamma a\Gamma a\Gamma S]$. If $u \in a\Gamma S\Gamma a\Gamma a$, then $a \leq a\alpha x\beta a\gamma a$ for some $x \in S$ and $\alpha, \beta, \gamma \in \Gamma$. So, we have

$$a \leq a\alpha x\beta a\gamma a \leq (a\alpha x\beta a\gamma a)\alpha x\beta a\gamma a = (a\alpha x)\beta a\gamma a\alpha(x\beta a\gamma a) \subseteq S\Gamma a\Gamma a\Gamma S.$$

Hence, $a \in (S\Gamma a\Gamma a\Gamma S]$. If $u \in S\Gamma a\Gamma a\Gamma S$, then $a \in (S\Gamma a\Gamma a\Gamma S]$. Therefore, S is intra-regular. □

4 The relation $\mathcal{B}$ and minimal bi-Γ-hyperideals in ordered Γ-semihypergroups

Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. An equivalence relation $\mathcal{L}$ is defined on S by $a\mathcal{L}b$ if and only if $(a \cup S\Gamma a] = (b \cup S\Gamma b]$ for any $a, b \in S$. Similarly,

$$a\mathcal{R}b \text{ if and only if } (a \cup a\Gamma S] = (b \cup b\Gamma S],$$

$$a\mathcal{I}b \text{ if and only if } (a \cup S\Gamma a \cup a\Gamma S \cup S\Gamma a\Gamma S] = (b \cup S\Gamma b \cup b\Gamma S \cup S\Gamma b\Gamma S],$$

$$a\mathcal{B}b \text{ if and only if } (a \cup a\Gamma a \cup a\Gamma S\Gamma a] = (b \cup b\Gamma b \cup b\Gamma S\Gamma b].$$

These equivalence relations are called Green's relations on an ordered Γ-semihypergroup $(S, \Gamma, \leq)$.

Theorem 4.1. *Let $(S, \Gamma, \leq)$ be an intra-regular ordered Γ-semihypergroup. Then, for any $a, b \in S$, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$.*

Proof. Let $a \in S$. First of all, we show that $(S\Gamma a\Gamma S]$ is a Γ-hyperideal of S. We have

$$\begin{aligned} S\Gamma(S\Gamma a\Gamma S] &= (S]\Gamma(S\Gamma a\Gamma S] \\ &\subseteq (S\Gamma(S\Gamma a\Gamma S)] \\ &= (S\Gamma S(\Gamma a\Gamma S)] \\ &\subseteq (S\Gamma a\Gamma S]. \end{aligned}$$

Similarly, we have $(S\Gamma a\Gamma S]\Gamma S \subseteq (S\Gamma a\Gamma S]$. Now, let $x \in S$ and $y \in (S\Gamma a\Gamma S]$ such that $x \leq y$. Then $x \leq y \leq z$ for some $z \in S\Gamma a\Gamma S$. Hence, $x \leq z$ and so $x \in (S\Gamma a\Gamma S]$. Therefore, $(S\Gamma a\Gamma S]$ is a Γ-hyperideal of S. By hypothesis, S is intra-regular. We have $a \in (S\Gamma a\Gamma a\Gamma S] \subseteq (S\Gamma a\Gamma S]$. Since $(S\Gamma a\Gamma S]$ is a Γ-hyperideal of S containing a, we have $I_S(a) \subseteq (S\Gamma a\Gamma S]$. Clearly, $(S\Gamma a\Gamma S] \subseteq I_S(a)$. Thus $I_S(a) = (S\Gamma a\Gamma S]$. Similarly, we have $I_S(b) = (S\Gamma b\Gamma S]$. Therefore, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$. □

In view of Theorem 4.1, we have the following corollaries.

Corollary 4.1. *Let $(S, \Gamma, \leq)$ be a commutative ordered Γ-semihypergroup. If S is regular, then, for any $a, b \in S$, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$.*

Proof. Let S be regular and $a \in S$. Then, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$. Again Since S is regular, there exist $y \in S$ and $\theta, \lambda \in \Gamma$ such that $x \leq x\theta y\lambda x$. By assumption, we have

$$\begin{aligned} a &\leq a\alpha x\beta a \\ &\leq a\alpha(x\theta y\lambda x)\beta a \\ &= (x\alpha a)\theta y\lambda(x\beta a) \\ &= (x\alpha a)\theta y\lambda(a\beta x) \\ &= (x\alpha a)\theta(y\lambda a)\beta x \\ &= (x\alpha a)\theta(a\lambda y)\beta x \\ &= x\alpha(a\theta a)\lambda y\beta x \\ &\subseteq S\Gamma a\Gamma a\Gamma S. \end{aligned}$$

So, $a \in (S\Gamma a\Gamma a\Gamma S]$ and this implies that S is intra-regular. By Theorem 4.1, for any $a, b \in S$, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$. □

Corollary 4.2. *Let $(S, \Gamma, \leq)$ be an idempotent ordered Γ-semihypergroup. Then, for any $a, b \in S$, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$.*

Proof. Let $a \in S$. Then, $a \in a\gamma a$ for every $\gamma \in \Gamma$. So, we have

$$a \in a\gamma a \subseteq (a \in a\gamma a)\gamma(a \in a\gamma a) = a\gamma(a\gamma a)\gamma a.$$

Since $\leq$ is reflexive, we have $a \leq a\gamma(a\gamma a)\gamma a$ This implies that $a \in (S\Gamma a\Gamma a\Gamma S]$. Therefore, S is intra-regular. By Theorem 4.1, for any $a, b \in S$, $a\mathcal{I}b$ if and only if $(S\Gamma a\Gamma S] = (S\Gamma b\Gamma S]$. □

Theorem 4.2. *Let B be a bi-Γ-hyperideal of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. If B is B-simple, then B is a minimal bi-Γ-hyperideal of S.*

Proof. If C is a bi-Γ-hyperideal of S contain in B, then

$$C\Gamma B\Gamma C \subseteq C\Gamma S\Gamma C \subseteq C.$$

This means that C is a bi-Γ-hyperideal of B. By hypothesis, we have $C = B$. Hence, B is a minimal bi-Γ-hyperideal of S. □

Theorem 4.3. *Let B be a bi-Γ-hyperideal of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. Then, B is a minimal bi-Γ-hyperideal of S if and only if B is a $\mathcal{B}$-class.*

Proof. Suppose that B is a minimal bi-Γ-hyperideal of S. Let $a, b \in B$. If $a = b$, then $a\mathcal{B}b$. Assume that $a \neq b$. Since B is a bi-Γ-hyperideal of S containing a, we have $B_S(a) \subseteq B$. By the minimality of B, we obtain $B_S(a) = B$. Similarly, we have $B_S(b) = B$. Thus, $B_S(a) = B_S(b)$. This means that $a\mathcal{B}b$. So we have shown that B is a $\mathcal{B}$-class.

Conversely, suppose that B is a $\mathcal{B}$-class. Fix $a \in B$. Take any $b \in B$; then $a\mathcal{B}b$. Thus, $b \in B_S(b) = B_S(a)$. Since b was chosen arbitrarily, we obtain $B \subseteq B_S(a)$. By hypothesis, we vhave

$$B_S(a) = (a \cup a\Gamma a \cup a\Gamma S\Gamma a] \subseteq B.$$

Hence, $B = B_S(a)$ for all $a \in B$. Let C be a bi-Γ-hyperideal of S and $C \subseteq B$. If $x \in C$, then $x \in B$ and $B = B_S(x)$. Since $x \in C$ and C is a bi-Γ-hyperideal of S, we have

$$B = B_S(x) = (x \cup x\Gamma x \cup x\Gamma S\Gamma x] \subseteq C.$$

Therefore, B is a minimal bi-Γ-hyperideal of S. □

Corollary 4.3. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. Then S is a B-simple ordered Γ-semihypergroup if and only if S is a $\mathcal{B}$-class.*

Proof. It follows by Theorems 4.2 and 4.3. □

Theorem 4.4. *Let A be a non-empty subset of an ordered Γ-semihyergroup $(S, \Gamma, \leq)$. We consider the relation on S as follows:*

$$\sigma_A := \{(x, y) \in S \times S \mid x, y \in A \text{ or } x, y \notin A\}$$

Let Θ be the set of bi-Γ-hyperideals of S. Then we have

$$\mathcal{B} = \bigcap_{B \in \Theta} \sigma_B.$$

Proof. Let $(x,y) \in \mathcal{B}$ and $B \in \Theta$. If $x \in B$, then we have

$$\begin{aligned} y \in B_S(y) &= B_S(x) \\ &= (x \cup x\Gamma x \cup x\Gamma S\Gamma x] \\ &\subseteq (B \cup B\Gamma B \cup B\Gamma S\Gamma B] \\ &\subseteq (B] \\ &= B. \end{aligned}$$

Then, $x, y \in B$ and so $(x,y) \in \sigma_B$. If $x \notin B$, then $y \notin B$. Thus, $x, y \notin B$. This implies that $(x,y) \in \sigma_B$. Therefore, $\mathcal{B} \subseteq \bigcap_{B\in\Theta} \sigma_B$. Now, let $(x,y) \in \sigma_B$ for every $B \in \Theta$. Since $x \in B_S(x)$ and $(x,y) \in \sigma_{B_S(x)}$, we have $y \in B_S(x)$. Since $B_S(x)$ is a bi-Γ-hyperideal of S containing y, we have $B_S(y) \subseteq B_S(x)$. Similarly, we have $B_S(x) \subseteq B_S(y)$. Then, $B_S(x) = B_S(y)$ and so $(x,y) \in \mathcal{B}$. Hence, $\bigcap_{B\in\Theta} \sigma_B \subseteq \mathcal{B}$. Thus, $\mathcal{B} = \bigcap_{B\in\Theta} \sigma_B$. □

We conclude this paper with the following observation.

Corollary 4.4. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. Then S is a B-simple ordered Γ-semihypergroup if and only if S has only one $\mathcal{B}$-class.*

Proof. Suppose that S does not contain proper bi-Γ-hyperideals. Let $a \in S$. Then, for any $x \in S$ such that $a \notin x$, we have $B_S(a) = S$ and $B_S(x) = S$. Hence, $(a,x) \in \mathcal{B}$. Then, $B_S(a)$ is the only $\mathcal{B}$-class of S.

Conversely, suppose that S has only one $\mathcal{B}$-class. If C is a bi-Γ-hyperideal of S and $x \in S$, then we have $x \in C$. Let $x \notin C$. Take any $a \in C$; then $(x,a) \notin \sigma_C$. Therefore, $(x,a) \notin \mathcal{B}$. Then $x \neq a$ and $B_S(x) \neq B_S(a)$, which is a contradiction. This leads to $x \in C$. This means that $S \subseteq C$. Therefore, S is B-simple. □

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