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NURZHAN BOKAYEV

(to the 60th birthday)

On January 5, 2016 was the 60th birthday of Doctor of Physical-Mathematical Sciences (1996), Professor Nurzhan Adilkhanovich Bokayev. Professor Bokayev is the head of the department "Higher Mathematics" of the L.N. Gumilyov Eurasian National University (since 2009), the Vice-President of Mathematical Society of the Turkic World (since 2014), and a member of the Editorial Board of our journal.

N.A. Bokayevwas born in the Urnek village, Karabalyk district, Kostanay region. He graduated from the E.A. Buketov Karaganda State University in 1977 and the M.V. Lomonosov Moscow State University's full-time postgraduate study in 1984.

Scientific works of Professor Bokayev are devoted to studying problems of the theory of functions, in particular of the theory of orthogonal series.

He proved renewal and uniqueness theorems for series with respect to periodic multiplicative systems and Haar-type systems, constructed continual sets of uniqueness (U-sets) and sets of non-uniqueness (M-sets) for multiplicative systems; investigated Besov-type function spaces with respect to the Price bases; studied the Price - and Haar-type p-adic operators; introduced new classes of Faber-Schauder-type systems of functions and spaces of multivariable functions of bounded p-variation and of bounded p-fluctuation, obtained estimates for the best approximation of functions in these spaces by polynomials with respect to the Walsh and Haar systems, established weighted integrability conditions of the sum of multiple trigonometric series and series with respect to multiplicative systems, found the embedding criterion for the Nikol'skii-Besov spaces with respect to multiplicative bases and the coefficient criterion for belonging of functions to such spaces.

His scientific results have made essential contribution to the theory of series with respect to the Walsh and Haar systems and multiplicative systems.

N.A. Bokayev has published more than 150 scientific papers. Under his supervision 8 dissertations have been defended: 4 candidate of sciences dissertations and 4 PhD dissertations.

The Editorial Board of the Eurasian Mathematical Journal congratulates Nurzhan Adilkhanovich Bokayev on the occasion of his 60th birthday and wishes him good health and successful work in mathematics and mathematical education.

The EMJ has been included in the Emerging Sources Citation Index

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On behalf of the Editorial Board of the EMJ
V.I. Burenkov, T.V. Tararykova, A.M. Temirkhanova

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UNIQUENESS OF AN INVERSE SOURCE NON-LOCAL PROBLEM FOR FRACTIONAL ORDER MIXED TYPE EQUATIONS

M.S. Salakhitdinov, E.T. Karimov

Communicated by V.I. Burenkov

Key words: inverse source problem, fractional order mixed type equation, Caputo fractional derivative.

AMS Mathematics Subject Classification: 35M10, 35R11, 35R30.

Abstract. In the present work, we investigate the uniqueness of a solution to the inverse source problem with non-local conditions for a mixed parabolic-hyperbolic type equation with the Caputo fractional derivative. Solution of the problem we represent as bi-orthogonal series with respect to space variable and will get fractional order differential equations with respect to time-variable. Using boundary and gluing conditions, we deduce system of algebraic equations regarding unknown constants and imposing condition to the determinant of this system, we prove the uniqueness of the considered problem. Moreover, we find some non-trivial solutions to the problem in the case, in which the imposed conditions are not satisfie.

1 Formulation of a problem

Consider the equation

$$f(x)=\begin{cases} {}_CD_{0t}^{\alpha}u-u_{xx}, & t>0,\\ {}_CD_{t0}^{\beta}u-u_{xx}, & t<0\end{cases} \tag{1.1}$$

in the rectangular domain $\Omega=\{(x,t):\,0<x<1,\,-p<t<q\}$. Here $\alpha,\beta,p,q\in\mathbb{R}$ are such that $0<\alpha\le 1$, $1<\beta\le 2$, f is the unknown function,

$$ {}_CD_{0t}^{\alpha}g=\begin{cases} \dfrac{1}{\Gamma(1-\alpha)}\displaystyle\int_0^t \frac{g'(z)}{(t-z)^{\alpha}}dz, & 0<\alpha<1,\\ \dfrac{dg}{dt}, & \alpha=1,\end{cases}$$

$$ {}_CD_{t0}^{\beta}g=\begin{cases} \dfrac{1}{\Gamma(2-\beta)}\displaystyle\int_t^0 \frac{g''(z)}{(z-t)^{\beta-1}}dz, & 1<\beta<2,\\ \dfrac{d^2g}{dt^2}, & \beta=2\end{cases}$$

are the Caputo fractional differential operators [1, page 92, formular (2.4.16)].

Problem. Find a pair of functions $(u(x,t), f(x))$ in the domain Ω, satisfying

i) the regularity conditions $u(x,t) \in C\left(\overline{\Omega}\right), u_{xx}(x,t) \in C^2\left(\Omega^+ \cup \Omega^-\right)$, ${}_CD_{0t}^{\alpha}u \in C\left(\Omega^+\right)$,
${}_CD_{t0}^{\beta}u \in C(\Omega^-)$, $f(t) \in C(0,1)$;

ii) equation (1.1) in Ω^+, Ω^-;

iii) the boundary conditions

$$u(0,t) = u(1,t),\ u_x(0,t) = 0,\ \ -p \le t \le q, \tag{1.2}$$

$$u(x,-p) = 0,\ u(x,q) = 0,\ 0 \le x \le 1, \tag{1.3}$$

and

iv) the transmitting condition

$$\lim_{t\to+0} {}_CD_{0t}^{\alpha}u(x,t) = \lim_{t\to-0} \frac{\partial u(x,t)}{\partial(-t)},\ \ 0 < x < 1, \tag{1.4}$$

where $\Omega^+ = \Omega \cap \{t > 0\},\ \Omega^- = \Omega \cap \{t < 0\}$.

Solution of this problem we represent as follows:

$$u(x,t) = V_0(t) + \sum_{k=1}^{\infty} V_{1k}(t)\cos 2k\pi x + \sum_{k=1}^{\infty} V_{2k}(t)\cdot x\sin 2k\pi x,\ \ t \ge 0, \tag{1.5}$$

$$u(x,t) = W_0(t) + \sum_{k=1}^{\infty} W_{1k}(t)\cos 2k\pi x + \sum_{k=1}^{\infty} W_{2k}(t)\cdot x\sin 2k\pi x,\ \ t \le 0, \tag{1.6}$$

$$f(x) = f_0 + \sum_{k=1}^{\infty} f_{1k}\cos 2k\pi x + \sum_{k=1}^{\infty} f_{2k}\cdot x\sin 2k\pi x, \tag{1.7}$$

where

$$V_0(t) = 2\int\limits_0^1 u(x,t)(1-x)\,dx,\ t\geq 0,$$

$$V_{1k}(t) = 4\int\limits_0^1 u(x,t)(1-x)\cos 2k\pi x\,dx,\ t\geq 0,$$

$$V_{2k}(t) = 4\int\limits_0^1 u(x,t)\sin 2k\pi x\,dx,\ t\geq 0,$$

$$W_0(t) = 2\int\limits_0^1 u(x,t)(1-x)\,dx,\ t\leq 0,$$

$$W_{1k}(t) = 4\int\limits_0^1 u(x,t)(1-x)\cos 2k\pi x\,dx,\ t\leq 0, \tag{1.8}$$

$$W_{2k}(t) = 4\int\limits_0^1 u(x,t)\sin 2k\pi x\,dx,\ t\leq 0,$$

$$f_0 = 2\int\limits_0^1 f(x)(1-x)\,dx,$$

$$f_{1k} = 4\int\limits_0^1 f(x)(1-x)\cos 2k\pi x\,dx,$$

$$f_{2k} = 4\int\limits_0^1 f(x)\sin 2k\pi x\,dx.$$

Detailed explanation of this representation can be found in [2, p.62], which is based on [3,4].

We would like to note some works [5-7], where local and non-local inverse source problems for time-fractional diffusion and diffusion-wave equations were studied. Especially, the work by M. Kirane and S.A. Malik [8], where similar non-local conditions were in use.

Based on (1.8), we introduce similar functions with small shift into the interior of the considered domain. Then applying appropriate Caputo fractional operators, after integrating by parts, we deduce

$${}_C D_{0t}^{\alpha} V_0(t) = f_0, \quad t\geq 0, \tag{1.9}$$

$${}_C D_{t0}^{\beta} W_0(t) = f_0, \quad t< 0, \tag{1.10}$$

$${}_C D_{0t}^{\alpha} V_{1k}(t) + (2k\pi)^2 V_{1k}(t) = f_{1k} + 4k\pi V_{2k}(t), \quad t\geq 0, \tag{1.11}$$

$${}_C D_{t0}^{\beta} W_{1k}(t) + (2k\pi)^2 W_{1k}(t) = f_{1k} + 4k\pi W_{2k}(t), \quad t< 0, \tag{1.12}$$

$$ {}_CD_{0t}^{\alpha}V_{2k}(t)+(2k\pi)^2V_{2k}(t)=f_{2k},\quad t\geq 0, \tag{1.13}$$
$$ {}_CD_{t0}^{\beta}W_{2k}(t)+(2k\pi)^2W_{2k}(t)=f_{2k},\quad t<0. \tag{1.14}$$

The general solutions of (1.9) and (1.13) can be written as [1, p.231, form. (4.1.66)]

$$V_0(t)=V_0(0)+\frac{f_0}{\Gamma(\alpha+1)}t^{\alpha}, \tag{1.15}$$

$$V_{2k}(t)=V_{2k}(0)E_{\alpha,1}\left(-(2k\pi)^2t^{\alpha}\right)+f_{2k}t^{\alpha}E_{\alpha,\alpha+1}\left(-(2k\pi)^2t^{\alpha}\right), \tag{1.16}$$

respectively. Here

$$E_{\alpha,\beta}(t)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\quad \alpha,\beta>0$$

is the Mittag-Leffler function of two parameters [9, p.17].

We write the general solution of equation (1.11) in the following form

$$\begin{aligned}
V_{1k}(t)&=V_{1k}(0)E_{\alpha,1}\left(-(2k\pi)^2t^{\alpha}\right)+f_{1k}\cdot t^{\alpha}\cdot E_{\alpha,\alpha+1}\left(-(2k\pi)^2t^{\alpha}\right)\\
&+4k\pi\cdot V_{2k}(0)\cdot t^{\alpha}\cdot E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2t^{\alpha}\\ \alpha+1,\alpha,\alpha;1,1;1,1 & |-(2k\pi)^2t^{\alpha}\end{pmatrix}\\
&+4k\pi\cdot f_{2k}\cdot t^{2\alpha}\cdot E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2t^{\alpha}\\ 2\alpha+1,\alpha,\alpha;1,1;1,1 & |-(2k\pi)^2t^{\alpha}\end{pmatrix}.
\end{aligned} \tag{1.17}$$

Based on [1, p. 232, formula (4.1.74)], the general solution of (1.10) can be written as

$$W_0(t)=W_0(0)-tW_0'(0)+\frac{f_0}{\Gamma(\beta+1)}(-t)^{\beta}. \tag{1.18}$$

Similarly, we can write the general solution of (1.14) as follows

$$\begin{aligned}
W_{2k}(t)&=W_{2k}(0)E_{\beta,1}\left(-(2k\pi)^2(-t)^{\beta}\right)-tW_{2k}'(0)E_{\beta,2}\left(-(2k\pi)^2(-t)^{\beta}\right)\\
&+f_{2k}(-t)^{\beta}E_{\beta,\beta+1}\left(-(2k\pi)^2(-t)^{\beta}\right).
\end{aligned} \tag{1.19}$$

The general solution of (1.12) has the form

$$\begin{aligned}
W_{1k}(t)&=W_{1k}(0)E_{\beta,1}\left(-(2k\pi)^2(-t)^{\beta}\right)-tW_{1k}{}'(0)E_{\beta,2}\left(-(2k\pi)^2(-t)^{\beta}\right)\\
&+f_{1k}\cdot(-t)^{\beta}E_{\beta,\beta+1}\left(-(2k\pi)^2(-t)^{\beta}\right)\\
&+4k\pi\cdot W_{2k}(0)\cdot(-t)^{\beta}E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\\ \beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix}\\
&+4k\pi\cdot W_{2k}{}'(0)\cdot(-t)^{\beta+1}E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\\ \beta+2,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix}\\
&+4k\pi\cdot f_{2k}\cdot(-t)^{2\beta}E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\\ 2\beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix},
\end{aligned} \tag{1.20}$$

Here

$$E_1\left(\begin{matrix}\gamma_1,\alpha_1;\gamma_2,\beta_1 & |x\\ \delta_1,\alpha_2,\beta_2;\delta_2,\alpha_3;\delta_3,\beta_3 & |y\end{matrix}\right)=\sum_{m,n=0}^{\infty}\frac{(\gamma_1)_{\alpha_1 m}(\gamma_2)_{\beta_1 n}}{\Gamma(\delta_1+\alpha_2 m+\beta_2 n)}\cdot\frac{x^m}{\Gamma(\delta_2+\alpha_3 m)}\cdot\frac{y^n}{\Gamma(\delta_3+\beta_3 n)}$$

is the Mittag-Leffler type function in two variables, introduced by Garg et al in [10, formula (11)].

Now, using conditions (1.2)-(1.4) we find the unknown constants f_0, f_{1k}, f_{2k}, $V_0(0), V_{1k}(0), V_{2k}(0)$, $W_0(0), W_{1k}(0), W_{2k}(0), W_0'(0), W_{1k}'(0), W_{2k}'(0)$.

We substitute the obtained solutions in condition (1.3) and deduce that

$$W_0(0) + p{W_0}'(0) + \frac{f_0}{\Gamma(\beta+1)}p^\beta = 0, \tag{1.21}$$

$$\begin{aligned}
&W_{1k}(0)E_{\beta,1}\left(-(2k\pi)^2p^\beta\right) + p{W_{1k}}'(0)E_{\beta,2}\left(-(2k\pi)^2p^\beta\right) + f_{1k}p^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2p^\beta\right)\\
&+ 4k\pi\cdot W_{2k}(0)\cdot p^\beta E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2p^\beta \\ \beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2p^\beta\end{pmatrix}\\
&+ 4k\pi\cdot {W_{2k}}'(0)\cdot p^{\beta+1} E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2p^\beta \\ \beta+2,\beta,\beta;1,1;1,1 & |-(2k\pi)^2p^\beta\end{pmatrix}\\
&+ 4k\pi\cdot f_{2k}\cdot p^{2\beta} E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2p^\beta \\ 2\beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2p^\beta\end{pmatrix} = 0,
\end{aligned} \tag{1.22}$$

$$\begin{aligned}
&W_{2k}(0)E_{\beta,1}\left(-(2k\pi)^2p^\beta\right) + p{W_{2k}}'(0)E_{\beta,2}\left(-(2k\pi)^2p^\beta\right)\\
&+ f_{2k}p^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2p^\beta\right) = 0,
\end{aligned} \tag{1.23}$$

$$V_0(0) + \frac{f_0}{\Gamma(\alpha+1)}q^\alpha = 0, \tag{1.24}$$

$$\begin{aligned}
&V_{1k}(0)E_{\alpha,1}\left(-(2k\pi)^2q^\alpha\right) + f_{1k}q^\alpha E_{\alpha,\alpha+1}\left(-(2k\pi)^2q^\alpha\right)\\
&+ 4k\pi\cdot V_{2k}(0)\cdot q^\alpha E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2q^\alpha \\ \alpha+1,\alpha,\alpha;1,1;1,1 & |-(2k\pi)^2q^\alpha\end{pmatrix}\\
&+ 4k\pi\cdot f_{2k}\cdot q^{2\alpha} E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2q^\alpha \\ 2\alpha+1,\alpha,\alpha;1,1;1,1 & |-(2k\pi)^2q^\alpha\end{pmatrix} = 0,
\end{aligned} \tag{1.25}$$

$$V_{2k}(0)E_{\alpha,1}\left(-(2k\pi)^2q^\alpha\right) + f_{2k}q^\alpha E_{\alpha,\alpha+1}\left(-(2k\pi)^2q^\alpha\right) = 0. \tag{1.26}$$

Taking into account that $u(x,+0) = u(x,-0)$, which follows from $u(x,t)\in C\left(\overline{\Omega}\right)$, we get

$$V_0(0) = W_0(0),\ V_{1k}(0) = W_{1k}(0),\ V_{2k}(0) = W_{2k}(0). \tag{1.27}$$

Based on (1.9), (1.11), (1.13), we deduce

$$\begin{aligned}
&\lim_{t\to+0} {}_CD_{0t}^\alpha V_0(t) = f_0,\\
&\lim_{t\to+0} {}_CD_{0t}^\alpha V_{1k}(t) = f_{1k} + 4k\pi V_{2k}(0) - (2k\pi)^2 V_{1k}(0),\\
&\lim_{t\to+0} {}_CD_{0t}^\alpha V_{2k}(t) = f_{2k} - (2k\pi)^2 V_{2k}(0).
\end{aligned} \tag{1.28}$$

Now we calculate $\frac{d}{d(-t)}W_0(t)$, $\frac{d}{d(-t)}W_{1k}(t)$ and $\frac{d}{d(-t)}W_{2k}(t)$. One can easily deduce that

$$\frac{d}{d(-t)}W_0(t) = W_0'(0) + \frac{f_0}{\Gamma(\beta)}(-t)^{\beta-1}. \tag{1.29}$$

From (1.19) we find

$$\frac{d}{d(-t)}W_{2k}(t)=W_{2k}(0)\frac{d}{d(-t)}E_{\beta,1}\left(-(2k\pi)^2(-t)^\beta\right)$$
$$+W'_{2k}(0)\frac{d}{d(-t)}\left(-tE_{\beta,2}\left(-(2k\pi)^2(-t)^\beta\right)\right)$$
$$+f_{2k}\frac{d}{d(-t)}\left((-t)^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2(-t)^\beta\right)\right).$$

We use the following differentiation formula (see [9], p.21, formula (1.82))

$${}_{RL}D_{0t}^{\gamma}\left(t^{\alpha k+\beta-1}E_{\alpha,\beta}^{(k)}\left(\lambda t^\alpha\right)\right)=t^{\alpha k+\beta-\gamma-1}E_{\alpha,\beta-\gamma}\left((\lambda t^\alpha\right),$$

where ${}_{RL}D_{0t}^{\gamma}(\cdot)$ is the Riemann-Liouville fractional derivative of order γ [1], $E_{\alpha,\beta}^{(k)}(t)=\frac{d^k}{dt^k}E_{\alpha,\beta}(t)$. After some computations we deduce that

$$\begin{aligned}&\frac{d}{d(-t)}W_{2k}(t)=-W_{2k}(0)(2k\pi)^2(-t)^{\beta-1}E_{\beta,\beta}\left(-(2k\pi)^2(-t)^\beta\right)\\&+W'_{2k}(0)E_{\beta,1}\left(-(2k\pi)^2(-t)^\beta\right)\\&+f_{2k}(-t)^{\beta-1}E_{\beta,\beta}\left(-(2k\pi)^2(-t)^\beta\right).\end{aligned}\tag{1.30}$$

In a similar way, using the following formula (see [10], formula (33))

$${}_{RL}D_{ax}^{\gamma}\left\{(x-a)^{\delta_1-1}E_1\left(\begin{matrix}\gamma_1,\alpha_1;\gamma_2,\beta_1 & |w_1(x-a)^{\alpha_2}\\ \delta_1,\alpha_2,\beta_2;\delta_2,\alpha_3,\delta_3,\beta_3 & |w_2(x-a)^{\beta_2}\end{matrix}\right)\right\}$$
$$=(x-a)^{\delta_1-\gamma-1}E_1\left(\begin{matrix}\gamma_1,\alpha_1;\gamma_2,\beta_1 & |w_1(x-a)^{\alpha_2}\\ \delta_1-\gamma,\alpha_2,\beta_2;\delta_2,\alpha_3;\delta_3,\beta_3 & |w_2(x-a)^{\beta_2}\end{matrix}\right),$$

we deduce that

$$\begin{aligned}&\frac{d}{d(-t)}W_{1k}(t)=-(2k\pi)^2W_{1k}(0)(-t)^{\beta-1}E_{\beta,\beta}\left(-(2k\pi)^2(-t)^\beta\right)\\&+W'_{1k}(0)E_{\beta,1}\left(-(2k\pi)^2(-t)^\beta\right)+f_{1k}(-t)^{\beta-1}E_{\beta,\beta+1}\left(-(2k\pi)^2(-t)^\beta\right)\\&+4k\pi W_{2k}(0)(-t)^{\beta-1}E_1\left(\begin{matrix}1,1;1,1 & |-(2k\pi)^2(-t)^\beta\\ \beta,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^\beta\end{matrix}\right)\\&+4k\pi W'_{2k}(0)(-t)^{\beta}E_1\left(\begin{matrix}1,1;1,1 & |-(2k\pi)^2(-t)^\beta\\ \beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^\beta\end{matrix}\right)\\&+4k\pi f_{2k}(-t)^{2\beta-1}E_1\left(\begin{matrix}1,1;1,1 & |-(2k\pi)^2(-t)^\beta\\ 2\beta,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^\beta\end{matrix}\right).\end{aligned}\tag{1.31}$$

From (1.29)-(1.31) we get

$$\lim_{t\to-0}\frac{d}{d(-t)}W_0(t)=W'_0(0),\ \lim_{t\to-0}\frac{d}{d(-t)}W_{1k}(t)=W'_{1k}(0),\ \lim_{t\to-0}\frac{d}{d(-t)}W_{2k}(t)=W'_{2k}(0).\tag{1.32}$$

Considering (1.28) and (1.32) we have

$$\begin{aligned} &f_0 = W_0'(0),\\ &f_{1k} + 4k\pi V_{2k}(0) - (2k\pi)^2 V_{1k}(0) = W_{1k}'(0),\\ &f_{2k} - (2k\pi)^2 V_{2k}(0) = W_{2k}'(0). \end{aligned} \tag{1.33}$$

From (1.21), (1.24) and first relations of (1.27), (1.33) we get the following system of equations

$$\begin{cases} W_0(0) + \left(\dfrac{p^\beta}{\Gamma(\beta+1)} + p\right) W_0'(0) = 0,\\ W_0(0) + \dfrac{q^\alpha}{\Gamma(\alpha+1)} W_0'(0) = 0,\\ f_0 = W_0'(0). \end{cases} \tag{1.34}$$

If

$$\Delta_0 = p + \frac{p^\beta}{\Gamma(\beta+1)} - \frac{q^\alpha}{\Gamma(\alpha+1)} \neq 0, \tag{1.35}$$

then we find that

$$f_0 = W_0'(0) = V_0(0) = W_0(0) = 0. \tag{1.36}$$

Now from (1.22), (1.23), (1.25), (1.26) and last two relations in (1.27), (1.33), we obtain another system of equations

$$\begin{cases} f_{1k} = W_{1k}'(0) + (2k\pi)^2 W_{1k}(0) - 4k\pi W_{2k}(0),\\ f_{2k} = W_{2k}'(0) + (2k\pi)^2 W_{2k}(0),\\ W_{2k}(0) + W_{2k}'(0)\left[p^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2 p^\beta\right) + pE_{\beta,2}\left(-(2k\pi)^2 p^\beta\right)\right] = 0,\\ W_{2k}(0) + W_{2k}'(0) q^\alpha E_{\alpha,\alpha+1}\left(-(2k\pi)^2 q^\alpha\right) = 0,\\ W_{1k}(0) + W_{1k}'(0)\left[p^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2 p^\beta\right) + pE_{\beta,2}\left(-(2k\pi)^2 p^\beta\right)\right] = 0,\\ W_{1k}(0) + W_{1k}'(0) q^\alpha E_{\alpha,\alpha+1}\left(-(2k\pi)^2 q^\alpha\right) = 0. \end{cases} \tag{1.37}$$

Further, assuming that

$$\Delta_k = p^\beta E_{\beta,\beta+1}\left(-(2k\pi)^2 p^\beta\right) + pE_{\beta,2}\left(-(2k\pi)^2 p^\beta\right) - q^\alpha E_{\alpha,\alpha+1}\left(-(2k\pi)^2 q^\alpha\right) \neq 0, \tag{1.38}$$

we get

$$V_{1k}(0) = V_{2k}(0) = W_{1k}(0) = W_{2k}(0) = W_{1k}'(0) = W_{2k}'(0) = f_{1k} = f_{2k} = 0. \tag{1.39}$$

If conditions (1.35), (1.38) hold, then based on (1.5)-(1.7), due to (1.36), (1.39) we have

$$\int_0^1 u(x,t)(1-x)\,dx = 0, \ \int_0^1 u(x,t)\sin 2k\pi x\,dx = 0,$$

$$\int_0^1 u(x,t)(1-x)\cos 2k\pi x dx = 0, \ \int_0^1 f(x)(1-x)\,dx = 0,$$

$$\int_0^1 f(x)\sin 2k\pi x\,dx = 0, \ \int_0^1 f(x)(1-x)\cos 2k\pi x\,dx = 0, \ k = 1, 2, ...$$

According to the completeness of the system $\{1, \cos 2k\pi x, x \sin 2k\pi x\}$ in $L_2[0,1]$, we can state that $u(x,t) = 0$ a.e. in $[0,1]$ for $t \in [-p, q]$ and $f(x) = 0$ a.e. in $[0,1]$.

Now we formulate the obtained result as the following

Theorem 1.1. *If conditions (1.35), (1.38) hold, then problem has only trivial solution.*

2 Nontrivial solutions of the problem

We consider case, when conditions (1.35), (1.38) are not satisfied. Let $\Delta_0 = 0$ for some p, q. Then problem has a nontrivial solution of the form

$$u(x,t) = u_0(t), \;\; f(x) = f_0, \tag{2.1}$$

where

$$u_0(t) = \begin{cases} \dfrac{t^\alpha - q^\alpha}{\Gamma(\alpha+1)} f_0, \;\; t \geq 0, \\ \left[\dfrac{(-t)^\beta - p^\beta}{\Gamma(\beta+1)} - t - p\right] f_0, \;\; t \leq 0, \end{cases}$$

$f_0 \neq 0$ is an arbitrary constant.

If $\Delta_k = 0$ for $k = m \in \mathbb{N}$, i.e. $\Delta_m = 0$, then the considered problem has nontrivial solutions of the form

$$u_m(x,t) = \begin{cases} V_{1m}(t) \cos 2m\pi x + V_{2m}(t) x \sin 2m\pi x, \;\; t \geq 0, \\ W_{1m}(t) \cos 2m\pi x + W_{2m}(t) x \sin 2m\pi x, \;\; t \leq 0, \end{cases} \tag{2.2}$$

$$\begin{aligned} f_m(x) = &\left\{E_{\alpha,1}\left(-(2m\pi)^2 q^\alpha\right) W'_{1m}(0)\right. \\ &\left.+4k\pi q^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^\alpha\right) W'_{2m}(0)\right\} \cos 2m\pi x \\ &+ E_{\alpha,1}\left(-(2m\pi)^2 q^\alpha\right) W'_{2m}(0) x \sin 2m\pi x, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} V_{1m}(t) = &W'_{1m}(0)\left\{t^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 t^\alpha\right) E_{\alpha,1}\left(-(2m\pi)^2 q^\alpha\right) - q^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^\alpha\right)\right. \\ &\left.\times E_{\alpha,1}\left(-(2m\pi)^2 t^\alpha\right)\right\} + 4k\pi t^\alpha W'_{2m}(0)\left\{q^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^\alpha\right)\right. \\ &\times\left[E_{\alpha,\alpha+1}\left(-(2m\pi)^2 t^\alpha\right) - E_1\left(\begin{matrix} 1,1;1,1 & \mid -(2k\pi)^2 t^\alpha \\ \alpha+1,\alpha,\alpha;1,1;1,1 & \mid -(2k\pi)^2 t^\alpha \end{matrix}\right)\right] \\ &\left.+t^\alpha E_{\alpha,1}\left(-(2m\pi)^2 q^\alpha\right) E_1\left(\begin{matrix} 1,1;1,1 & \mid -(2k\pi)^2 t^\alpha \\ 2\alpha+1,\alpha,\alpha;1,1;1,1 & \mid -(2k\pi)^2 t^\alpha \end{matrix}\right)\right\}, \end{aligned}$$

$$\begin{aligned} V_{2m}(t) = &\left\{t^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 t^\alpha\right) E_{\alpha,1}\left(-(2m\pi)^2 q^\alpha\right)\right. \\ &\left.-q^\alpha E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^\alpha\right) E_{\alpha,1}\left(-(2m\pi)^2 t^\alpha\right)\right\} W'_{2m}(0), \end{aligned}$$

$$
\begin{aligned}
&W_{1m}(t) = \left\{(-t)^{\beta} E_{\alpha,1}\left(-(2m\pi)^2 q^{\alpha}\right) E_{\beta,\beta+1}\left(-(2m\pi)^2(-t)^{\beta}\right)\right.\\
&-q^{\alpha} E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^{\alpha}\right) E_{\beta,1}\left(-(2m\pi)^2(-t)^{\beta}\right)\Big\} W'_{1m}(0)\\
&+ 4k\pi(-t)^{\beta} W'_{2m}(0)\left\{q^{\alpha} E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^{\alpha}\right)\right.\\
&\times\left[E_{\beta,\beta+1}\left(-(2m\pi)^2(-t)^{\beta}\right) - E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta} \\ \beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix}\right]\\
&- tE_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta} \\ \beta+2,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix}\\
&\left.+(-t)^{\beta} E_{\alpha,1}\left(-(2m\pi)^2 q^{\alpha}\right) E_1\begin{pmatrix} 1,1;1,1 & |-(2k\pi)^2(-t)^{\beta} \\ 2\beta+1,\beta,\beta;1,1;1,1 & |-(2k\pi)^2(-t)^{\beta}\end{pmatrix}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&W_{2m}(t) = \left\{(-t)^{\beta} E_{\alpha,1}\left(-(2m\pi)^2 q^{\alpha}\right) E_{\beta,\beta+1}\left(-(2m\pi)^2(-t)^{\beta}\right)\right.\\
&\left.- q^{\alpha} E_{\alpha,\alpha+1}\left(-(2m\pi)^2 q^{\alpha}\right) E_{\beta,1}\left(-(2m\pi)^2(-t)^{\beta}\right) - -tE_{\beta,2}\left(-(2m\pi)^2(-t)^{\beta}\right)\right\} W'_{2m}(0).
\end{aligned}
$$

Here $W'_{1m}(0), W_{2m}(0)$ are arbitrary non-zero constants.
Conclusion If conditions (1.35), (1.38) are not satisfied, there exist nontrivial solutions of the considered problem and they have form (2.1) or (2.2)-(2.3).

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