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ASYMPTOTIC ANALYSIS OF THE GENERALIZED CONVECTION PROBLEM

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Abstract. The averaging method is justified and the complete asymptotics of a solution periodic in time is constructed and justified for an evolutional system of partial differential equations with quickly oscillating in time junior terms, some of which are proportional to the frequency of oscillations. The considered system generalizes the well-known thermal liquid convection problem (in Oberdeck-Boussinesc approach) when a vessel with a liquid vibrates with high frequency.

1 Introduction

In the work [11] the Krylov-Bogoliubov averaging method [1] is applied to the problem of the thermal liquid convection beginning in a vessel subjected to small in amplitude vibrations at high frequency ω. In [6, 4] a rigorous justification of the applicability of this method to a wide class of thermal convection problems in the field of quickly oscillating forces (in particular, to the problem in [11]) is provided. In the work [5] for similar problems the complete asymptotics of a solution (as $\omega \to \infty$) is constructed and justified, where the leading term is a solution of the homogenized problem. In our paper the considered problem is much more general than in papers [11, 6, 4, 5]. We call it a *generalized problem of vibrational convection.* For this problem the complete asymptotics (as $\omega \to \infty$) of a solution $2\pi/\omega$-periodic in time of the original (perturbed) problem is constructed in a small neighbourhood of a stationary solution $\mathring{v}$ of the corresponding averaged problem. Note that a specificity of the problem of the thermal convection (see, eg., [4]), approaches of the averaging method theory [1] and the boundary layer method [9] are used. It is proved that the construction of every partial sum of the asymptotics is reduced (if $\mathring{v}$ is known) to solving a finite number of ω-independent linear stationary uniquely solvable problems of five specific types. Two of these types are related to partial differential equations and three of them are related to ordinary differential equations. Existence and relative uniqueness of the mentioned $2\pi\omega^{-1}$-periodic in time solutions and justification of constructed in this work complete asymptotics may be verified as in [4, 5].

2 The problem statement and the result formulation

Let Ω be a bounded domain in $\mathbb{R}^3$ with an infinitely smooth boundary $\partial\Omega$. In the cylinder $\overline{Q} = \overline{\Omega} \times \mathbb{R}, \overline{\Omega} = \Omega \cup \partial\Omega$, we consider the problem of finding solutions $2\pi\omega^{-1}$-periodic in t of the system of equations

$$\begin{aligned}
&\frac{\partial v}{\partial t} + (v,\nabla)v = -\nabla p + \nu\Delta v + \omega \sum_{0<|k|\le m} a_k(x)\ exp(ik\omega t)T \\
&+ \sum_{0\le|k|\le m} f_{1k}\left(x, v, \frac{\partial v}{\partial x}, T, \frac{\partial T}{\partial x}\right)\ exp(ik\omega t), \\
&\frac{\partial T}{\partial t} + (v,\nabla T) = \chi\Delta T + \sum_{0\le|k|\le m} f_{2k}\left(x, v, \frac{\partial v}{\partial x}, T, \frac{\partial T}{\partial x}\right)\ exp(ik\omega t), \\
&div\ v = 0,\ v|_{\partial\Omega} = 0,\ T|_{\partial\Omega} = h.
\end{aligned} \tag{2.1}$$

Here $m \in \mathbb{N}$, ω is a large parameter, $\nu, \chi > 0$, $a_k(x), f_{1k}(x,v,w,T,S) \in \mathbb{C}^3$, $f_{2k}(x,v,w,T,S) \in \mathbb{C}, h(x) \in \mathbb{R}$, and $a_k(x) = \overline{a_{-k}(x)}$,

$$f_{1k}(x,v,w,T,S) = \overline{f_{1,-k}(x,v,w,T,S)},$$

$$f_{2k}(x,v,w,T,S) = \overline{f_{2,-k}(x,v,w,T,S)},$$

where $(x,v,w,T,S) \in \overline{\Omega} \times \mathbb{R}^3 \times \mathbb{R}^9 \times \mathbb{R} \times \mathbb{R}^3$, and the overlined vector-functions are complex conjugate. Let f_{1k}, f_{2k}, a_k, h be infinitely differentiable in all their arguments. We suppose also that the components f_{ikr} of the vector-functions f_{ik} depend on components v, w and S in a polynomial way and they are linear functions in w:

$$f_{ikr}(x,v,w,T,S) = \sum_{|\alpha|=0}^{Q} \psi_{ikr\alpha}(x,T) v_1^{\alpha_1} v_2^{\alpha_2} v_3^{\alpha_3} w_1^{\alpha_4} w_2^{\alpha_5} ... w_9^{\alpha_{12}} S_1^{\alpha_{13}} S_2^{\alpha_{14}} S_3^{\alpha_{15}}, \tag{2.2}$$

where $Q \in \mathbb{N}, \alpha = \alpha_{ikr} = (\alpha_1, ..., \alpha_{15})$ is a multi-index, and $\alpha_4 + \alpha_5 + ... + \alpha_{12} \le 1$.

In the present work solutions to problem (2.1) and all other problems are understood in the classic sense.

Let us consider the system

$$\begin{aligned}
&\frac{\partial u}{\partial t} - \nu\Delta u + \nabla q + (u,\nabla)u + \sum_{0<|k|\le m} k^{-2}[(\Pi a_k W, \nabla)\Pi a_{-k} W - a_k(\Pi a_{-k} W, \nabla W)] \\
&= \sum_{0<|k|\le m} < a_k \Psi\left(x, u, \frac{\partial u}{\partial x}, W, \frac{\partial W}{\partial x}, \tau\right) exp(ik\tau) > \\
&+ \sum_{0\le|k|\le m} < \varphi_{1k}\left(x, u, \frac{\partial u}{\partial x}, W, \frac{\partial W}{\partial x}, \tau\right) >, \\
&div\ u = 0, \\
&\frac{\partial W}{\partial t} - \chi\Delta W + (u,\nabla W) = \sum_{0\le|k|\le m} < \varphi_{2k}\left(x, u, \frac{\partial u}{\partial x}, W, \frac{\partial W}{\partial x}, \tau\right) >, x \in \Omega, \\
&u|_{\partial\Omega} = 0, W|_{\partial\Omega} = h(x).
\end{aligned} \tag{2.3}$$

We call it an averaged (limiting) problem. Here and below the symbol $<\varphi(...,\tau)>$ denotes the φ averaging in τ $:<\varphi(...,\tau)>=\frac{1}{2\pi}\int\limits_0^{2\pi}\varphi(...,\tau)d\tau$, where $\varphi(...,\tau)$ is a continuous 2π-periodic in τ vector-function,

$$\varphi_{rk}(x,u,\tfrac{\partial u}{\partial x},W,S,\tau)=f_{rk}(x,u+u_0,\tfrac{\partial(u+u_0)}{\partial x},W,S)e^{ik\tau}\equiv\sum_s l_{rks}e^{is\tau},r=1,2,$$

$$u_0=\sum_{0<|k|\le m}(ik)^{-1}\Pi a_kWe^{ik\tau}.$$

Here Π is a known in mathematical hydrodynamics orthoprojector in $L_2(\Omega)$ onto the subspace of solenoid vectors $S_2(\Omega)$ (see [10, 3, 8]),

$$\Psi(x,u,\frac{\partial u}{\partial x},W,S,\tau)\equiv\sum_{\substack{s\ne0\\0\le|k|\le m}}(is)^{-1}l_{2ks}e^{is\tau}.$$

Let problem (2.3) have a real stationary solution $(u,q,W)=(u_0,p_0,T_0)$, and the elliptic system

$$\begin{gathered}L(v,q,T)\equiv-\nu\Delta v+(v_0,\nabla)v+(v,\nabla)v_0+\nabla q\\
+\sum_{0<|k|\le m}k^{-2}[(\Pi a_kT_0,\nabla)\Pi a_{-k}T+(\Pi a_kT,\nabla)\Pi a_{-k}T_0-a_k(\Pi a_{-k}T,\nabla T_0)\\
-a_k(\Pi a_{-k}T_0,\nabla T)]-\sum_{0<|k|\le m}<a_k\left[\frac{\partial\Psi}{\partial u}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)v\right.\\
+\frac{\partial\Psi}{\partial w}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial v}{\partial x}+\frac{\partial\Psi}{\partial T}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)T\\
\left.+\frac{\partial\Psi}{\partial S}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial T}{\partial x}\right]exp(ik\tau)>\\
-\sum_{0\le|k|\le m}\left[<\frac{\partial\varphi_{1k}}{\partial u}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)v\right.\\
+\frac{\partial\varphi_{1k}}{\partial w}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial v}{\partial x}+\frac{\partial\varphi_{1k}}{\partial T}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)T\qquad(2.4)\\
\left.+\frac{\partial\varphi_{1k}}{\partial S}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial T}{\partial x}>\right]=0,\\
div\ v=0,\\
R(v,T)\equiv-\chi\Delta T+(v_0,\nabla T)+(v,\nabla T_0)\\
-\sum_{0\le|k|\le m}\left[<\frac{\partial\varphi_{2k}}{\partial u}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)v\right.\end{gathered}$$

$$+\frac{\partial\varphi_{2k}}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v}{\partial x} + \frac{\partial\varphi_{2k}}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)T$$

$$+\frac{\partial\varphi_{2k}}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T}{\partial x} >\Big] = 0,\, v|_{\partial\Omega} = 0,$$

$$T|_{\partial\Omega} = 0, \int\limits_{\Omega} q dx = 0$$

have no nontrivial solutions.

Let the above mentioned conditions be satisfied. Then problem (2.1) for large values of ω has a relatively unique (in the usual Hölder norm $C^{2,1}(\overline{Q})$) $2\pi\omega^{-1}$-periodic in time solution (v_ω, T_ω) (the corresponding function p_ω is defined here uniquely up to numerical term).

To construct the asympthotic expansion the solution of the stated problem we replace in (2.1) ωt by τ and $\omega^{-1/2}$ by ϵ. Then we get the problem of finding 2π-periodic in τ solutions of the system

$$\begin{aligned}
&\frac{\partial v}{\partial \tau} - \epsilon^2\nu\Delta v + \epsilon^2\nabla p + \epsilon^2(v,\nabla)v = \sum_{0<|k|\le m} a_k(x)\; exp(ik\tau)T\\
&+\epsilon^2\sum_{0\le|k|\le m} f_{1k}\left(x, v, \frac{\partial v}{\partial x}, T, \frac{\partial T}{\partial x}\right)\; exp(ik\tau),\\
&\frac{\partial T}{\partial \tau} - \epsilon^2\chi\Delta T + \epsilon^2(v,\nabla T) = \epsilon^2\sum_{0\le|k|\le m} f_{2k}\left(x, v, \frac{\partial v}{\partial x}, T, \frac{\partial T}{\partial x}\right)\; exp(ik\tau),\\
&div\; v = 0,\, v|_{\partial\Omega} = 0,\, T|_{\partial\Omega} = h.
\end{aligned} \tag{2.5}$$

In order to apply the boundary-layer method [9] to problem (2.5) we use the curviliniar coordinate system (ψ, r) on the closure $\overline{\Omega}_\eta$ of the boundary subdomain Ω_η of the domain Ω (η is the width of the layer). We define a mapping $\partial\Omega\times[0,\eta]\to\overline{\Omega}_\eta$ by the rule $(\psi, r)\to\psi + n_\psi r$, where $\psi = (\psi_1, \psi_2)$ is a point on $\partial\Omega$ with local coordinates ψ, and n_ψ is an inward normal vector to $\partial\Omega$ at the point ψ. We choose the number η to be sufficiently small, so that the mentioned normals in Ω_η do not intersect.

Then let us introduce the new independent variable $\rho = \sqrt{\omega}r$, $r \le \eta$, express the derivatives in x_j, $1\le j\le 3$, via the derivatives in ψ_1, ψ_2 and ρ and expand the coefficients in the powers of $\omega^{-1/2}$. We get the following equalities:

$$\frac{\partial}{\partial x_j} = \frac{\partial r}{\partial x_j}\frac{\partial}{\partial r} + \sum_{k=1}^{2}\frac{\partial\psi_k}{\partial x_j}\frac{\partial}{\partial\psi_k} = \sqrt{\omega}\frac{\partial r}{\partial x_j}\frac{\partial}{\partial\rho} + \sum_{k=1}^{2}\frac{\partial\psi_k}{\partial x_j}\frac{\partial}{\partial\psi_k}, \tag{2.6}$$

$$\begin{aligned}
\frac{\partial r}{\partial x_j} &\equiv b_j(x) = b_j(\psi, r) = b_j(\psi, \frac{\rho}{\sqrt{\omega}})\\
&= b_j(\psi, 0) + \frac{\partial b_j(\psi,0)}{\partial r}\frac{\rho}{\sqrt{\omega}} + \frac{1}{2}\frac{\partial^2 b_j(\psi,0)}{\partial r^2}\frac{\rho^2}{\omega} + \ldots\\
&\equiv b_{j0} + b_{j1}\frac{\rho}{\sqrt{\omega}} + b_{j2}\frac{\rho^2}{\omega} + \ldots
\end{aligned} \tag{2.7}$$

$$\frac{\partial\psi_k}{\partial x_j} \equiv c_{kj}(x) = c_{kj}(\psi, r) = c_{kj}(\psi, \frac{\rho}{\sqrt{\omega}}) = c_{kj}(\psi, 0) + \frac{\partial c_{kj}(\psi,0)}{\partial r}\frac{\rho}{\sqrt{\omega}} + \frac{1}{2}\frac{\partial^2 c_{kj}(\psi,0)}{\partial r^2}\frac{\rho^2}{\omega} + ... \equiv c_{kj0} + c_{kj1}\frac{\rho}{\sqrt{\omega}} + c_{kj2}\frac{\rho^2}{\omega} + ... \quad (2.8)$$

Hence

$$\frac{\partial}{\partial x_j} = \left(\sqrt{\omega}b_{j0} + b_{j1}\rho + b_{j2}\frac{\rho^2}{\sqrt{\omega}} + b_{j3}\frac{\rho^3}{\omega} + \dots\right)\frac{\partial}{\partial\rho} + \sum_{k=1}^{2}\left(c_{kj0} + c_{kj1}\frac{\rho}{\sqrt{\omega}} + c_{kj2}\frac{\rho^2}{\omega} + \dots\right)\frac{\partial}{\partial\psi_k}. \quad (2.9)$$

We seek the asymptotics of the 2π-periodic in τ solution of problem (2.5) in the following form

$$\begin{aligned} v(x,\tau,\epsilon) &= \sum_{k=0}^{\infty}\epsilon^k v_k(x) + \sum_{k=0}^{\infty}\epsilon^k u_k(x,\tau) + \sum_{k=0}^{\infty}\epsilon^k w_k(\psi,\rho) + \sum_{k=0}^{\infty}\epsilon^k z_k(\psi,\rho,\tau),\\ p(x,\tau,\epsilon) &= \sum_{k=0}^{\infty}\epsilon^k p_k(x) + \sum_{k=0}^{\infty}\epsilon^{k-2} s_k(x,\tau) + \sum_{k=0}^{\infty}\epsilon^k h_k(\psi,\rho) + \sum_{k=0}^{\infty}\epsilon^{k-1} g_k(\psi,\rho,\tau),\\ T(x,\tau,\epsilon) &= \sum_{k=0}^{\infty}\epsilon^k T_k(x) + \sum_{k=0}^{\infty}\epsilon^{k+2} R_k(x,\tau) + \sum_{k=0}^{\infty}\epsilon^{k+1} W_k(\psi,\rho) + \sum_{k=0}^{\infty}\epsilon^{k+1} Z_k(\psi,\rho,\tau),\\ &\rho = \epsilon^{-1} r, \end{aligned} \quad (2.10)$$

Let us consider five types of linear uniquely solvable problems: the construction of coefficients of series (2.10) is reduced to them (see Section 3). The first two problems are related to partial differential equations.

(A1) The Neyman problem for the following system

$$\Delta s = f(x), \left.\frac{\partial s}{\partial n}\right|_{\partial\Omega} = \varphi(x), \quad (2.11)$$

where f, φ are the infinitely differentiable in Ω and $\partial\Omega$ three-dimensional vector-functions, n is an inward normal to the boundary $\partial\Omega$ and

$$\int\limits_{\Omega} f dx + \int\limits_{\partial\Omega} \varphi ds = 0.$$

(A2) The Dirichlet problem for the following system

$$\begin{aligned} &L(v,q,T) = f(x), div\ v = 0, \int\limits_{\Omega} q dx = 0,\\ &R(v,T) = g(x), v|_{\partial\Omega} = 0, T|_{\partial\Omega} = 0, \end{aligned} \quad (2.12)$$

where f, g are the infinitely differentiable vector-functions.

The next three are the problems of limited on the ray $\rho > 0$ solutions to the following ordinary differential equations with the parameter $\psi \in \partial\Omega$:

(A3)

$$\begin{aligned} &ikz(\psi,\rho) = \nu\frac{\partial^2 z(\psi,\rho)}{\partial\rho^2} + F(\psi,\rho)\ exp(\lambda\rho),\\ &z(\psi,0) = z_0(\psi),\\ &z|_{\rho=\infty} = 0. \end{aligned} \tag{2.13}$$

(A4)

$$\begin{aligned} &\frac{\partial^2 w(\psi,\rho)}{\partial\rho^2} + F(\psi,\rho)\ exp(\lambda\rho) = 0,\\ &w\,|_{\rho=\infty} = 0. \end{aligned} \tag{2.14}$$

(A5)

$$\begin{aligned} &\frac{\partial w(\psi,\rho)}{\partial\rho} + F(\psi,\rho)\ exp(\lambda\rho) = 0,\\ &w\,|_{\rho=\infty} = 0. \end{aligned} \tag{2.15}$$

Here $k \in \mathbb{Z} \setminus \{0\}, Re\lambda < 0, F$ is a polynomial in ρ with infinitely differentiable in ψ coeffitients.

Remark. The presented list of five types of auxiliary problems fully characterizes our algorithm used for construction the asympthotics of the solution to problem (2.1). Problems (A3)-(A5) are also included in this algorithm just because of it, though their solution consists of finite number of arithmetic operations.

Theorem 2.1. *Let the conditions of this section be satisfied, $\omega > \omega_0$, where ω_0 is a sufficiently large number, $(u_\omega, p_\omega, T_\omega)$ is the specified in this section $2\pi/\omega^{-1}$-periodic in t solution of system* (2.1). *Then the following statements are true.*

1) Construction of any partial sum (u^n, p^n, T^n) of the complete formal asymptotics of the solution of the stated problem for $\omega >> 1$ is reduced to solving a finite number of linear uniquely solvable problems of types (A1)-(A5). *All the terms of partial sums of series* (2.10) *are real and infinitely smooth.*

2) For all non-negative numbers l, m $(u_\omega, p_\omega, T_\omega) \in \mathbb{C}^{l,m}(\overline{Q})$ and following estimates hold

$$\|u_\omega - u^n\|_{l,m} + \|T_\omega - T^n\|_{l,m} \le c_{l,m,n}\omega^{-[n+1-\max(l-2,2m-2,0)]/2}, \tag{2.16}$$

$$\|\nabla p_\omega - \nabla p^n\|_{l,m} \le c_{l,m,n}\omega^{-[n+1-\max(l,2m)]/2}, \tag{2.17}$$

where $c_{l,m,n}$ are constants independent of ω.

Here $C^{l,m}(\overline{Q})$ is a usual Hölder space on the cylinder $\overline{Q}$ of all vector-functions $u(x,t)$, which have continuous derivatives in x up to order $[l]$ and in t up to order $[m]$, satisfying the Hölder condition with the appropriate indices.

3 Construction of a formal asymptotic expansion

In this section we prove the first statement of Theorem 2.1.

Proof. First we establish that the construction of the coefficients in equalities (2.1) is indeed reduced to solving a finite number of problems of types (A1)-(A5).

For $x \in \Omega_\eta$ we denote the components of an arbitrary vector $v(x) \in \mathbb{R}^3$ in the curvilinear coordinates (ψ_1, ψ_2, r) as $v^{(s)}(x), s = 1, 2, 3$. If $v(x) = v_1(x)+v_2(x)i, v_1, v_2 \in \mathbb{R}^3$, then we write $v^{(s)}(x) = v_1^{(s)}(x)+v_2^{(s)}(x)i$ and use the known representations of Δ, ∇ and div in the coordinates (ψ, ρ) (see [4], [2]):

$$\Delta v = \sum_{j=-2}^{N} \epsilon^j L_j(\psi, \rho)v + [R_{1,N}(x, \epsilon)](v), \tag{3.1}$$

$$\nabla p = \sum_{j=-1} \epsilon^j P_j(\psi, \rho)p + [R_{2,N}(x, \epsilon)](p) \tag{3.2}$$

$$\begin{aligned} div\ \ v = \epsilon^{-1}\frac{\partial}{\partial\rho}v^{(3)} + \sum_{j=0}^{N} \epsilon^j (D_{j,1}(\psi, \rho)v^{(1)} + D_{j,2}(\psi, \rho)v^{(2)} \\ +D_{j,3}(\psi, \rho)v^{(3)}) + [R_{3,N}(x, \epsilon)](v). \end{aligned} \tag{3.3}$$

Here $L_j, P_j, D_{j,i}$ are linear differential expressions in ψ_1, ψ_2, ρ and their coefficients are polynomials in ρ with (ψ_1, ψ_2)-dependent coefficients; $R_{j,N}$ are linear differential expressions in ψ_1, ψ_2, ρ with (x, ϵ)-dependent coefficients. We write down only the expressions for the leading terms of expansions (3.1) and (3.2):

$$\begin{aligned} (L_{-2}(\psi, \rho)v)^{(s)} = -\partial^2 v^{(s)}/\partial\rho^2, s = 1, 2, 3, \\ (P_{-1}(\psi, \rho)p)^{(s)} = 0, s = 1, 2, (P_{-1}(\psi, \rho))^{(3)} = \partial p/\partial\rho. \end{aligned} \tag{3.4}$$

Let us formally substitute series (2.10) in equalities (2.5):

$$\begin{aligned} &\sum_{k=0}^{\infty} \epsilon^k \left[\frac{\partial u_k(x,\tau)}{\partial\tau} + \frac{\partial z_k(\psi,\rho,\tau)}{\partial\tau}\right] \\ &-\epsilon^2\nu\Delta \sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x, \tau) + w_k(\psi, \rho) + z_k(\psi, \rho, \tau)\right] \\ &+\epsilon^2\nabla \sum_{k=0}^{\infty} \epsilon^k \left[p_k(x) + \epsilon^{-2}s_k(x, \tau) + h_k(\psi, \rho) + \epsilon^{-1}g_k(\psi, \rho, \tau)\right] \\ &+\epsilon^2 \left(\sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x, \tau) + w_k(\psi, \rho) + z_k(\psi, \rho, \tau)\right], \nabla\right) \\ &\times \sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x, \tau) + w_k(\psi, \rho) + z_k(\psi, \rho, \tau)\right] \\ &= \sum_{0<|k|\le m} a_k(x) \sum_{k=0}^{\infty} \epsilon^k \left[T_k(x) + \epsilon^2 R_k(x, \tau) + \epsilon W_k(\psi, \rho) + \epsilon^2 Z_k(\psi, \rho, \tau)\right] \\ &\times exp(ik\tau) \end{aligned} \tag{3.5}$$

$$+\epsilon^2 \sum_{0\le|k|\le m} f_{1k}\left(x, v(x,\tau), \frac{\partial v(x,\tau)}{\partial x}, T(x,\tau), \frac{\partial T(x,\tau)}{\partial x}\ exp(ik\tau)\right);$$

$$\sum_{k=0}^{\infty} \epsilon^{k+2}\left[\frac{\partial R_k(x,\tau)}{\partial \tau} + \frac{\partial Z_k(\psi,\rho,\tau)}{\partial \tau}\right]$$

$$-\epsilon^2\chi\Delta \sum_{k=0}^{\infty} \epsilon^k \left[T_k(x) + \epsilon^2 R_k(x,\tau) + \epsilon W_k(\psi,\rho) + \epsilon^2 Z_k(\psi,\rho,\tau)\right]$$

$$+\epsilon^2 \left(\sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x,\tau) + w_k(\psi,\rho) + z_k(\psi,\rho,\tau)\right], \right. \tag{3.6}$$

$$\left. \nabla \sum_{k=0}^{\infty} \epsilon^k \left[T_k(x) + \epsilon^2 R_k(x,\tau) + \epsilon W_k(\psi,\rho) + \epsilon^2 Z_k(\psi,\rho,\tau)\right]\right)$$

$$= \epsilon^2 \sum_{0\le|k|\le m} f_{2k}\left(x, v(x,\tau), \frac{\partial v(x,\tau)}{\partial x}, T(x,\tau), \frac{\partial T(x,\tau)}{\partial x}\right)\ exp(ik\tau),$$

$$div\left(\sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x,\tau) + w_k(\psi,\rho) + z_k(\psi,\rho,\tau)\right]\right) = 0, \tag{3.7}$$

$$\sum_{k=0}^{\infty} \epsilon^k \left[v_k(x) + u_k(x,\tau) + w_k(\psi,\rho) + z_k(\psi,\rho,\tau)\right]\bigg|_{\partial\Omega} = 0, \tag{3.8}$$

$$\sum_{k=0}^{\infty} \epsilon^k \left[T_k(x) + \epsilon^2 R_k(x,\tau) + \epsilon W_k(\psi,\rho) + \epsilon^2 Z_k(\psi,\rho,\tau)\right]\bigg|_{\partial\Omega} = h(x). \tag{3.9}$$

Let us expand vector-functions f_{ik}, where $v, \frac{\partial v}{\partial x}, T, \frac{\partial T}{\partial x}$ are represented by series (2.10), in Taylor series with $(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x})$ as the center, and equate the coefficients at the same degrees of ϵ separately for regular and boundary layer vector-functions. Applying averaging in τ, we split the obtained equalities into problems for stationary and non-stationary vector-functions.

From equations (3.5), (3.7), by taking into account equalities (3.3), (3.1) and (3.4), we find first of all that

$$\begin{aligned} &\frac{\partial w_0^{(3)}}{\partial \rho} = 0, \frac{\partial z_0^{(3)}}{\partial \rho} = 0,\\ &\frac{\partial^2 w_0^{(s)}}{\partial \rho^2} = 0, s = 1, 2,\\ &w_0|_{\rho=\infty} = z_0|_{\rho=\infty} = 0, < z_0^{(3)} >= 0. \end{aligned} \tag{3.10}$$

For the first oscillating regular coefficients in equations (3.5), (3.6) we find that

$$
\begin{aligned}
&\frac{\partial u_0}{\partial \tau} + \nabla s_0 = \sum_{0<|k|\le m} a_k T_0\ exp(ik\tau), div\ u_0 = 0,\ u_0^{(3)}\Big|_{\partial\Omega} = -z_0^{(3)}\Big|_{\partial\Omega},\\
&\frac{\partial R_0}{\partial \tau} + (u_0, \nabla T_0) = \sum_{0\le|k|\le m}\left[f_{2k}\left(x, v_0+u_0, \frac{\partial(v_0+u_0)}{\partial x}, T_0, \frac{\partial T_0}{\partial x}\right)\ exp(ik\tau)\right.\\
&\left. - < f_{2k}\left(x, v_0+u_0, \tfrac{\partial(v_0+u_0)}{\partial x}, T_0, \tfrac{\partial T_0}{\partial x}\right)\ exp(ik\tau) >\right] \qquad (3.11)\\
&= \sum_{0\le|k|\le m}\left[\varphi_{2k}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) - < \varphi_{2k}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) >\right],\\
&< R_0 >=< u_0 >=< s_0 >= 0.
\end{aligned}
$$

The last representation takes into account equality (3.14) which follows from the first equation (3.11) (see below).

We have furthermore

$$
\begin{aligned}
&\frac{\partial z_0^{(s)}}{\partial \tau} - \nu\frac{\partial^2 z_0^{(s)}}{\partial \rho^2} = 0, < z_0^{(s)} >= 0,\ z_0^{(s)}\Big|_{\partial\Omega} = -u_0^{(s)}\Big|_{\partial\Omega}, s = 1, 2,\\
&\frac{\partial^2 W_0}{\partial \rho^2} = -\chi^{-1}\sum_{0\le|k|\le m}\sum_{l=0}^{2}\sum_{j=1}^{3} < \frac{\partial f_{2k}}{\partial w_{j+3l}}\left(x, u_0+v_0, \frac{\partial(u_0+v_0)}{\partial x}, T_0, \frac{\partial T_0}{\partial x}\right)\Big|_{r=0}\ exp(ik\tau)\\
&\times\frac{\partial(w_{0,l+1}+z_{0,l+1})}{\partial \rho} b_{j0} >\equiv -\chi^{-1} < A >,\\
&\frac{\partial g_0}{\partial \rho} = -\frac{\partial z_0^{(3)}}{\partial \tau}, < g_0 >= 0,\\
&\frac{\partial Z_0}{\partial \tau} - \chi\frac{\partial^2 Z_0}{\partial \rho^2} = A- < A >, < Z_0 >= 0,\ Z_0|_{\partial\Omega} = 0,\\
&W_0|_{\rho=\infty} = Z_0|_{\rho=\infty} = z_0|_{\rho=\infty} = g_0|_{\rho=\infty} = 0.
\end{aligned} \qquad (3.12)
$$

We find for main stationary regular coefficients (2.10)

$$
\begin{aligned}
&-\nu\Delta v_0 + \nabla p_0 + (v_0, \nabla)v_0 + < (u_0, \nabla)u_0 >\\
&= \sum_{0\le|k|\le m} < \varphi_{1k}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) > + \left\langle \sum_{0<|k|\le m} a_k R_0\ exp(ik\tau)\right\rangle,\\
&div\ v_0 = 0, \qquad (3.13)\\
&-\chi\Delta T_0 + (v_0, \nabla T_0) = \sum_{0\le|k|\le m} < \varphi_{2k}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) >,\\
&v_0|_{\partial\Omega} = -w_0|_{\partial\Omega},\ T_0|_{\partial\Omega} = h.
\end{aligned}
$$

Following [9], we consider all boundary layer vector-functions vanishing out of the boundary strip $\Omega_{2\eta/3}$, and in Ω they are multiplied by the cutting-off function $\chi \in C^\infty(\Omega)$:

$$\chi(x) = \begin{cases} 1, 0 \le r \le \eta/3, \\ 0, x \in \Omega \setminus \Omega_{2\eta/3}. \end{cases}$$

From (3.10) we find that $w_0 = 0, z_0^{(3)} = 0$. From the first equality (3.11) we have

$$u_0 = \sum_{0<|k|\le m} (ik)^{-1} \Pi a_k T_0 \; exp(ik\tau), \tag{3.14}$$

$$\Delta s_0 = div \sum_{0<|k|\le m} a_k T_0 \; exp(ik\omega t), \tag{3.15}$$

$$\left.\frac{\partial s_0}{\partial n}\right|_{\partial\Omega} = \sum_{0<|k|\le m} (a_k T_0)^{(3)} \;\; exp(ik\omega t)|_{\partial\Omega}, < s_0 >= 0. \tag{3.16}$$

From the second equality (3.11) and (3.14) we find

$$R_0 = \Psi\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) + \sum_{0<|k|\le m} k^{-2} (\Pi a_k T_0, \nabla T_0) \; exp(ik\tau). \tag{3.17}$$

Let us replace u_0 and R_0 in system (3.13) by their expressions (3.14), (3.17) found just now. So, we obtain problem (2.3) with $u = v_0, q = p_0, W = T_0$ as the unknowns. It is solvable by the hypothesis.

By the mathematical induction we prove that the construction of each coefficient of series (2.10) is reduced to solving a finite number of problems of types (A1)-(A5). Let for $n \ge 1$ and for all $k < n$ the inductive assumption be satisfied for the set of all coeffitients P_k: $v_k, u_k, p_k, u_k, s_k, w_k, z_k, h_{k-1}, g_k, T_k, R_k, W_k, Z_k$. By using (3.1)-(3.9), let us write out the problems for the set of coefficients P_n.

According to (3.7), (3.5), (3.1)-(3.4) and the inductive assumption, we have

$$\frac{\partial w_n^{(3)}}{\partial \rho} = \alpha_n(\psi, \rho), \; w_n^{(3)}\Big|_{\rho=\infty} = 0, \tag{3.18}$$

$$\frac{\partial^2 w_n^{(s)}}{\partial \rho^2} = \beta_{n,s}(\psi, \rho), \; w_n^{(s)}\Big|_{\rho=\infty} = 0, s = 1, 2, \tag{3.19}$$

$$\frac{\partial z_n^{(3)}}{\partial \rho} = \lambda_n(\psi, \rho, \tau), \; z_n^{(3)}\Big|_{\rho=\infty} = 0, < z_n^{(3)} >= 0. \tag{3.20}$$

Here $\alpha_n, \beta_{n,s}, \lambda_n$ are the known at this step boundary layer functions, and λ_n is quickly oscillating in τ having zero average in τ. We note that in this case α_n, β_{ns} have the form $\sum_{s=1}^{k} F_s(\psi, \rho) \; exp(\lambda_s \rho)$ and λ_n the form $\sum_{r=1}^{p} \sum_{s=1}^{k} F_s(\psi, \rho) \; exp(\lambda_s \rho) \; exp(ir\omega t)$. Here F_s and λ_s are of the same nature as F and λ in problems (A3)-(A5).

Now for regular nonstationary vector-functions we find

$$\frac{\partial u_n}{\partial \tau} + \nabla s_n - \sum_{0<|k|\le m} a_k T_n \, exp(ik\tau) = f_n(x,\tau), div \; u_n = 0, \; u_n^{(3)}\big|_{\partial\Omega} = -z_n^{(3)}\big|_{\partial\Omega}, \quad (3.21)$$

$$\frac{\partial R_n}{\partial \tau} + (u_0, \nabla T_n) + (u_n, \nabla T_0) - \Bigg[\sum_{0\le|k|\le m} \Bigg\{ \frac{\partial \varphi_{2k}}{\partial u}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) v_n$$
$$+\frac{\partial \varphi_{2k}}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v_n}{\partial x} + \frac{\partial \varphi_{2k}}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) T_n \quad (3.22)$$
$$+\frac{\partial \varphi_{2k}}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T_n}{\partial x}\Bigg\} - < ... > \Bigg] = \varphi_n(x,\tau),$$
$$< u_n > = < R_n > = < s_n > = 0.$$

The known vector-functions $f_n(x,\tau), \varphi_n(x,\tau)$ having zero average in τ are in the right-hand sides of equalities (3.21), (3.22).

Once again for the boundary layer coefficients we find

$$\chi \frac{\partial h_n}{\partial \rho} = \delta_n(\psi,\rho), \; h_n|_{\rho=\infty} = 0, \quad (3.23)$$

$$\frac{\partial z_n^{(s)}}{\partial \tau} - \nu \frac{\partial^2 z_n^{(s)}}{\partial \rho^2} = \mu_{n,s}(\psi,\rho,\tau), \; z_n^{(s)}\big|_{\partial\Omega} = -\, u_n^{(s)}\big|_{\partial\Omega}, \, s = 1,2, \quad (3.24)$$

$$\chi \frac{\partial^2 W_n}{\partial \rho^2} = \gamma_n(\psi,\rho), \; W_n|_{\rho=\infty} = 0, \quad (3.25)$$

$$\frac{\partial g_n}{\partial \rho} = \xi_n(\psi,\rho,\tau), \quad (3.26)$$

$$\frac{\partial Z_n}{\partial \tau} - \chi \frac{\partial^2 Z_n}{\partial \rho^2} = \upsilon_n(\psi,\rho,\tau), \; Z_n|_{\partial\Omega} = -R_{n-1}|_{\partial\Omega},$$
$$< Z_n > = < z_n^{(s)} > = < g_n > = 0, \; W_n|_{\rho=\infty} = Z_n|_{\rho=\infty} = z_n|_{\rho=\infty} = g_n|_{\rho=\infty} = 0. \quad (3.27)$$

Here $\upsilon_n, \mu_{n,s}, \xi_n, \gamma_n, \delta_n$ are the known boundary layer vector-functions, having the above noted structure after formula (3.20).

For the regular coeffitients we have further

$$-\nu\Delta v_n+\nabla p_n+(v_0,\nabla)v_n+(v_n,\nabla)v_0+<(u_0,\nabla)u_n>+<(u_n,\nabla)u_0>$$

$$-\left\langle\sum_{0<|k|\le m}a_kR_n\ exp(ik\tau)\right\rangle-\sum_{0\le|k|\le m}\left[<\frac{\partial\varphi_{1k}}{\partial u}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)v_n\right.$$

$$+\frac{\partial\varphi_{1k}}{\partial w}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial v_n}{\partial x}+\frac{\partial\varphi_{1k}}{\partial T}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)T_n$$

$$\left.+\frac{\partial\varphi_{1k}}{\partial S}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial T_n}{\partial x}>\right]=\psi_n(x),div\ v_n=0,$$

$$-\chi\Delta T_n+(v_0,\nabla T_n)+(v_n,\nabla T_0) \tag{3.28}$$

$$-\sum_{0\le|k|\le m}\left[<\frac{\partial\varphi_{2k}}{\partial u}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)v_n\right.$$

$$+\frac{\partial\varphi_{2k}}{\partial w}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial v_n}{\partial x}+\frac{\partial\varphi_{2k}}{\partial T}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)T_n$$

$$\left.+\frac{\partial\varphi_{2k}}{\partial S}\left(x,v_0,\frac{\partial v_0}{\partial x},T_0,\frac{\partial T_0}{\partial x},\tau\right)\frac{\partial T_n}{\partial x}>\right]=\chi_n(x),$$

$$v_n|_{\partial\Omega}=-\left.w_n\right|_{\partial\Omega},\left.T_n\right|_{\partial\Omega}=-\left.W_n\right|_{\partial\Omega},$$

where $\psi_n(x),\chi_n(x)$ are the known regular vector-functions.

We find w_n from (3.18), (3.19), and W_n from (3.25). Now after excluding the variable τ from (3.20), (3.27) we can uniquely define $z_n^{(3)}$. Let $(\overset{0}{u}_n,\overset{0}{s}_n)$ be the solution of the problem

$$\frac{\partial u}{\partial\tau}+\nabla s=f_n,div\ u=0,\left.u^{(3)}\right|_{\partial\Omega}=-\left.z_n^{(3)}\right|_{\partial\Omega},<u>=<s>=0, \tag{3.29}$$

which is solvable due to the equality

$$\int_{\partial\Omega}z_n^{(3)}ds=0 \tag{3.30}$$

(see [4, Lemma 4]).

Next by (3.21)

$$u_n=\sum_{0<|k|\le m}(ik)^{-1}\Pi a_kT_n\ exp(ik\tau)+\overset{0}{u}_n\ . \tag{3.31}$$

Substituting expression (3.31) for u_0, u_n in (3.22), we find

$$R_n = \sum_{0<|k|\le m} [k^{-2}\{(\Pi a_k T_n, \nabla T_0) + (\Pi a_k T_0, \nabla T_n)\}\ exp(ik\tau)]$$
$$+\frac{\partial \Psi}{\partial u}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) v_n + \frac{\partial \Psi}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v_n}{\partial x}$$
$$+\frac{\partial \Psi}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) T_n \tag{3.32}$$
$$+\frac{\partial \Psi}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T_n}{\partial x} + d_n(x,\tau),$$

where $d_n(x,\tau)$ is the known function. Let us substitute expressions (3.31), (3.32) for u_n, R_n in (3.28). We get the problem

$$-\nu\Delta v_n + \nabla p_n + (v_0,\nabla)v_n + (v_n,\nabla)v_0$$
$$+\sum_{0<|k|\le m} k^{-2}(\Pi a_k T_0,\nabla)\Pi a_{-k}T_n + \sum_{0<|k|\le m} k^{-2}(\Pi a_k T_n,\nabla)\Pi a_{-k}T_0$$
$$-\sum_{0<|k|\le m} k^{-2}a_k(\Pi a_{-k}T_0,\nabla T_n) - \sum_{0<|k|\le m} k^{-2}a_k(\Pi a_{-k}T_n,\nabla T_0)$$
$$-\sum_{0<|k|\le m} < a_k\left[\frac{\partial \Psi}{\partial u}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) v_n\right.$$
$$+\frac{\partial \Psi}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v_n}{\partial x} + \frac{\partial \Psi}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) T_n$$
$$\left.+\frac{\partial \Psi}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T_n}{\partial x}\right]\ exp(ik\tau) >$$

$$-\sum_{0\le|k|\le m}\left[< \frac{\partial \varphi_{1k}}{\partial u}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) v_n\right.$$
$$+\frac{\partial \varphi_{1k}}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v_n}{\partial x} + \frac{\partial \varphi_{1k}}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) T_n \tag{3.33}$$
$$\left.+\frac{\partial \varphi_{1k}}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T_n}{\partial x} >\right] = l_n(x),$$

$$div\ v_n = 0,$$
$$-\chi\Delta T_n + (v_0,\nabla T_n) + (v_n,\nabla T_0)$$
$$-\sum_{0\le|k|\le m}\left[< \frac{\partial \varphi_{2k}}{\partial u}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right) v_n + \right.$$

$$+\frac{\partial \varphi_{2k}}{\partial w}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial v_n}{\partial x} + \frac{\partial \varphi_{2k}}{\partial T}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)T_n +$$
$$+\frac{\partial \varphi_{2k}}{\partial S}\left(x, v_0, \frac{\partial v_0}{\partial x}, T_0, \frac{\partial T_0}{\partial x}, \tau\right)\frac{\partial T_n}{\partial x} >\Bigg] = \chi_n(x),$$
$$v_n|_{\partial\Omega} = -w_n|_{\partial\Omega}\,,\ T_n|_{\partial\Omega} = -W_n|_{\partial\Omega}\,.$$

In view of the equality

$$\int\limits_{\partial\Omega} w_n^{(3)} ds = 0 \tag{3.34}$$

[4, Lemma 4] the heterogeneity in boundary conditions is removed. So, one can construct infinitely smooth couple $(\overset{0}{v}_n, \overset{0}{T}_n)$, which satisfies the boundary conditions of problem (3.33), and the condition $div\ \overset{0}{v}_n = 0$. As a result we get the problem like (2.4), which is solvable by the hypothesis. We find functions $Z_n, z_n^{(s)}, s = 1, 2, g_n, h_n$ from equations (3.23), (3.24), (3.26), (3.27). It is obvious now that our induction assumption is satisfied for the set of coeffitients P_n. Thus construction of the formal asymthotic expansion of the solution of problem (2.1) is completed. Its justification and the proof of the second statement of Theorem 2.1 can be carried out as in [5]. □

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