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A SUZUKI TYPE FIXED POINT THEOREM FOR A HYBRID PAIR OF MAPS IN PARTIAL HAUSDORFF METRIC SPACES

K.P.R. Rao, K.R.K. Rao

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Abstract. In this paper, we introduce the notion of (θ, L) generalized weak contraction for a hybrid pair of mappings in a partial metric space by using partial Hausdorff metric. The main result of the paper generalizes the main theorem of H. Aydi et al [6] .

1 Introduction and preliminaries

There are a lot of generalizations of the Banach fixed point principle in the literature. One of the most interesting generalizations is that given by T. Suzuki [33]. This interesting fixed point result is the following:

Theorem 1.1. ([33]) *Let (X, d) be a complete metric space, let T be a mapping on X, and let a non-increasing function θ from [0,1) into $(\frac{1}{2}, 1]$ be defined by*

$$\theta(r) = \begin{cases} 1\ , & 0 \le r \le \frac{\sqrt{5}-1}{2}, \\ \frac{1-r}{r^2}\ , & \frac{\sqrt{5}-1}{2} \le r \le \frac{1}{\sqrt{2}}, \\ \frac{1}{1+r}\ , & \frac{1}{\sqrt{2}} \le r < 1. \end{cases}$$

Assume that $r \in [0, 1)$ is such that

$$\theta(r) d(x, Tx) \le d(x, y) \quad \text{implies} \quad d(Tx, Ty) \le r\ d(x, y)$$

for all $x, y \in X$.

Then there exists a unique fixed point z of T. Moreover, $\lim_{n\to\infty} T^n x = z$ for all $x \in X$.

This result has lead to some important contributions in the metric fixed point theory (see for instance [26, 31, 32, 33, 34]).

S.B. Nadler [24] proved the following multi-valued extension of the Banach contraction theorem.

Theorem 1.2. ([24]) *Let (X, d) be a complete metric space and $T : X \to CB(X)$ be a mapping satisfying $H(Tx, Ty) \le k\ d(x, y)$ for all $x, y \in X$, where $k \in [0, 1)$. Then there exists $x \in X$ such that $x \in Tx$.*

Later an interesting and rich fixed point theory was developed and Theorem 1.2 was extended by using weak and generalized contraction mappings (see [13, 30, 22, 12]). The theory of multi-valued maps has application in control theory, convex optimization, differential equations and economics (see also [13]). The notion of a partial metric space was introduced by S.G. Mathews [23], as a part of the study of denotational semantics of data flow networks. Recently many authors proved some fixed point theorems for a one, two and four mappings for weak and generlized contractions in partial metric spaces, see, for example, [29, 9, 10, 25, 15, 16, 17, 18, 19, 20, 21, 7, 8, 4, 5, 27, 28, 11, 26, 1, 2, 3].

Very recently H. Aydi et al. [6] generalized the Hausdorff metric by introducing the partial Hausdorff metric in a partial metric space and extended Nadler's fixed point theorem as follows.

Theorem 1.3. ([6]) *Let (X,p) be a complete partial metric spce and $T : X \to CB^p(X)$ be a multi-valued mapping such that for all $x, y \in X$, we have $H_p(Tx,Ty) \le kp(x,y)$ where $k \in (0,1)$, then T has a fixed point.*

In this paper we consider the generalized (θ, L) weak contraction for a hybrid pair of maps to obtain a Suziki type fixed point theorem in partial metric spaces which generalizes the theorem of H. Aydi et al. [6].

Consistent with [14, 6, 4, 23], now we consider the following definitions and results which are needed in the sequel.

Definition 1.1. ([23]). A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p_2) $p(x,x) \le p(x,y)$

(p_3) $p(x,y) = p(y,x)$,

(p_4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.

In this case (X,p) is called a partial metric space.

It is clear that $|p(x,y) - p(y,z)| \le p(x,z)\ \forall x, y, z \in X$. Also it is clear that $p(x,y) = 0$ implies $x = y$ by (p_1) and (p_2). But if $x = y$, $p(x,y)$ may not be zero.

A basic example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p(x,y) = \max\{x,y\}$ for all $x, y \in \mathbb{R}^+$.

Each partial metric p on X generates the topology τ_p on X which has as a base the family of open p - balls $\{B_p(x,\epsilon) \mid x \in X,\ \epsilon > 0\}$ for all $x \in X$ and $\epsilon > 0$, where $B_p(x,\epsilon) = \{y \in X \mid p(x,y) < p(x,x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

If p is a partial metric on X, then the function $d_p : X \times X \to \mathbb{R}^+$ given by $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ is a metric on X.

Definition 1.2. ([23]) Let (X,p) be a partial metric space.

(i) A sequence $\{x_n\}$ in (X,p) is said to converge to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) A sequence $\{x_n\}$ in (X,p) is said to be a Cauchy sequence if $\lim_{n\to\infty} p(x_n,x_m)$ exists and is finite.

(iii) (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that

$$p(x, x) = \lim_{m,n\to\infty} p(x_n, x_m).$$

Lemma 1.1. ([23]). *Let (X, p) be a partial metric space.*

(a) $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p).

(b) (X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore,
$\lim_{n\to\infty} d_p(x_n, x) = 0$ if and only if $p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{m,n\to\infty} p(x_n, x_m)$.

Lemma 1.2. ([4]). *Let (X, p) be a partial metric space and A any nonempty set in (X, p), then $a \in \overline{A}$ if and only if $p(a, A) = p(a, a)$, where $\overline{A}$ denotes the closure of A with respect to the partial metric p.*

Note that A is closed in (X, p) if and only if $A = \overline{A}$.

In [6], H. Aydi et al. introduced the following definitions.

Let (X, p) be a partial metric space. Let $CB^p(X)$ be the family of all nonempty, closed and bounded subsets of the partial metric space (X, p), induced by the partial metric p. For $A, B \in CB^p(X)$ and $x \in X$, define

$$p(x, A) = \inf\{p(x, a) : a \in A\}, \quad \delta_p(A, B) = \sup\{p(a, B) : a \in A\},$$

$$\delta_p(B, A) = \sup\{p(b, A) : b \in B\}$$

and

$$H_p(A, B) = \max\{\delta_p(A, B), \quad \delta_p(B, A)\}.$$

H_p is called the partial Hausdorff metric induced by the partial metric p.

H. Aydi et al. proved that any Hausdorff metric is a partial Hausdorff metric. The converse is not true (see Remark 2.7 in [6]).

Lemma 1.3. ([6]). *Let (X, p) be a partial metric space. For any $A, B, C \in CB^p(X)$, we have*
(i) $\delta_p(A, A) = \sup\{p(a, a) : a \in A\}$,
(ii) $\delta_p(A, A) \le \delta_p(A, B)$,
(iii) $\delta_p(A, B) = 0$ *implies that* $A \subseteq B$,
(iv) $\delta_p(A, B) \le \delta_p(A, C) + \delta_p(C, B) - \inf_{c\in C} p(c, c)$.

Lemma 1.4. ([6]). *Let (X, p) be a partial metric space. For, any $A, B, C \in CB^p(X)$, we have*
(i) $H_p(A, A) \le H_p(A, B)$,
(ii) $H_p(A, B) = H_p(B, A)$,
(iii) $H_p(A, B) \le H_p(A, C) + H_p(C, B) - \inf_{c\in C} p(c, c)$.

Lemma 1.5. ([6]). *Let (X,p) be a partial metric space. For, any $A,B \in CB^p(X)$, $H_p(A,B)=0$ implies that $A=B$.*

Remark 1.1. The converse of Lemma 1.5, in general, is not true as the following example shows.

Let $X=[0,1]$ be endowed with the partial metric $p: X \times X \to R^+$ defined by $p(x,y)=\max\{x,y\}$. By (i) of Lemma 1.3, we have

$$H_p(X,X)=\delta_p(X,X)=sup\{x: 0 \leq x \leq 1\}=1 \neq 0.$$

Lemma 1.6. ([6]). *Let (X,p) be a partial metric space, $A,B \in CB^p(X)$ and $h>1$. For any $a \in A$, there exists $b \in B$ such that $p(a,b) \leq hH_p(A,B)$.*

Definition 1.3. ([14]). Mappings $f: X \to X$ and $T: X \to CB(X)$ are said to be weakly compatible if they commute at their coincidence points, i.e., if $f(Tx)=T(fx)$ whenever $fx \in Tx$.

2 Main results

We start with the following lemma.
Lemma 2.1. Let $x_n \to x$ as $n \to \infty$ in a partial metric space (X,p) such that $p(x,x)=0$. Then $\lim\limits_{n\to\infty} p(x_n,B)=p(x,B)$ for any $B \in CB^p(X)$.

Proof. Since $x_n \to x$ we have $\lim\limits_{n\to\infty} p(x_n,x)=p(x,x)=0$. By the triangle inequality for $x_n \in X$ and $y \in B$ we have

$$p(x_n,y) \leq p(x_n,x)+p(x,y)-p(x,x)=p(x_n,x)+p(x,y)$$

which gives that $p(x_n,B) \leq p(x_n,x)+p(x,B)$.
Therefore

$$\lim_{n\to\infty} p(x_n,B) \leq p(x,B). \tag{2.1}$$

Also

$$p(x,y) \leq p(x,x_n)+p(x_n,y)-p(x_n,x_n) \leq p(x,x_n)+p(x_n,y).$$

So $p(x,B) \leq p(x,x_n)+p(x_n,B)$. Therefore

$$p(x,B) \leq \lim_{n\to\infty} p(x_n,B). \tag{2.2}$$

From (2.1) and (2.2) we have $\lim\limits_{n\to\infty} p(x_n,B)=p(x,B)$. □

Now, we give our main result.

Theorem 2.1. *Let (X,p) be a partial metric space and let $T: X \to CB^p(X)$ and $f: X \to X$ be mappings satisfying the $\eta(\theta)p(fx,Tx) \leq p(fx,fy)$ implies*

$$H_p(Tx,Ty) \leq \theta p(fx,fy)+L[p(fy,Tx)-p(fy,fy)-H_p(Tx,Tx)]$$

where $\theta \in [0,1)$, $L \geq 0$ *for all* $x, y \in X$ *and* $\eta : [0,1) \to (\frac{1}{2+L}, \frac{1}{1+L}]$ *defined by* $\eta(\theta) = \frac{1}{1+\theta+L}$ *is strictly decreasing function. Also let* $T(X) \subset f(X)$ *and* $f(X)$ *be complete. Then* f *and* T *have a coincidence point.*

Furthermore, if T *and* f *are weakly compatible and* $f(f(u)) = f(u)$, *then* f *and* T *have a common fixed point.*

Proof. Choose $q > 1$ be such that $h = q\,\theta < 1$. Let $x_0 \in X$ and $x_1 \in X$ such that $fx_1 \in Tx_0$. Then

$$\eta(\theta)p(fx_0, Tx_0) \leq \eta(\theta)p(fx_0, fx_1) \leq p(fx_0, fx_1).$$

Hence from given hypothesis we have

$$\begin{aligned} H_p(Tx_0, Tx_1) &\leq \theta p(fx_0, fx_1) + L[p(fx_1, Tx_0) - p(fx_1, fx_1) - H_p(Tx_0, Tx_0)] \\ &\leq \theta p(fx_0, fx_1) + L[p(fx_1, fx_1) - p(fx_1, fx_1)] \\ &= \theta p(fx_0, fx_1). \end{aligned}$$

But by Lemma 1.6, there exists $fx_2 \in Tx_1$ such that

$$p(fx_1, fx_2) \leq qH_p(Tx_0, Tx_1).$$

So

$$p(fx_1, fx_2) \leq q\theta\ p(fx_0, fx_1).$$

Thus

$$p(fx_1, fx_2) \leq hp(fx_0, fx_1).$$

Now we have,

$$\eta(\theta)p(fx_1, Tx_1) \leq \eta(\theta)p(fx_1, fx_2) \leq p(fx_1, fx_2)$$

So by the assumptions of the theorem, we have

$$\begin{aligned} H_p(Tx_1, Tx_2) &\leq \theta p(fx_1, fx_2) + L[p(fx_2, Tx_1) - p(fx_2, fx_2) - H_p(Tx_1, Tx_1)] \\ &\leq \theta p(fx_1, fx_2) + L[p(fx_2, fx_2) - p(fx_2, fx_2)] \\ &= \theta p(fx_1, fx_2) \end{aligned}$$

Again by using Lemma 1.6, there exists $fx_3 \in Tx_2$ such that

$$p(fx_2, fx_3) \leq qH_p(Tx_1, Tx_2) \leq q\theta p(fx_1, fx_2).$$

Hence we have

$$p(fx_2, fx_3) \leq hp(fx_1, fx_2) \leq h^2 p(fx_0, fx_1).$$

Proceeding in this way we can obtain a sequence $\{fx_n\}$ in X such that

$$p(fx_n, fx_{n+1}) \leq h^n p(fx_0, fx_1).$$

If $fx_n = fx_{n+1}$ for some n, then $fx_n \in Tx_n$ hence x_n is a coincidence point of T and f. Assume that $fx_n \neq fx_{n+1}$ for all n. By Property (p_4) of a partial metric space for any $n > m$ we have

$$\begin{aligned} p(fx_n, fx_m) &\leq p(fx_n, fx_{n+1}) + p(fx_{n+1}, fx_{n+2}) + ... + p(fx_{m-1}, fx_m) \\ &\leq h^n p(fx_0, fx_1) + h^{n+1} p(fx_0, fx_1) + ... + h^{m-1} p(fx_0, fx_1) \\ &= (h^n + h^{n+1} + ... + h^{m-1}) p(fx_0, fx_1) \\ &\leq \tfrac{h^n}{1-h} p(fx_0, fx_1) \to 0 \end{aligned}$$

as $n \to \infty$ since $h < 1$.

Thus $\lim_{n,\ m\to\infty} p(fx_n, fx_m) = 0$ hence by (p_2), we have

$$\lim_{n\to\infty} p(fx_n, fx_n) = 0. \tag{2.3}$$

By the definition of d_p, for any $n > m$ we get

$$d_p(fx_n, fx_m) \le 2p(fx_n, fx_m) \to 0 \quad \text{as } n \to \infty.$$

This yields that $\{fx_n\}$ is a Cauchy sequence in $(f(X), d_p)$.

Since $(f(X), p)$ is complete, by (b) of Lemma 1.1, we have $(f(X), d_p)$ that is a complete metric space. Therefore the sequence $\{fx_n\}$ converges to some $f(u) \in f(X)$ with respect to the metric d_p, that is, $\lim_{n\to\infty} d_p(fx_n, f(u)) = 0$.
Also by (b) of Lemma 1.1, we have

$$p(f(u), f(u)) = \lim_{n\to\infty} p(fx_n, f(u)) = \lim_{n,m\to\infty} p(fx_n, fx_m) = 0. \tag{2.4}$$

Since $fx_n \to f(u)$, $fx_n \ne fx_{n+1}$ for all n, it follows that $fx_n \ne f(u)$ for sufficiently large n.

So by (2.4), there exists a positive integer n_0 such that

$$p(f(u), fx_n) \le \frac{1}{3} p(f(u), fx)$$

for all $n \ge n_0$ and for all $x \in X - \{u\}$.

Now we have

$$\begin{aligned} \eta(\theta)p(fx_n, Tx_n) &\le p(fx_n, Tx_n) \le p(fx_n, fx_{n+1}) \\ &\le p(fx_n, f(u)) + p(f(u), fx_{n+1}) - p(f(u), f(u)). \end{aligned}$$

So

$$\begin{aligned} \eta(\theta)p(fx_n, Tx_n) &\le \tfrac{1}{3}p(f(u), fx) + \tfrac{1}{3}p(f(u), fx) = \tfrac{2}{3}p(f(u), fx) \\ &= p(f(u), fx) - \tfrac{1}{3}p(f(u), fx) \\ &\le p(fx, f(u)) - p(fx_n, f(u)) \\ &\le p(fx, fx_n) + p(fx_n, f(u)) - p(fx_n, fx_n) - p(fx_n, f(u)) \\ &\le p(fx_n, fx) \end{aligned}$$

which implies that

$$\begin{aligned} p(fx_{n+1}, Tx) &\le H_p(Tx_n, Tx) \\ &\le \theta p(fx_n, fx) + L[p(fx, Tx_n) - p(fx, fx) - H_p(Tx_n, Tx_n)] \\ &\le \theta p(fx_n, fx) + Lp(fx, Tx_n) \\ &\le \theta p(fx_n, fx) + Lp(fx, fx_{n+1}). \end{aligned}$$

Letting $n \to \infty$ by Lemma 2.1, we get

$$p(fu, Tx) \le \theta p(fx, fu) + Lp(fx, fu) = (\theta + L)p(fx, fu) \tag{2.5}$$

Since $p(fu, Tx) = \inf_{y\in Tx} p(fu, y)$ we have for every $n \in N$, there exists
$y_n \in Tx$ such that $p(fu, y_n) < p(fu, Tx) + \frac{1}{n}p(fx, fu)$.

Now consider,

$$\begin{aligned} p(fx,Tx) &\leq p(fx,y_n) \leq p(fx,fu)+p(fu,y_n)-p(fu,fu) \\ &\leq p(fx,fu)+p(fu,Tx)+\tfrac{1}{n}p(fx,fu) \\ &\leq p(fx,fu)+(\theta+L)p(fx,fu)+\tfrac{1}{n}p(fx,fu) \quad \text{by (2.5)} \\ &= (1+\theta+L+\tfrac{1}{n})p(fx,fu). \end{aligned}$$

This implies

$$\frac{1}{1+\theta+L}p(fx,Tx) \leq \left[1+\frac{1}{n(1+\theta+L)}\right]p(fx,fu).$$

Letting $n \to \infty$ we get

$$\eta(\theta)p(fx,Tx) \leq p(fx,fu).$$

Then by the assumptions of the theorem we have,

$$H_p(Tx,Tz) \leq \theta p(fx,fu)+L[p(fu,Tx)-p(fu,fu)-H_p(Tx,Tx)]$$

Thus

$$H_p(Tx,Tz) \leq \theta p(fx,fu)+Lp(fu,Tx) \tag{2.6}$$

Now by Lemma 2.1, we have

$$\begin{aligned} p(f(u),Tu) &= \lim_{n\to\infty} p(fx_{n+1},Tu) \leq \lim_{n\to\infty} H_p(Tx_n,Tz) \\ &\leq \lim_{n\to\infty} [\theta p(fx_n,fu)+Lp(Tx_n,fu)], \quad \text{by (2.6)} \\ &\leq \lim_{n\to\infty} [\theta p(fx_n,fu)+Lp(fx_{n+1},fu)] \\ &= 0. \end{aligned}$$

So we have $p(fu,Tu)=p(fu,fu)=0$. By Lemma 1.2, we have $f(u) \in \overline{Tu} = Tu$, since Tu is closed. So u is a coincidence point of f and T.

Suppose f and T are weakly compatible then we have $T(fu)=f(Tu)$. Also by the assumptions of the theorem we have $f(fu)=fu$. So $f(fu) \in f(Tu) = T(fu)$ i.e., $fu \in T(fu)$. Hence fu is a common fixed point of f and T. □

If f is an identity map in Theorem 2.1, we have the following corollary.

Corollary 2.1. *Let (X,p) be a complete partial metric space and let $T: X \to CB^p(X)$ be a mapping satisfying*

$$\eta(\theta)p(x,Tx) \leq p(x,y) \Rightarrow H_p(Tx,Ty) \leq \theta p(x,y)+L[p(y,Tx)-p(y,y)-H_p(Tx,Tx)],$$

where $\theta \in [0,1)$, $L \geq 0$ for all $x,y \in X$, and $\eta : [0,1) \to (\frac{1}{2+L},\frac{1}{1+L}]$ defined by $\eta(\theta) = \frac{1}{1+\theta+L}$ is a strictly decreasing function.

Then there exists a point $x \in X$ such that $x \in Tx$.

Corollary 2.2. *Let (X,p) be a complete partial metric space and let $T: X \to CB(X)$ be a mapping such that for all $x,y \in X$,*

$$H_p(Tx,Ty) \leq k\ p(x,y),$$

where $k \in [0,1)$.

Then there exists a point $x \in X$ such that $x \in Tx$.

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K.P.R. Rao
Department of Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510, A.P., India
E-mail: kprrao2004@yahoo.com.

K.R.K. Rao
Department of Mathematics
GITAM University
Hyderabad-502 329, A.P., India
E-mail: krk08@gmail.com.

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