ISSN 2077–9879

Eurasian Mathematical Journal

2014, Volume 5, Number 2

Founded in 2010 by
the L.N. Gumilyov Eurasian National University
in cooperation with
the M.V. Lomonosov Moscow State University
the Peoples' Friendship University of Russia
the University of Padua

Supported by the ISAAC
(International Society for Analysis, its Applications and Computation)
and
by the Kazakhstan Mathematical Society

Published by

the L.N. Gumilyov Eurasian National University
Astana, Kazakhstan

EURASIAN MATHEMATICAL JOURNAL

EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 5, Number 2 (2014), 52 – 59

MATHEMATICAL MODEL OF MULTIFRACTAL DYNAMICS AND GLOBAL WARMING

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Communicated by V.I. Burenkov

Key words: Fractal, multifractal dynamics, global warming, climate, global temperature.

AMS Mathematics Subject Classification: 28A80

Abstract. In this work the variations of global temperature that have occurred in the period from 1860 up to now are analyzed on the basis of the concept of multifractal dynamics. The multifractal curve describing dynamics of global temperature for this period of time has the following values of fractal dimensions over 5 periods lasting for 30-31 years each, accordingly: $D_1 = 1,140; D_2 = 1,166; D_3 = 1,141; D_4 = 1,203; D_5 = 1,183$. Such relatively small values of fractal dimensions are indicative of essentially determined character of processes responsible for variations of global temperature. Our predictive estimates provide $0,5^oC$ increase in global temperature by 2072, thereby confirming maintenance of the tendency of global warming in the near future.

1 Introduction

The meteorological data for the period from 1850 till now, let P. D. Jones and T.M.L. Wigley deduce that the Planet climate had warmed, and this change in temperature is $0,5^oC$[1]. Naturally, the question now arises of whether this conclusion is true. A more intricate question is the one of possibility of further warming. There are a lot of facts which undoubtedly have an effect on measurement data thereby causing an apparent warming effect.

Under these circumstances, mathematical climate modeling is of great importance.

Currently there are no models which describe with sufficient degree of accuracy complex atmospheric and ocean physics and which can prove that the greenhouse gas emission essentially affects the Earth climate fluctuation. In whole, temperature increase is in keeping with the previous industrial revolution in consequence of which the concentration of carbon dioxide and other greenhouse gases was essentially increased in the atmosphere.

About fifty years ago attempts to determine the Earth temperature trends were taken. But initially, this was impossible because of the fewness of the view points. Since 1850 all National Weather Services have been concordantly collecting and maintaining temperature data. Gradually, the weather surveillance network extended over the world, and by the end of 1950 it covered Antarctica. However, even at the present time in some regions especially in ocean areas having rugged depth contour, measurements are taken rather seldomly. However, the partial covering does not constitute a serious problem; it is compensated by satellite observations.

Notwithstanding the fact that the observational data indicates the temperature rise over the last 120 years, there are a lot of questions to decide. Namely, how considerable the warming trend is? What is the reason for it? Is it connected with the greenhouse effect?

2 Mathematical model of multi-fractal dynamics and catastrophes in this model

We shall summarize the elements of multi-fractal dynamics [2, 3].

Definition. Let $y(t)$ be a multi-fractal curve describing the dynamics of the quantity we are interested in and having the fractal dimension D_i on the intervals of time T_i $(i = 1, 2, 3 \dots n)$.

If the rate of change X_i of the linear trend $\overline{y}_i(t)$ approximating this function on the interval T_i with the required degree of accuracy, only depends on D_i then the given type of dynamics we will designate as multi-fractal.

In this case in [3] we offer the following approach: the multi-fractal process dynamics on the intervals T_i $(t_{0i} < t < t_{0i+1}, T_i = t_{0i+1} - t_{0i})$ can be divided into two components by means of using the idea of the linear trend:

$$y_i(t) = \overline{y}_i(t) + \tilde{y}_i(t) \tag{2.1}$$

where $\overline{y}_i(t)$ are linear trend of the process varying with time smoothly; $\tilde{y}_i(t)$ - are fast oscillations with respect to a trend. It is assumed that $|\overline{y}_i(t)| \gg |\tilde{y}_i(t)|$, and that the curve $y_i(t)$ is a multi-fractal one. The trend line $\overline{y}_i(t)$ has the fractal dimension, equal to 1, and $\tilde{y}_i(t)$ has the fractal dimension D_i.

As the measure of error for the model we will take a value of $\triangle_i = \max|\widetilde{y_i}(t)|$ on the interval T_i. The total error is $\Delta = \max\Delta_i, i = 1...n$.

It is suggested in the multi-fractal dynamics model that the tangent of the linear trend angle $\overline{y}(t)$ is a function of the fractal dimension D:

$$\bar{y}_i(t) = X_i(D_i)(t - t_{0i}) + \bar{y}_i(t_{0i})$$

In our situation $y(t)$ is the global average annual temperature.

A significant moment of the approach [2] is the possibility of describing catastrophes in its framework.

In the segments of the multi-fractal curve with a constant value of D the slope ratio of the linear trend (the average velocity of the corresponding process) according to [3], is a function of D_i and is to be determined from the solution of the cubic equation:

$$A_i(D_i) X_i + B_c X_i^3 = \eta \tag{2.2}$$

It is convenient to choose a scale enabling to meet the condition $|X_i| \ll 1$. The parameter η describes an effective action of external factors on the system under investigation.

For the function $A(D)$, let us choose the following analytic representation [3]:

$$A(D) = \begin{cases} (D_0 - D)^{-1} & \text{if} \quad 1 \leq D \leq D_0, \\ (D_0 - D_c)^{-1}(D_0 - D)^{-1}(D - D_c) & \text{if} \quad D_0 \leq D \leq 2. \end{cases} \tag{2.3}$$

Formula (2.3) allows to describe the variety of behaviours of the linear trend $X(D)$.

The model parameters $D_0,\ D_c,\ B_c$ and η are to be selected from the best possible fit with experimental results.

In the case $D < D_k$, we can neglect the member containing B_c; in this case the following linear approximation is true:

$$X = \eta (D_0 - D). \tag{2.4}$$

In this range of values of D equation (2.2) has one real root determined by formula (2.4).

When D goes to D_k, the situation is altering essentially, and we cannot neglect the member in (2.2) containing B_c.

Equation (2.2) can be obtained as extreme points of the Fractal Determining Function (FDF):

$$V(X) = \frac{1}{4}X^4 + \frac{a}{2}X^2 + bX. \tag{2.5}$$

The factor $\frac{1}{4}$ is chosen for the reasons of convenience. The control parameters in (2.5) will be a and b.

The extreme points of (2.5) satisfy the equation

$$X^3 + aX + b = 0. \tag{2.6}$$

The critical point X_c corresponds to $a = 0$. In case of $b \neq 0$, the analytic formula for $X_c(a,\ b)$ is given in [3].

In (2.5) the catastrophe term is equal to $(1/4)X^4$, and this implies that if $a = b = 0$, then the catastrophe of A_3 type exists in this model by Tom‘s classification [3]. Moreover if the parameter b depends on the parameter η, namely $b = -\frac{\eta}{B_c}$ then a will be a complicated function of the parameters $D_0,\ D_c,\ B_c$, D of the fractal model. From (2.3) we obtain

$$a = \begin{cases} B_c^{-1}(D_0 - D)^{-1} & \text{if} \quad 1 \leq D \leq D_0, \\ B_c^{-1}(D_0 - D_c)^{-1}(D_0 - D)^{-2}(D - D_c) & \text{if} \quad D_0 \leq D \leq 2. \end{cases} \tag{2.7}$$

Formula (2.7) this implies that the catastrophe of A_3 type takes place if $D = D_c$ and $\eta = 0$. In this case the separatrix equation has the following form:

$$\eta = \pm \frac{2}{\sqrt{27}} \left(-\frac{A(D)}{B_c} \right)^{\frac{3}{2}}. \tag{2.8}$$

3 Global temperature analysis in the multi-fractal dynamics model with a linear trend

The data observed for the past 160 years show [4] that the global temperature T_oC oscillates near the equilibrium value $T_0 = 15^oC$. Let u be the deviation from the equilibrium value:

$$T = u + T_0.$$

The global temperature curve showing the temperature starting with 1850 is given on Figure 1, on the basis of data in [4].

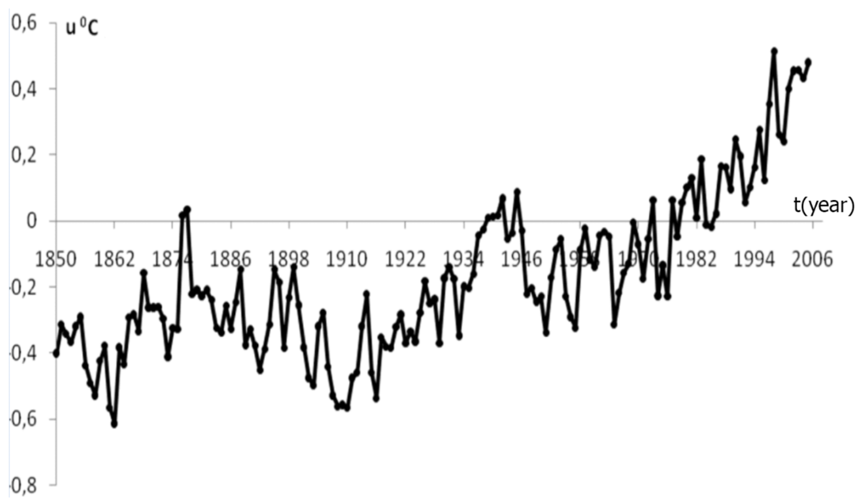

Fig. 1. Graph of yearly average temperature variations over the period from 1850 to 2005

Based on the multi-fractal dynamics model [3], let us split the total time interval (160 years) into 5 segments T_i $(i = 1,\ 2,\ 3,\ 4,\ 5)$, and perform a linear trend approximation of u $\bar{u}$ in each of them:

$$u_i = \bar{u}_i + \tilde{u}_i = \bar{u}_{0i} + X_i\left(D_i\right)\left(t - t_{0i}\right) + \tilde{u}_i. \tag{3.1}$$

$\triangle_i = \max|\tilde{u}_i|$. is the approximation error.

The function $X_i\left(D_i\right)$ should satisfy equation (2.2).

The computational results for T_i, D_i, X_i are given in Table 1, and the linear trend approximation is shown on Figure 2.

i	1	2	3	4	5
T_i $(year)$	30	31	30	31	30
$X_i\left(\frac{^oC}{year}\right)$	0,0089	-0,0081	0,0135	-0,0016	0,0183
D_i	1,140	1,166	1,141	1,203	1,183
Δ_i (^0C)	0,2624	0,229	0,1987	0,2281	0,1976

Table 1. Computational results

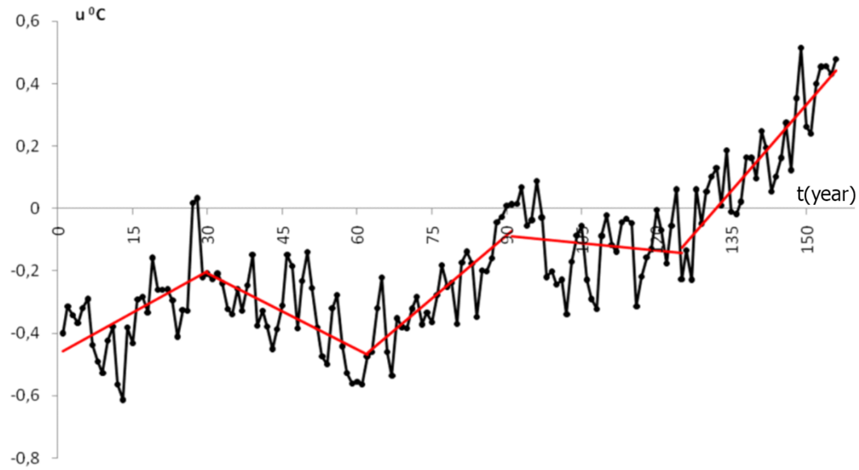

Fig. 2. Graph of linear trend approximation of yearly average temperature variations

The experimental results are well enough in keeping with the multi-fractal dynamics model [3] if for the first three periods $i = 1,\ 2,\ 3$ there have been chosen D_0=1,157 and $\eta = 0,862\ \frac{^0\mathrm{C}}{\mathrm{year}}$, and over the last two period $i = 5,6$ ($i > 5$ for the forecast periods) there have been chosen D_0=1,201 and $\eta = 0,995\ \frac{^0\mathrm{C}}{\mathrm{year}}$. The difference can be noticeable for X_1 only. This may be due to all types of inaccuracies of global temperature measurements in the initial period of instrumental measurements. Some increase of the parameter η in periods 4 and 5, in comparison with periods 1, 2, and 3 takes place, and this event characterizes the Sun activity and the dynamic activity of the atmosphere of the Earth providing for redistribution of resulting solar energy.

From the existing experimental results, the critical value of D_c whereby $A\left(D_c\right) = 0$ cannot be defined. Its value is assumed to be known like in the financial processes [4] belonging to the interval $1,67 \le D_c \le 1,75$. By virtue of the fact that all D_i ($i = 1, ..., 5$) values are significantly less than D_c, we can suggest there should not be any catastrophes in the global temperature dynamics in accordance with the multi-fractal dynamics model. There is no reason to wait for their appearance now.

The sufficiently low values of $\mathrm{D_i} \le 1,203$ fractal dimensions, in comparison with the Gaussian value of 1,5 are indicative of the essentially deterministic nature of processes responsible for the global temperature dynamics. The increase in the parameter η, namely $\Delta\eta = \frac{^oC}{year}$, in 1950 in our model counts in favour of a global warming trend at present.

A slight growth of D_0 from 1,157 to 1,201 over the same period, gives evidence of a slight increase of chaotization of global temperature.

Considering that the global temperature dynamics process is an oscillating one, in the adjacent time intervals T_i the rates of changes of X_i of linear trends change sign, in other words, there is good reason to believe that for the next interval $T_6 \approx 31$ year we shall have $X_6 < 0$. This points to the possibility of reduction of the global temperature trend in this period. The estimated specific value of reduction rate of u we will present in the next section.

4 Global temperature analysis in the multi-fractal dynamics model with a nonlinear trend

This section integrates the results obtained in earlier sections for a nonlinear trend. This will allow the trend function to be not only continuous throughout the observation interval but a differentiable one. It should be done in such a way that all primary virtues of the linear trend be preserved. For that purpose we suggest to change (3.1) in the following way

$$u_i = \overline{u}_{oi} + X_i\left(D_i\right)\left(t - t_{oi}\right) + \frac{X_i^{(nl)}\left(D_i, D_{i+1}\right)}{7T_i^6} \cdot \left(t - t_{oi}\right)^7 + \tilde{u}_i = \overline{u}_i + \tilde{u}_i. \qquad (4.1)$$

It follows by (4.1) that in the major parts of all intervals T_i the values of (3.1) and (4.1) are close except for the field near t_{0i+1}. In this field the nonlinear seventh degree term is significant enough the parameters and allows to perform a smooth junction from the slope ratio X_i to X_{i+1}. Let the values of X_i be the same as in the preceding section in the case of a linear trend parameters. $\bar{u}_{oi}$ and $X_i^{(nl)}$ factors will be calculated from the condition for continuity and smoothness of $\overline{u}_i$. Hence

$$\overline{u}_{0i+1} = \overline{u}_{0i} + X_i T_i + \tfrac{1}{7} \cdot X_i^{(nl)} T_i, \quad X_{i+1} = X_i + X_i^{(nl)}. \qquad (4.2)$$

The values of $\bar{u}_{oi}$ and $X_i^{(nl)}$ are to be determined from best possible fit with the experimental results according to the least square method. The resultant numerical values of $X_i^{(nl)}$ are shown in Table 2.

i	1	2	3	4	5
$X_{\mathbf{i}}^{(nl)}$ $(^0C/year)$	-0,0170	0,0216	-0,0151	0,0199	-0,0170

Table 2. The resultant numerical values.

The nonlinear trend approximation of the experimental results (4.1) is given on Figure 3.

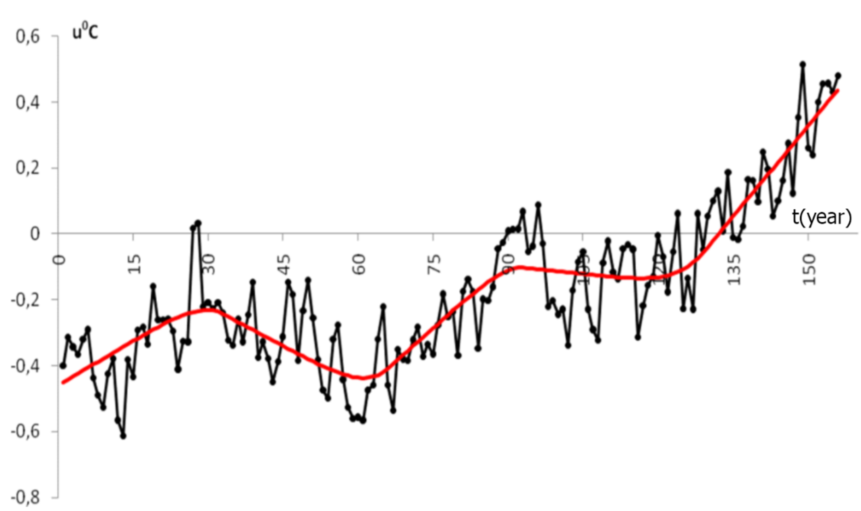

Fig. 3. Graph of nonlinear trend approximation of yearly average temperature variations

The values $X_i^{(nl)}$ have turned out to be the values of the same order as the values of the linear trend values X_i, including the sign-changing behavior. In our opinion the non-linear model determines dynamics of a trend of global temperature more precisely, but it contains the parameter $X_i^{(nl)}$ and therefore the forecast is possible only for one more period in comparison with the observed periods.

5 Forecast for global temperature linear trend dynamics

The periods of global temperature linear trend variations T_i have turned out to be the values of the order of 30 – 31 years. This value is close to the tripled period of 11-year period of solar activity. There is no any reliable proof of relation of these processes in view.

In this model the forecast periods $i = 6,\ 7$ must have the following values: T_6=31 years, T_7=30 years. On the supposition of X_6=X$_4$ and X_7=X_5, which is equivalent of maintenance of a trend for last two periods $i = 4,\ 5$, we find out $\triangle\overline{u}_6 \approx -0,0016 \cdot 31\ ^0C = -0,05^0C$ and $\triangle\overline{u}_7 \approx 0,0183 \cdot 30\ ^0C = 0,55^0C$. By summing the resultant values up we find that $\Delta u_6 + \Delta u_7 = 0,50^oC$, that is in 61 years the average global temperature should increase by 0,50^0C. This is an added reason for the global warming trend.

A somewhat different forecast follows from the nonlinear trend model. According to this model, X_6=X_5+$X_5^{(nl)}$$= 0,0013^0C/year$, and thus we have $\triangle\overline{u}_6 = 0,040^0C$. Instead of temperature drop of 0,05^0C, we have a slight 0,04^0C growth for average global temperature in 31 years or in 2042. As we do not know $X_i^{(nl)}$, for the nonlinear model it is not possible to define $\Delta\bar{u}_7$.

6 Conclusion

The analysis and the forecast for yearly average temperature change we have carried out within the framework of the multi-fractal dynamics, show that the global warming trend should continue for the coming 60 years. Over this period the global temperature linear trend should rise by about $0,5^0C$. At the same time over the period of 30 years the prediction is that the global temperature nonlinear trend should drop by about $0,05^oC$.

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Received: 30.01.2012.

EURASIAN MATHEMATICAL JOURNAL

2014 – Том 5, № 2 – Астана: ЕНУ. – 139 с.

Подписано в печать 29.06.2014 г. Тираж – 120 экз.

Адрес редакции: 010008, Астана, ул. Мирзояна, 2,
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Дизайн: К. Булан

Отпечатано в типографии ЕНУ имени Л.Н.Гумилева

Свидетельство о постановке на учет печатного издания
Министерства культуры и информации Республики Казахстан
№ 10330 – Ж от 25.09.2009 г.