To the 70th birthday of Professor V.I. Burenkov

The conference is dedicated to the 70th birthday of the outstanding mathematician Victor Ivanovich Burenkov. He has made major contributions to many areas of mathematics. His interests are wide and varied in both pure and applied mathematics, with more than 150 publications to his name in leading international journals. He is a leading authority on real analysis and the theory of function spaces, especially Sobolev spaces and spaces with fractional order of smoothness and their applications. He has also produced notable works on partial differential equations and integral equations, in particular, on the theory of hypoelliptic equations, spectral theory of partial differential operators and ill-posed problems.

Victor started scientific research being a student of Moscow Institute of Physics and Technology, together with his student-mate, now a recognized professor A.G. Aslanyan, in the area of ordinary differential equations and related topics in non-commutative algebra.

He had excellent mathematical training. Among his teachers were outstanding world recognized mathematicians A.A. Abramov, M.A Aiserman, O.M. Belotserkovskii, A.A. Desin, A.A. Dorodnitsin, F.R. Gantmaher, L.D. Kudryavtsev, V.B. Lidskii, N.N. Moiseev, M.A. Naimark, S.M. Nikol'skii, G.E. Shilov, V.A. Trenogin.

Victor's research interests were formed under the influence of his scientific supervisor, the great Russian mathematician Sergei Mikhailovich Nikol'skii.

V.I. Burenkov has developed a number of ingenious approaches and methods that led to significant breakthroughs.

His method of mollifiers with variable step and translation allowed him to obtain seminal results in the approximation of functions belonging to general function spaces by various classes of infinitely differentiable functions (with preservation of boundary values in particular) and especially in the extension problem for Sobolev spaces. Later on this method was successfully applied by V.I. Korzyuk to some problems in partial differential equations.

His research on inequalities for intermediate derivatives with sharp constants led to active investigation of a new case of a finite interval, and he is quoted as a pioneer of this new direction. Sharp constants were also found in certain inequalities of different metrics and of Markov type for polynomials, and in certain embedding theorems for Sobolev spaces. Sharp constants in some Hardy-type inequalities were obtained jointly with Swedish mathematicians J. Bergh and L.-E. Persson.

New types of Fourier multiplier theorems were proved for the weighted Lebesgue spaces with exponential weights in which sharp conditions on a function ensuring that it is a Fourier multiplier were expressed in terms of Gevrey classes.

His paper on the composition of absolutely continuous functions was communicated by A.N. Kolmogorov, and the result is quoted as Burenkov's theorem.

A considerable part of Burenkov's research was carried out jointly with M.L. Goldman. In particular they constructed a nonlinear extension operator for the limiting case of the trace theorem for Nikol'skii-Besov spaces and proved non-existence of a linear extension operator. They investigated interconnection between the norms of

a wide class of local operators in general normed function spaces and the norms of their periodic analogues. These results allow transferring many statements from the non-periodic case to the periodic one and vice versa.

The characteristic feature of Burenkov's research is his interest in non-standard effective applications of the theory of function spaces to other areas.

One of such applications is the detailed study of the problem of conditional hypoellipticity (hypoellipticity depending on behaviour of solutions at infinity). He developed the method of fractional differentiation of apriori inequalities, which allowed him to obtain necessary and sufficient conditions of conditional hypoellipticity of partial differential operators with constant coefficients. This direction was further developed by the group of researchers from Yerevan headed by H.G. Ghazaryan. In their papers the aforementioned result is quoted as "Burenkov's theorem on conditional hypoellipticity" and this type of hypoellipticity is called "Burenkov's hypoellipticity".

Another application belongs to the field of integral convolution-type equations (ill-posed problem). Application of the theory of spaces with low fractional smoothness allowed him to develop new flexible methods of constructing regularized approximate solutions of equations related to geophysical problems; these methods appeared to be more effective than the traditional approaches based on using Sobolev spaces.

He also worked in some areas of industrial mathematics (radars, medical equipment etc). In this area his main co-worker was Yu.I. Khudak.

V.I. Burenkov worked at several universities, in particular for more than 10 years at Moscow Institute of Electronics, Radio-engineering, and Automation, at Peoples' Friendship University of Russia and at Cardiff University (United Kingdom). He also worked at his alma mater Moscow Institute of Physics and Technology, the University of Padua (Italy), and L.N. Gumilyov Eurasian National University (Astana, Kazakhstan).

At Cardiff University he had fruitful collaboration with W.D. Evans which resulted in publishing several significant papers on weighted integral inequalities and a frequently cited paper on quantum mechanics.

In the last decade Burenkov's research focused mainly on the following two perspective directions.

The first one is dedicated to the operator theory in general Morrey-type spaces. V.I. Burenkov together with V.S. Guliyev and his son H.V. Guliyev were first to obtain, for a certain range of the numerical parameters, necessary and sufficient conditions on the functional parameters, ensuring the boundedness of the maximal operator and fractional maximal operator from one general local Morrey-type space to another one. These conditions appeared to be rather "stable": under the appropriate assumptions they are also necessary and sufficient for the boundedness of the Riesz potential and of genuine singular integral operators. Now they are often called, at this conference in particular, the Burenkov-Guliyevs conditions.

Many mathematicians shared Victor's enthusiasm in developing this theory. He has joint papers on it with A. Gogatishvili, M.L. Goldman, H.V. Guliyev, V.S. Guliyev, P. Jain, R. Mustafaev, E.D. Nursultanov, R. Oinarov, A. Serbetci, T.V. Tararykova. Most of them participated in OMTSA 2011.

Due to Victor's efforts and the efforts of his co-researchers, the necessary and sufficient conditions on the functional parameters characterizing these spaces were obtained,

for a wide range of the numerical parameters (but currently not for all admissible values) ensuring the boundedness of many classical operators of real analysis (the maximal operator, the fractional maximal operator, the Riesz potential, the Hardy operator, genuine singular integrals) from one general local Morrey-type space to another one. Some of these results are exhaustive, however, there are still many interesting open problems in this area.

The second research direction is dedicated to sharp spectral stability estimates for the eigenvalues of selfadjoint elliptic partial differential operators. These developments have started after joint research with E.B. Davies on spectral stability of the Neumann Laplacian. Later on V.I. Burenkov, together with P.D. Lamberti, developed the method of transition operators. This method allowed them to obtain sharp spectral stability estimates for the variation of the eigenvalues via effective geometric characteristics of the vicinity of open sets for elliptic operators of arbitrary even order defined on open sets admitting arbitrarily strong degeneration for both Dirichlet and Neumann boundary conditions, thus giving a complete solution to the main aspects of this problem. The case of the Robin boundary conditions was considered together with M. Lanza de Cristoforis. Some results on the stability of the singular numbers of non-selfadjoint elliptic operators were obtained jointly with M. Otelbaev.

An important open problem in this area is related to obtaining sharp spectral stability estimates for the variation of the eigenfunctions upon domain perturbation. Some preliminary results in this direction are obtained by V.I. Burenkov jointly with G. Barbatis, P.D. Lamberti, and E. Feleqi.

Victor is open to constructive scientific collaboration and he has more than 50 co-authors.

V.I. Burenkov participated in numerous research projects supported by various grants of several countries, and led some of them. In particular, through 2006-2009 he was the co-ordinator of the large international project on function spaces and applications supported by INTAS (11 teams from 11 countries, 65 participants with more than 30 professors among them).

In addition to his research, Victor has also made a significant contribution to mathematical education through his teaching and supervision of postgraduate students.

He is a talented and sought-after lecturer. Students of several countries enjoyed his lectures on calculus, real, complex and functional analysis and differential equations. The distinguishing traits of Victor's character – creativity, relentless pursuit of perfection have always inspired his students. He is able to develop the student's independent thinking, jointly explore ideas and develop new insights. There are many mathematicians who have uncovered whole new fields of mathematical research after discussions with him. 25 of his research students have gained PhD degrees and are working in many countries of the world (on all continents).

V.I. Burenkov has published several text-books for students and developed the course "The main ideas in the theory of Sobolev spaces" with flexible structure from 2 to 30 lectures which he delivered in many universities around the world.

His monograph "Sobolev spaces on domains" became a popular text for both experts in the theory of function spaces and a wide range of mathematicians interested in applying the theory of Sobolev spaces. The monograph covers the subject at the right

level of technical details and focuses on thorough exposition of the main results and ideas of the theory.

Victor is also known for his interest in languages, gave regular lecture courses in three languages (Russian, English, and Italian) and made short introductory speeches in several other languages. Many mathematicians are thankful to him for vivid and energetic interpretations of their talks.

V.I. Burenkov has always been a keen editor and played a leading role in numerous editorial projects. He was a scientific editor of major books on the theory of function spaces published in Russia, organized and participated in translation of several books from English and Japanese into Russian, he is a member of the editorial boards of several international journals. Recently V.I. Burenkov became one of the organizers and subsequently one of the editors-in-chief, together with V.A. Sadovnichy and M. Otelbaev, of the Eurasian Mathematical Journal.

Victor takes active part in the international mathematical life, participates in and contributes to organizing many international conferences. He gave more than 100 invited talks at conferences and by invitations of universities in more than 30 countries, and he is an honorary professor of L.N. Gumilyov Eurasian National University, an honorary doctor of Russian-Armenian (Slavonic) University, and a foreign member of the Crimean Academy of Sciences.

He is one of active members of the International Society for Analysis, its Application and Computation (ISAAC) and has been Vice-President of this society since 2003. In August 2011 he was one of the main organizers of the 8th Congress of the ISAAC which was held in Moscow at Peoples' Friendship University of Russia.

In his student years Victor played basketball for his university, but he is also keen on football, volleyball, tennis, table tennis. Not only does he play himself but encourages others to participate. During his visits he likes to organize football matches of a team of professors versus a team of postgraduate students. For this reason at a number of conferences in which he participates, football is included in the social part of the programme, at this conference in particular.

We congratulate Victor Ivanovich with the jubilee and wish him good health and fruitful further research.

Participants of the workshop OMTSA 2011