# To the 70th birthday of Professor Mukhtarbai Otelbaev

This issue of the Eurasian Mathematical Journal is dedicated to the 70th birthday of Mukhtarbai Otelbaev, professor of the Department of fundamental and applied mathematics of the L.N. Gumilyov Eurasian National University, director of the Eurasian Mathematical Institute at the L.N. Gumilyov Eurasian National University, deputy director of the Kazakhstan branch of the M.V. Lomonosov Moscow State University, Doctor of Sciences in physics and mathematics, academician of the National Academy of Sciences of the Republic of Kazakhstan.

He was born on October 3, 1942 in the village Karakemer of the Kordai district of the Zhambyl region. He started his labour life as a machine operator in his native village. After graduating from the evening school in 1962 in the village Karakonyz (now Masanchi), he entered the Kyrgyz State University in Frunze (now Bishkek). In 1962-1965, he served in the Soviet Army. After that he continued studies at the Faculty of mechanics and mathematics of the M.V. Lomonosov Moscow State University and graduated in 1969. In the same year he entered postgraduate studies at the same faculty under supervision of the famous scientists, Professors B.M. Levitan and A.G. Kostyuchenko. In 1972 he defended the PhD thesis titled "About the spectrum of some differential operators".

Since 1973, M. Otelbaev was in Alma-Ata, worked as a junior researcher, a senior researcher, the head of a laboratory at the Institute of mathematics and mechanics of the Academy of Sciences of the Kazakh SSR. In 1978, he brilliantly defended the Doctor of Sciences thesis titled "Estimates of the spectrum of elliptic operators and related embedding theorems" at the Dissertation Council number 1 of the Faculty of mechanics and mathematics of the M.V. Lomonosov Moscow State University headed by Professor A.N. Kolmogorov, a prominent mathematician, academician of the Academy of Sciences of the USSR.

In 1989, M. Otelbaev was elected a corresponding member of the Academy of Sciences of the Kazakh SSR, and in 2004 he became a real member of the National Academy of Sciences of Republic of Kazakhstan. M. Otelbaev is an expert in the field of functional analysis and its applications, the author of 3 monographs and over 210 scientific papers and inventions widely recognized both in Kazakhstan and abroad. More than 70 of his works were published in rating international scientific journals (with the impact-factor ISI or included in the SCOPUS database).

His main works are grouped around the following fields:

### I. Spectral theory of differential operators

M. Otelbaev developed new methods for studying the spectral properties of differential operators, which are the result of consistent and skilled implementation of the general idea of the localization of the considered problems. In particular, he invented a construction of averaging coefficients well describing those features of their behaviour which influence the spectral properties of a differential operator. This construction known under the notation $q^*$ made it possible to answer many of the hitherto open questions of the spectral theory of the Schrцdinger operator and its generalizations. The function $q^*$ and its different variants have a number of remarkable properties, which allowed applying this function to a wide range of problems. Here we note some problems for the first time solved by M. Otelbaev by using the function $q^*$ on the basis of sophisticated analysis of the properties of differential operators.

1) A criterion for belonging of the resolvent of the Schrödinger type operator with a non-negative potential to the class $\mathfrak{S}_p(1 \le p \le \infty)$ was found (previously only a criterion for belonging to $\mathfrak{S}_\infty$ was known) and two-sided estimates for the eigenvalues of this operator were obtained with the minimal assumptions of the smoothness of the coefficients.

2) The general localization principle was proved for the problems of selfadjointness and of the maximal dissipativity (simultaneously with the American mathematician P. Chernov) which provided significant progress in this area.

3) Examples were given showing the classical Carleman-Titchmarsh formula for the distribution function $N(\lambda)$ of the eigenvalues of the Sturm-Liouville operator is not always correct even in the class of monotonic potentials and a new formula was found valid for all monotonic potentials.

4) The following result of M. Otelbaev is principally important: for $N(\lambda)$ there is no universal asymptotic formula.

5) From the time of Carleman, who found the asymptotics for $N(\lambda)$ and, by using it, the asymptotics of the eigenvalues themselves, all mathematicians started with finding the asymptotics for $N(\lambda)$ and as a result they could not get rid of the so-called Tauberian conditions. M. Otelbaev was the first who, when looking for the asymptotics of the eigenvalues, omitted the interim step of finding the asymptotics for $N(\lambda)$ , which allowed getting rid of all unessential conditions for the problem including Tauberian conditions.

6) The two-sided asymptotics for $N(\lambda)$ for the Dirac operator was for the first time found when $N_-(\lambda)$ and $N_+(\lambda)$ are not equivalent.

The results of M. Otelbaev on the spectral theory were included as separate chapters in the monographs of B.M. Levitan and I.S. Sargsyan "Sturm-Liouville and Dirac operators" (Moscow: Nauka, 1985), and of A.G. Kostyuchenko and I.S. Sargsyan "Distribution of eigenvalues" (Moscow: Nauka, 1979), which became classical.

Recently, M. Otelbaev, jointly with Professor V.I. Burenkov, described a wide class of non-selfadjoint elliptic operators of order $2l$ with general boundary conditions, whose singular numbers have the same asymptotics as the eigenvalues of the $l$th power of the Laplace operators with the Dirichlet boundary conditions.

### II. Embedding theory and approximation theory

This field of mathematics has developed as a separate branch in the works of S.L. Sobolev in 1930s. Beginning with the works of L.D. Kudryavtsev (around 1960) a new era arises of weighted function spaces used in the theory of differential operators with variable coefficients. M. Otelbaev began research in this field being a mature mathematician and managed to create a new method of proving embedding theorem which is, in form and essence, a local approach to such problems according. In the theory of weighted Sobolev spaces, most used weighted function spaces, M. Otelbaev obtained the following fundamental results.

1) A criterion for an embedding and for the compactness of an embedding.

2) Two-sided estimates for the norm of an embedding operator.

3) Two-sided estimates for Kolmogorov's width and for the approximation numbers of an embedding operator, and a criterion for belonging of an embedding operator to the classes $\mathfrak{S}_p(1 \leq p \leq \infty$ . It turned out that one of the variants of the function$q^*$ is an adequate tool for description of the exact conditions ensuring an embedding. For applications it is particularly important that all the estimates are given in terms of weight functions and allow taking into account the characteristics of their local behavior.

**III. Separability and coercive estimates for differential operators**

The term "separability" was suggested by the famous English mathematicians Everitt and Geertz around 1970s, who investigated the smoothness of solutions to the Sturm-Liouville equation. Soon after that, M. Otelbaev was involved in research on this topic. He developed a method for studying the separability of more general, multi-dimensional operators and variable type operators, as well for the smoothness of solutions to nonlinear equations. In particular, by using this method one can study the separability of general differential operators in weighted, not necessarily Hilbert spaces. With his interest in solving problems in the most general setting, M. Otelbaev obtained

1) weighted estimates not only of the derivatives of solutions of the highest order, but also of intermediate derivatives for a wide class of linear and nonlinear equations,

2) estimates of the approximation numbers of separable operators exact in a certain class of coefficients.

**IV. General theory of boundary problems**

The classical formulation of the boundary value problem is as follows: given an equation and boundary conditions, to investigate the solvability of this problem and the properties of the solution, if it exists (in the sense of belonging to a certain space).

Beginning with M.I. Vishik (1951), there is another, more general approach: given an equation and a space to which the right-hand side and the solution should belong, to describe all boundary conditions for which the problem is correctly solvable in this space. In this problem as well, despite the numerous previous studies,

M. Otelbaev has obtained new results remarkable in depth and transparency. The rich mathematical intuition, the depth of thinking and extensive knowledge, coupled with rejection of traditional constraints on the considered operators and spaces, allowed him to develop an abstract theory of extension and restriction of not necessarily linear operators in linear topological spaces. Using this theory, M. Otelbaev and his students were the first to describe all correct boundary value problems for such "pathological" operators as the Bitsadze-Samarsky operator, the ultrahyperbolic operator, the pseudoparabolic operator, the Cauchy-Riemann operator and others. (For some of them previously no correct boundary value problems were known!) Moreover, considerations were carried out in non-Hilbert spaces $L_p\,(p \neq 2)$ and $C$. This theory also allowed describing the structural properties of the spectrums of correct restrictions of a given differential operator.

### V. Theory of generalized analytic functions

In the theory of generalized analytic functions, built by the well-known scientist I.N. Vekua, a real member of the Academy of Sciences of the USSR, the main facts are: a) a theorem on the representation of a solution, b) a theorem on the continuity of a solution, c) a theorem on the Fredholm property. All other facts of the theory are deduced from a), b), and c).

Various authors have gradually widened the class of spaces in which the Vekua theory was valid. M. Otelbaev found the widest space among the spaces close to the so-called ideal spaces, to which the coefficients and the right-hand side should belong, so that the facts a), b) and c) remain valid.

### VI. Computational mathematics

1) M. Otelbaev proposed a new numerical method for solving boundary value problems (as well as general operator equations). By using the embedding and extension theorems, he reduced the considered boundary value problem to the problem of minimizing a functional. The boundary conditions and also nonlinearities are "hidden" in the integral expressions. Moreover, by this method the problem of "the choice of a basis" was solved, in which many prominent mathematicians have been interested for a long time. The method of M. Otelbaev can be easily algorithmized and allows finding the solution with the required accuracy. Moreover, the procedure of finding a numerical solution is stable. Computer calculations conducted by his students and students of Professor Sh. Smagulov showed the effectiveness of the method.

2) M. Otelbaev developed a method of approximate calculation of eigenvalues and eigenvectors of non-selfadjoint matrices, based on a variational principle. The method reduces the problem to the analogous problem for self-adjoint matrices, for which there is a well-developed theory. Unlike other methods, for example, the maximum gradient method, this method a) provides global convergence, b) is convenient for calculating the initial approximation, c) allows calculating the eigenvalues with the smallest real part, d) can be used in the general case of a compact non-selfadjoint operator.

3) M. Otelbaev obtained a two-sided estimate for the smallest eigenvalue of a difference operator which is important for computational mathematics.

4) Due to the need for cumbersome calculations, methods for parallelization are actively developed in the world. M. Otelbaev offered an effective algorithm of parallelization for approximate solutions of boundary value problems and the Cauchy problem for various classes of differential equations.

### VII. Nonlinear evolutional equations

In hydrodynamics for describing a laminar flow of an incompressible fluid, as well as a turbulent flow the system of the Navier–Stokes equations is used. However, mathematically, it is not well justified, since the existence of a global solution has not yet been proved. Therefore, there are some doubts about the rightness of using this system as a mathematical model.

M. Otelbaev managed to reduce the existence problem of a global solution to the Navie–Stokes equation to other equivalent problems, in particular, to the problem of the existence of the so-called "dividing function". He obtained a criterion for strong solvability of nonlinear evolution equations, similar to the Navier–Stokes equation, and also built the examples of equations not globally strongly solvable to which the system of Navier–Stokes equations reduces.

**VIII. Theoretical physics**

M. Otelbaev obtained a number of interesting mathematical results in this area. He

1) found explicit formulas for $n$-particle motion in the space (in the framework of Einstein's relativity theory);

2) derived an integral formula of the matter motion;

3) proposed a new transformation of the type of the well-known Lorentz transformation which works both for $|v| < c$ and for $|v| > c$. If $|v| < c$ Otelbaev's transformation of coincides with the Lorentz transformation;

4) proved mathematically that the results of physics arising from the special Einstein's relativity theory one can obtain while staying within the classical wave theory.

**IX. Other fields of mathematics**

The research interests of M. Otelbaev are extremely diverse. The following topics complete their partial characterization.

1) M. Otelbaev chose a certain nonlinear integral operator, for which he proved a criterion of continuity. This operator appeared to be an important model in the theory of nonlinear integral operators, based on which one can develop and test new methods. Due to this, M. Otelbaev together with Professor R. Oinarov obtained a necessary and sufficient condition ensuring the Lipschitz property (contractibility) of the Uryson operator in the spaces of summable and continuous functions.

2) He investigated spectral characteristics and smoothness of solutions to equations of mixed type. A criterion of coinciding of the generalized Neumann and Dirichlet problems for degenerate elliptic equations was found.

3) In recent years, the problem of oscillatory and non-oscillatory solutions to differential equations has become a fashionable topic in mathematics. Already in the late 80s, M. Otelbaev obtained a sufficient condition ensuring the non-oscillation property of solutions to the Sturm-Liouville problem, close to a necessary one.

4) M. Otelbaev studied the problem of controlling a laser heat source. He showed that under the usual formulation, it does not even have a generalized solution and proposed a new formulation of the problem in terms of "order" and "admittance precision" for surface treatment. He proved the solvability of this problem in such a formulation, and solved some optimization problems without using the known methods of optimal control. In addition, jointly with Professor A. Hasanoglu, he solved an inverse identifi-

cation problem of an unknown time source, on the bases of the measured output data, when the boundary conditions are given in the Dirichlet or Neumann form, as well as in the form of the final overdetermination.

Summing up the review of scientific creativity of M. Otelbaev, one should note as characteristics features of his work the diversity of his scientific interests, the fundamentality of research, the interest in solving problems in the most general formulation and obtaining solutions of the level of a criterion. A large number of publications of M. Otelbaev characterize his high efficiency, diligence, and research productivity.

He was a participant of a number of international scientific conferences, which took place in Kazakhstan, Russia, Ukraine, Poland, Czechoslovakia, Germany, Morocco, Turkey, Greece, and Japan.

M. Otelbaev has carried out great work in preparing highly qualified researchers and university teachers. Over 35 years he held lectures for students of various universities of the Republic of Kazakhstan, organized a series of seminars and study groups for graduate students, interns, master and PhD students. The courses "Extensions and restrictions of differential operators", "The theory of divisibility," "Embedding theorems," "Modern numerical methods," and many others, developed by M. Otelbaev, are well known.

He has created a large mathematical school in Kazakhstan. 70 postgraduate students have defended PhD theses under his supervision. 9 of them later defended Doctor of Sciences theses.

M. Otelbaev made a significant contribution to organization and development of science and education in Kazakhstan. In 1985-1986, he was the rector of the Zhambul Pedagogical Institute, from 1991 to 1993 organized and worked as the director of the new Institute of Applied Mathematics of the Academy of Sciences and the Ministry of Education and Science of the Republic of Kazakhstan in Karaganda, in 1994-1995, he was the head of a department at the Aerospace Agency of the Republic of Kazakhstan.

Since 2001 he is the deputy director of the Kazakhstan branch of the M.V. Lomonosov Moscow State University, and simultaneously the director of the Eurasian Mathematical Institute at the L.N. Gumilyov Eurasian National University.

For a number of years, M. Otelbaev was a member of the editorial boards of the "Proceedings of the Academy of Sciences of the Republic of Kazakhstan. Series in Physics and Mathematics" and of the international scientific journal "Applied and Computational Mathematics" of the National Academy of Sciences of the Republic of Azerbaijan. He was the editor-in-chief of the "Evraziiskii Matematicheskii Zhurnal", published by the L.N. Gumilyov Eurasian National University, together with the M.V. Lomonosov Moscow State University in 2003-2009. Since 2010 he is an editor-in-chief, together with academician V.A. Sadovnichy and Professor V.I. Burenkov, of the "Eurasian Mathematical Journal", which is published in English by the L.N. Gumilyov Eurasian National University, together with the M.V. Lomonosov Moscow State University, the Peoples' Friendship University of Russia, and the University of Padua.

He was the chairman of the international scientific conference "Modern Problems of Mathematics", held at the L.N. Gumilyov Eurasian National University in 2002, and was a member of program committees of 10 international scientific conferences devoted to problems of mathematics and computer science held at the Kazakh National University, the Karaganda State University, the Institute of Mathematics of the Ministry of Education and Sciences of the Republic of Kazakhstan, the Pavlodar State University, and the University Semei.

In 2007, he was elected the Vice-President of the Mathematical Society of Turkic-speaking countries.

In 2007, M. Otelbaev was awarded the state prize of the Republic of Kazakhstan in the field of science and technology. In 2004, he became a laureate of the Economic Cooperation Organization in the category "Science and technology". In 2006 and 2011, M. Otelbaev was awarded the state grant "The best university teacher". In 1985 - 1986, he was elected to the Zhambul City Council of people's deputies.

The Editorial Board of the Eurasian Mathematical Journal is happy to congratulate Mukhtarbai Otelbaev on occasion of his 70th birthday, wishes him good health and further productive work in mathematics and mathematical education.

Editorial Board