EURASIAN MATHEMATICAL JOURNAL
ISSN 2077-9879
Volume 2, Number 1 (2011), 112 – 119

ON SCORES IN MULTIPARTITE HYPERTOURNAMENTS

S. Pirzada

Communicated by M. Otelbaev

Key words: hypergraph, hypertournament, multipartite hypertournament, score, losing score.

AMS Mathematics Subject Classification: 05C20.

Abstract. In this paper, we discuss two types of hypertournaments, one $[\alpha_i]_1^k$-multipartite hypertournament ($[\alpha_i]_1^k$-MHT) and the second $(\alpha_i)_1^k$-multipartite hypertournament ($(\alpha_i)_1^k$-MHT). We obtain necessary and sufficient conditions for the k lists of non-negative integers in non-decreasing order to be the losing score lists (score lists) of $[\alpha_i]_1^k$-MHT and that of $(\alpha_i)_1^k$-MHT. We extend this concept to more general class of $[\alpha_i]_1^k$-multipartite multihypertournament ($[\alpha_i]_1^k$-MMHT) and $(\alpha_i)_1^k$-multipartite multihypertournament ($(\alpha_i)_1^k$-MMHT).

1 Introduction

A hypergraph is a generalization of a graph [1]. While an edge of a graph is a pair of vertices, an edge of a hypergraph is a subset of the vertex set. An edge consisting of k vertices is called a k-edge and a hypergraph whose all edges are k-edges is called a k-hypergraph. A k-hypertournament is a complete k-hypergraph with each k-edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge. In a k-hypertournament, the score $s(v_i)$ or s_i of a vertex v_i is the number of arcs contaning v_i and in which v_i is not the last element, the losing score $r(v_i)$ or r_i of a vertex v_i is the number of arcs containing v_i and in which v_i is the last element. The score sequence (losing score sequence) is formed by listing the scores (losing scores) in non-decreasing order.

The following characterization of score sequences and losing score sequences in k-hypertournaments are due to Zhou, Yao and Zhang [12]. These results are analogous to Landau's theorem [4] on tournament scores.

Theorem 1. *Given two non-negative integers n and k with $n \geq k > 1$, a non-decreasing sequence $R = [r_i]_1^n$ of non-negative integers is a losing score sequence of some k-hypertournament if and only if for each p,*

$$\sum_{i=1}^{p} r_i \geq \binom{p}{k},$$

with equality when $p = n$.

Theorem 2. *Given non-negative integers n and k with $n \geq k > 1$, a non-decreasing sequence $S = [s_i]_1^n$ of non-negative integers is a score sequence of some k-hypertournament if and only if for each p,*

$$\sum_{i=1}^{p} s_i \geq p\binom{n-1}{k-1} + \binom{n-p}{k} - \binom{n}{k},$$

with equality when $p = n$.

A new and short proof of Theorem 1 can be seen in Pirzada and Zhou [9]. Koh and Ree [2, 3] have given some properties of the hypertournament matrix and have also found some conditions for the existence of k-hypertournament matrix with constant score sequence. Various results on scores in bipartite, tripartite and multipartite hypertournament scores can be found in [5, 6, 7, 10]. The extension of score structure to oriented hypergraphs can be found in Pirzada and Zhou [8]. Further, various results on degrees and degree sequences in hypertournaments can be found in Wang and Zhou [11], and the extension of degrees to oriented hypergraphs can be seen in Zhou and Pirzada [13].

2 Scores in $[\alpha_i]_1^k$-multipartite hypertournaments

Given non-negative integers n_i and $\alpha_i (i = 1, 2, \cdots, k)$ with $n_i \geq \alpha_i \geq 1$ for each i, an $[\alpha_1, \alpha_2, \cdots, \alpha_k]$-$k$-partite hypertournament (or $[\alpha_i]_1^k$-multipartite hypertournament or $[\alpha_i]_1^k$-MHT) of order $\sum_1^k n_i$ consists of k vertex set U_i with $|U_i| = n_i$ for each i $(1 \leq i \leq k)$ together with an arc set E, a set of $\sum_1^k \alpha_i$ tuples of vertices, with exactly α_i vertices from U_i, called arcs such that any $\sum_1^k \alpha_i$ subset $\cup_1^k U_i'$ of $\cup_1^k U_i$, E contains exactly one of the $(\sum_1^k \alpha_i)!$ $\sum_1^k \alpha_i$-tuples whose α_i entries belong to U_i'. The score and losing score of a vertex in $[\alpha_i]_1^k$-hypertournament is defined in the same way as in k-hypertournaments. The following two results [7] characterize losing score lists and score lists in $[\alpha_i]_1^k$-MHT.

Theorem 3. *Given k non-negative integers n_i and k non-negative integers α_i with $n_i \geq \alpha_i \geq 1$, for each $i (1 \leq i \leq k)$, the k non-decreasing lists $R_i = [r_{ij_i}]_{j_i}^{n_i}$ of non-negative integers are the losing score lists of $[\alpha_i]_1^k$- MHT if and only if for each $p_i (1 \leq i \leq k)$ with $p_i \leq n_i$,*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} r_{ij_i} \geq \prod_{i=1}^{k}\binom{p_i}{\alpha_i} \tag{2.1}$$

with equality when $p_i = n_i$.

Theorem 4. *Given non-negative integers n_i and α_i with $n_i \geq \alpha_i \geq 1$, for each $i (1 \leq i \leq k)$, the k non-decreasing lists $S_i = [s_{ij_i}]_{j_i}^{n_i}$ of non-negative integers are the losing score lists of $[\alpha_i]_1^k$- MHT if and only if for each $p_i \leq n_i (1 \leq i \leq k)$ with $p_i \leq n_i$,*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} s_{ij_i} \geq \left(\sum_{i=1}^{k}\frac{\alpha_i p_i}{n_i}\right)\left(\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\right) + \prod_{i=1}^{k}\left(\frac{n_i - p_i}{\alpha_i}\right) - \prod_{i=1}^{k}\binom{n_i}{\alpha_i} \tag{2.2}$$

with equality when $p_i = n_i$.

If in $[\alpha_i]_1^k$-MHT H, the arc set E contains at most $(\sum_1^k \alpha_i)!\ \sum_1^k \alpha_i$-tuples (but at least one), then it is called $[\alpha_i]_1^k$-multipartite multihypertournament(or briefly $[\alpha_i]_1^k$-MMHT). If E contains exactly $(\sum_1^k \alpha_i)!\ \sum_1^k \alpha_i$-tuples, then $[\alpha_i]_1^k$-MMHT is said to be complete. The score and losing score of a vertex in an $[\alpha_i]_1^k$-MMHT is defined in the same way as in $[\alpha_i]_1^k$-MHT. The losing score lists R_i, $i = 1, 2, \cdots, k$ of an $[\alpha_i]_1^k$-MMHT H are the k non-decreasing sequences $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, where r_{ij_i} is the losing score of a vertex u_{ij_i} in U_i. Similarly the score lists S_i, $i = 1, 2, \cdots, k$ of an $[\alpha_i]_1^k$-MMHT H are the k non-decreasing sequences $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$, where s_{ij_i} is the score of a vertex u_{ij_i} in U_i. We note that there are exactly $(\sum_1^k \alpha_i)! \prod_{i=1}^k \binom{n_i}{\alpha_i}$ arcs in a complete $[\alpha_i]_1^k$-MMHT, and therefore

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} d_H^-(u_{ij_i}) = \Big(\sum_{i=1}^{k}\alpha_i\Big)!\prod_{i=1}^{k}\binom{n_i}{\alpha_i}$$

where $d_H^-(u_{ij_i})$ denotes the number of arcs in which u_{ij_i} is at the last entry and thus

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} r_{ij_i} = \Big(\sum_{i=1}^{k}\alpha_i\Big)!\prod_{i=1}^{k}\binom{n_i}{\alpha_i}. \tag{2.3}$$

Now we have the following result.

Theorem 5. *If H is a complete $[\alpha_i]_1^k$-MMHT of order $\sum_{i=1}^k n_i$ with score lists $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ for each i, $1 \le i \le k$, then*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} s_{ij_i} = \Big[\Big(\sum_{i=1}^{k}\alpha_i\Big) - 1\Big]\Big(\sum_{i=1}^{k}\alpha_i\Big)!\Big[\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\Big].$$

Proof. Since there are $(\sum_{i=1}^k \alpha_i)!\binom{n_i-1}{\alpha_i-1}[\prod_{i=1,t\neq i}^k \binom{n_t}{\alpha_t}]$ arcs containing a vertex $u_{ij_i} \in U_i$, for each i, $1 \le i \le k$ and $1 \le j_i \le n_i$, using equation (3), we get

$$\begin{aligned}
\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} s_{ij_i} &= \Big[\sum_{i=1}^{k}\Big(\sum_{i=1}^{k}\alpha_i\Big)!\binom{n_i-1}{\alpha_i-1}\Big]\Big[\prod_{i=1,t\neq i}^{k}\binom{n_t}{\alpha_t}\Big] - \Big(\sum_{i=1}^{k}\alpha_i\Big)!\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\\
&= \Big(\sum_{i=1}^{k}\alpha_i\Big)!\sum_{i=1}^{k}\binom{n_i-1}{\alpha_i-1}\prod_{i=1,t\neq i}^{k}\binom{n_t}{\alpha_t} - \Big(\sum_{i=1}^{k}\alpha_i\Big)!\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\\
&= \Big(\sum_{i=1}^{k}\alpha_i\Big)!\Big[\alpha_1\prod_{t=1}^{k}\binom{n_t}{\alpha_t} + \alpha_2\prod_{t=1}^{k}\binom{n_t}{\alpha_t} +\\
&\cdots + \alpha_k\prod_{t=1}^{k}\binom{n_t}{\alpha_t} - \prod_{i=1}^{k}\binom{n_i}{\alpha_i}\Big]\\
&= \Big(\sum_{i=1}^{k}\alpha_i\Big)!\Big[(\alpha_1 + \alpha_2 + \cdots + \alpha_k - 1)\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\Big]\\
&= \Big(\sum_{i=1}^{k}\alpha_i\Big)!\Big[\Big(\sum_{i=1}^{k}\alpha_i\Big) - 1\Big]\prod_{i=1}^{k}\binom{n_i}{\alpha_i},
\end{aligned}$$

and the proof is complete. □

The following two results are immediate consequences of the above observations.

Theorem 6. *The lists* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ *are the losing score lists of a complete* $[\alpha_i]_1^k$*-MMHT if and only if for each* p_i, $1 \leq i \leq k$

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} r_{ij_i} = \left[\left(\sum_{i=1}^{k}\alpha_i\right) - 1\right]!\left[p_i\binom{n_i-1}{\alpha_i-1}\prod_{t=1,t\neq i}^{k}\binom{n_t}{\alpha_t}\right].$$

Theorem 7. *The lists* $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ *are the score lists of a complete* $[\alpha_i]_1^k$*-MMHT if and only if for each* p_i, $1 \leq i \leq k$

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} s_{ij_i} = \left[\left(\sum_{i=1}^{k}\alpha_i\right) - 1\right]!\left[\left(\sum_{i=1}^{k}\alpha_i\right) - 1\right]\left[p_i\binom{n_i-1}{\alpha_i-1}\prod_{t=1,t\neq i}^{k}\binom{n_t}{\alpha_t}\right].$$

3 Scores in $(\alpha_i)_1^k$-multipartite hypertournaments

Let n_i and α_i $(i = 1, 2, \cdots, k)$ be non-negative integers with $n_i \geq \alpha_i \geq 1$ for each i. An $(\alpha_1, \alpha_2, \cdots, \alpha_k)$-$k$-partite hypertournament (or $(\alpha_i)_1^n$-multipartite hypertournament or $(\alpha_i)_1^n$-MHT) of order $\sum_{i=1}^k n_i$ consists of k vertex sets U_i with $|U_i| = n_i$ for each $i(1 \leq i \leq k)$ together with an arc set E, a set of $\sum_{i=1}^k \alpha_i$ tuples of vertices, with β_i vertices from U_i, for every $\beta_i = 1, 2, \cdots, \alpha_i$ and for every $i = 1, 2, \cdots, n$, called arcs such that any $\sum_{i=1}^k \beta_i$ subset $\cup_{i=1}^k U_i'$ of $\cup_{i=1}^k U_i$, E contains exactly one of the $(\sum_{i=1}^k \beta_i)!$ $\sum_{i=1}^k \beta_i$-tuples whose β_i entries belong to U_i'. We denote an $(\sum_{i=1}^k \beta_i)$-arc by e.

If H is an $(\alpha_i)_1^n$-MHT, for a given vertex $u_{ij_i} \in U_i$ for each $i(1 \leq i \leq k, 1 \leq j \leq \alpha_i)$, the score $d_H^+(u_{ij_i})$ or simply $d^+(u_{ij_i})$ respectively, is the number of $(\sum_{i=1}^k \beta_i)$-arcs containing u_{ij_i} as not the last element (as the last element). The score lists (losing score lists) of $(\alpha_i)_1^k$-MHT are k non-decreasing sequences of non-negative integers $Q_i = [q_{ij_i}]_{j_i}^{n_i}$ $(T_i = [t_{ij_i}]_{j_i}^{n_i})$ where q_{ij_i} (t_{ij_i}) is the score (losing score) of some vertex $u_{ij_i} \in U_i$, for each i $(1 \leq i \leq k)$.

Evidently, in $(\alpha_i)_1^k$-MHT H there are exactly

$$\sum_{t_1=1}^{\alpha_1}\sum_{t_2=1}^{\alpha_2}\cdots\sum_{t_k=1}^{\alpha_k}\binom{n_1}{t_1}\binom{n_2}{t_2}\cdots\binom{n_k}{t_k}$$

arcs, and in each arc only one vertex is at the last entry. Therefore,

$$\sum_{t_1=1}^{\alpha_1} d_H^-(u_{t_1}) + \sum_{t_2=1}^{\alpha_2} d_H^-(u_{t_2}) + \cdots + \sum_{t_k=1}^{\alpha_k} d_H^-(u_{i_k})$$
$$= \sum_{t_1=1}^{\alpha_1}\sum_{t_2=1}^{\alpha_2}\cdots\sum_{t_k=1}^{\alpha_k}\binom{n_1}{t_1}\binom{n_2}{t_2}\cdots\binom{n_k}{t_k}.$$

Now, we have the following observation about the score lists of $(\alpha_i)_1^k$-MHT of order $\sum_{i=1}^k n_i$.

Theorem 8. *If H is $(\alpha_i)_1^k$-MHT of order $\sum_{i=1}^k n_i$ with score lists $Q_i = [q_{ij_i}]_{j_i}^{n_i}$ for each i, $i \le i \le k$, then*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} q_{ij_i} = \Big[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\Big(\Big(\sum_{i=1}^{k}\beta_i\Big)-1\Big)\Big(\prod_{i=1}^{k}\binom{n_i}{\beta_i}\Big)\Big].$$

Proof. Let H be an $(\alpha_i)_1^k$-MHT of order $\sum_{i=1}^k n_i$ with score lists $Q_i = [q_{ij_i}]_{j_i}^{n_i}$ with $1 \le i \le k, 1 \le j \le \alpha_i$. Therefore, clearly

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} q_{ij_i} = \sum_{A}\Big(\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} s_{ij_i}\Big),$$

where $\sum_A$ is the sum of scores of $[\beta_i]_1^k$-MHT for all $\beta_i = 1, 2, \cdots, \alpha_i$ and for all $i = 1, 2, \cdots, k$.

Therefore,

$$\begin{aligned}\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} q_{ij_i} &= \sum_{A}\Big(\Big(\sum_{i=1}^{k}\beta_i\Big)-1\Big)\Big(\prod_{i=1}^{k}\binom{n_i}{\beta_i}\Big)\\ &= \sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\Big(\Big(\sum_{i=1}^{k}\beta_i\Big)-1\Big)\Big(\prod_{i=1}^{k}\binom{n_i}{\beta_i}\Big),\end{aligned}$$

completing the proof. □

The following result provides a characterization of losing score lists in $(\alpha_i)_1^k$-multipartite hypertournaments.

Theorem 9. *Given non-negative integers n_i and α_i with $n_i \ge \alpha_i \ge 1$, for each i $(1 \le i \le k)$, the k non-decreasing lists $T_i = [t_{ij_i}]_{j_i}^{n_i}$ of non-negative integers are the losing score lists of $[\alpha_i]_1^k$- MHT H if and only if for each $p_i \le n_i$ $(1 \le i \le k)$ with $p_i \le n_i$,*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} t_{ij_i} \ge \sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\Big(\prod_{i=1}^{k}\binom{p_i}{\beta_i}\Big),$$

with equality when $p_i = n_i$.

Proof. Obviously for $[\alpha_i]_1^k$- MHT H, we have

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} t_{ij_i} = \sum_{A}\Big(\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} r_{ij_i}\Big),$$

where $\sum_A$ is the sum of losing scores of all $[\beta_i]_1^k$-MHT for all $\beta_i = 1, 2, \cdots, \alpha_i$ and for all $i = 1, 2, \cdots, k$. By Theorem 3, T_i are score lists of $(\alpha_i)_1^k$-MHT if and only if

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} t_{ij_i} \ge \sum_{A}\Big(\prod_{i=1}^{k}\binom{p_i}{\beta_i}\Big)$$

that is if and only if

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} t_{ij_i} \geq \sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\left(\prod_{i=1}^{k}\binom{p_i}{\beta_i}\right),$$

and the proof is complete. □

By using the same argument as in Theorem 9, we have the following result on scores in $(\alpha_i)_1^k$-MHT.

Theorem 10. *Given non-negative integers n_i and α_i with $n_i \geq \alpha_i \geq 1$, $i = 1, 2, \cdots, k$,, the k non-decreasing lists $Q_i = [q_{ij_i}]_{j_i}^{n_i}$ of non-negative integers are the score lists of some $(\alpha_i)_1^k$-MHT if and only if for each $p_i \leq n_i (1 \leq i \leq k)$ with $p_i \leq n_i$*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} q_{ij_i} \geq \sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\left[\left(\sum_{i=1}^{k}\frac{\beta_i p_i}{n_i}\right)\left(\prod_{i=1}^{k}\binom{p_i}{\beta_i}\right) + \prod_{i=1}^{k}\left(\frac{n_i-p_i}{\beta_i}\right) - \prod_{i=1}^{k}\binom{p_i}{\beta_i}\right]$$

with equality when $p_i = n_i$.

If in $(\alpha_i)_1^k$-multipartite hypertournament H, the arc set E contains at most(at least one) $(\sum_{i=1}^{k}\alpha_i)!\sum_{i=1}^{k}\alpha_i$-tuples, then H is said to be $[\alpha_i]_1^k$-multipartite multi hypertournament (or briefly $(\alpha_i)_1^k$-MMHT). In case E contains exactly $(\sum_{i=1}^{k}\alpha_i)!\sum_{i=1}^{k}\alpha_i$-tuples, then H is said to be complete $(\alpha_i)_1^k$-MMHT. The score and losing score of a vertex in an $(\alpha_i)_1^k$-MMHT is defined in the same way as in $(\alpha_i)_1^k$-MHT. The scores (losing scores) arranged in k lists in non-decreasing order are then the score (losing score) lists of $(\alpha_i)_1^k$-MMHT and are denoted by $Q_i = [q_{ij_i}]_{j_i=1}^{n_i}$ and $T_i = [t_{ij_i}]_{j_i=1}^{n_i}$. Evidently there are exactly

$$\left(\sum_{i=1}^{k}\alpha_i\right)!\left[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_1}\cdots\sum_{\beta_k=1}^{\alpha_k}\prod_{i=1}^{k}\binom{n_i}{\beta_i}\right]$$

arcs in a complete $(\alpha_i)_1^k$-MMHT H, and therefore

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} d_H^-(u_{ij_i}) = \left(\sum_{i=1}^{k}\alpha_i\right)!\left[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\prod_{i=1}^{k}\binom{n_i}{\beta_i}\right].$$

So,

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} t_{ij_i} = \left[\left(\sum_{i=1}^{k}\alpha_i\right) - 1\right]\left(\sum_{i=1}^{k}\alpha_i\right)!\left[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_2}\cdots\sum_{\beta_k=1}^{\alpha_k}\prod_{i=1}^{k}\binom{n_i}{\beta_i}\right]. \tag{3.1}$$

Now, since there are

$$\left(\sum_{i=1}^{k}\alpha_i\right)!\left[\sum_{t_1=1}^{\alpha_1}\sum_{t_2=1}^{\alpha_1}\cdots\sum_{t_k=1}^{\alpha_k}\binom{n_i-1}{\alpha_i-1}\prod_{i=1,t\neq i}^{k}\binom{n_t}{\alpha_t}\right]$$

arcs containing a vertex $u_{ij_i} \in U_i$, by using equation (4), we obtain

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} q_{ij_i} = \Big(\sum_{i=1}^{k}\alpha_i\Big)!\Big[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_1}\cdots\sum_{\beta_k=1}^{\alpha_k}\Big(\Big(\sum_{i=1}^{k}\beta_i\Big)-1\Big)\prod_{i=1}^{k}\binom{n_i}{\beta_i}\Big].$$

The above observations lead to the following results, the proofs are immediate consequences.

Theorem 11. *The lists $T_i = [t_{ij_i}]_{j_i=1}^{n_i}$ are the losing score lists of a complete $(\alpha_i)_1^k$-MMHT if and only if*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} t_{ij_i} = \Big[\Big(\sum_{i=1}^{k}\alpha_i\Big)-1\Big]!\Big[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_1}\cdots\sum_{\beta_k=1}^{\alpha_k} p_i\binom{n_i-1}{\beta_i-1}\prod_{t=1,t\neq i}^{k}\binom{n_t}{\beta_t}\Big],$$

for each p_i, $1 \le i \le k$.

Theorem 12. *The lists $Q_i = [q_{ij_i}]_{j_i=1}^{n_i}$ are the score lists of a complete $(\alpha_i)_1^k$-MMHT if and only if*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} q_{ij_i} = \Big[\Big(\sum_{i=1}^{k}\alpha_i\Big)-1\Big]!\Big[\Big(\sum_{i=1}^{k}\alpha_i\Big)-1\Big]X$$
$$\Big[\sum_{\beta_1=1}^{\alpha_1}\sum_{\beta_2=1}^{\alpha_1}\cdots\sum_{\beta_k=1}^{\alpha_k} p_i\binom{n_i-1}{\beta_i-1}\prod_{t=1,t\neq i}^{k}\binom{n_t}{\beta_t}\Big],$$

for each p_i, $1 \le i \le k$.

References

[1] C. Berge, *Graphs and Hypergraphs.* North-Holland Mathematical Library 6, North-Holland Publishing Co., Amsterdam, London, 1973.

[2] Y. Koh, S. Ree, *Score sequences of hypertournament matrices.* J. Korea Soc. Math. Educ. Ser. B: Pure and Applied Math., 8, no. 2 (2001), 185 – 191.

[3] Y. Koh, S. Ree, *On k-hypertournament matrices.* Linear Algebra and its Applications, 373 (2003), 183 – 195.

[4] H.G. Landau, *On dominance relations and the structure of animal societies. The condition for a score structure.* Bull. Math. Biophys., 15 (1953), 143 – 148.

[5] S. Pirzada, T.A. Chishti, T.A. Naikoo, *Score lists in* $[h, k]$*-bipartite hypertournaments.* Disc. Math. and Appl., 19, no. 3 (2009), 321 – 328.

[6] S. Pirzada, G. Zhou, *Score lists in* (h, k)*-bipartite hypertournaments.* Applied Mathematics, J. Chinese Universities, 22, no. 4 (2007), 485 – 489.

[7] S. Pirzada, T.A. Naikoo, G. Zhou, *Score lists in tripartite hypertournaments.* Graphs and Combinatorics, 23 (2007), 445 – 454.

[8] S. Pirzada, G. Zhou, *Score sequences in oriented* k*-hypergraphs.* European J. Pure and Appl. Math., 1, no. 3 (2008), 10 – 20.

[9] S. Pirzada, G. Zhou, *On* k*-hypertournament losing scores.* Acta Univ. Sapientiae, Informatica, 2, no. 1 (2010), 5 – 9.

[10] S. Pirzada, G. Zhou, A. Ivanyi, *Score lists of multipartite hypertournaments.* Acta Univ. Sapien. Informatica, 2, no. 2 (2010), 184 – 193.

[11] C. Wang, G. Zhou, *Note on the degree sequences of k-hypertournaments.* Discrete Math., 308, no. 11 (2008), 2292 – 2296.

[12] G. Zhou, T. Yao, K. Zhang, *On score sequences of k-hypertournaments.* European J. Combin., 21, no. 8 (2000), 993 – 1000.

[13] G. Zhou, S. Pirzada, *Degree sequences in oriented* k*-hypergraphs.* J. Appl. Math. and Computing, 27 (2008), 149 – 158.

Shariefuddin Pirzada
Department of Mathematics
University of Kashmir
190006 Srinagar, India
E-mail: sdpirzada@yahoo.co.in

Received: 18.06.2010